Trisect a Line Segment (5)

Method:

1) Construct a segment AB
2) Draw an arbitrary segment AI on point A but not-coincident with AB
3) Take an arbitrary point C on segment AI
4) Construct segment AC, then segment AD on segment AI such that AC = CD
5) Construct segment BJ on point B that is parallel to segment AI
6) Construct segments BE and EF on segment BJ such that BE = EF = AC = CD
7) Connect points C and F and call it segment CF
8) Connect points D and E and call it segment DE
9) Segments CF and DE intersects segment AB at G and H respectively.

Claim: Points G and H trisects segment AB.

Proof:

Consider \( \triangle ACG \) and \( \triangle ADH \),

\[
\angle CAG = \angle DAH \quad \text{(Same angles)}
\]

\[AC = CD \quad \text{(By construction)}\]

\[
\angle ACG = \angle ADH \quad \text{(CF and DE are transversals to parallel lines AI and BJ)}
\]

So, \( \triangle ACG \) and \( \triangle ADH \) are similar triangles.

Since, \( AC = CD \), hence \( AG = GH \)

By similar argument we can show that \( \triangle BEH \) and \( \triangle BGF \) are similar triangles. Hence, \( BH = HG \) (as \( BE = EF \))

Now for, \( \triangle ACG \) and \( \triangle BEH \)

- \( \angle A = \angle B \) (AB is transversal to parallel lines AI and BJ)
- \( AC = BE \) (by construction)
- \( \angle ACG = \angle BEH \) (CF and DE are transversals to parallel lines AI and BJ)

Hence, \( \triangle ACG = \triangle BEH \) (A-A-S rule). So, \( AG = BH \)

Putting it all together, we have \( AG = GH = BH \)

Hence points G and H trisects segment AB.