The University of Georgia

Mathematics Education
EMAT 4680/6680 Mathematics with Technology
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An Exploration of the Binary System

by

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We now look what is arguably the second most prolific base system, the binary system.

Have you consider why the metric system is so convenient to use? What is it about the multiples of ten that are so convenient to use? Well, it's so convenient to use because we are in a base ten system which we are so accustomed to. Well, have you considered working in the opposite direction, ie. considering what base system would work best with a given measurement system?

Let's consider the the lowest possible base, the base 2 system, sometimes known as the binary system. Here 1 is a base 10 one, but from here it gets a little trippy. Since this is a base 2 system, directly after 1 comes 10. Ergo the phrase, " There are only 10 different types of people. Those who read that sentence in base ten, those who read it in base two, and those who know that this is a base three joke. Hopefully you just joined the latter group! Computer enthusiasts are usually quite fond of this sort of joke.

Let's consider how we might count in this system. We are accustomed to the base 10 system, which goes in the order of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. We notice that we go in order of the numbers until all of symbols are exhausted up to 9. Then we make a combined symbol of 1 and 0. This just happens a little earlier with the binary system. The first number exhausts all of the non-zero symbols right off the bat! So, we must use our combined symbol of 10 right off the bat for the second number. Then just like in the decimal system, 11 comes next. However, now we have used up all combinations of 2 digit numbers. This happens in the decimal system at number 99, which is followed by 100. So similarly, 11 is followed by 100 in the binary system, followed by 101 in both system. We see that the counting pattern continues in the binary system as it does in the decimal systems, but at a rapid pace due to the lack of numeric symbols. Counting goes on in binary as such: 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111, 10000, 10001.

The binary system can also be viewed in terms of an alternate place value system. In the decimal system, the right most number is the ones place. The place value to the left is the tens place, and to the left of that is the hundreds place. For example, in the decimal number, 357, the three represents three hundreds, the five represents five tens, and the seven represents 7 ones. If a number is n places to the left of the ones place, it represents the 10^n place. Similarly, in the binary system, the numbers from right to left represent the number of ones, two's four's eight's or sixteens. In other words, in the number 1101, there is a single one, no two's, one four, and one eight. If we were to represent each place in terms of binary exponents, they would be the 1 place, 10 place, $10^1$0 place, $10^1$1 place, and $10^1$00 place.

A great example of this is in the imperial system of measurement for volumes. Using the binary systems, A pottle is 10 quarts. A gallon is 100 quarts. A peck is 1,000 quarts. A Kenning is 10,000 quarts. A bushel is 100,000 quarts. A strike is 1,000,000 quarts. A coomb is 10,000,000 quarts. A hogshead is 100,000,000 quarts. And of course, one butt is equivelent to 1,000,000,000 quarts. So, if you were to submerge an object with a peck of volume in a full tub, you could expect 1,000 quarts to come out. Similarly, if we continue to use the binary system, then .1 quart is a pint. .01 quarts is a cup. .001 quarts is a gill. .0001 quarts is a jack. .00001 quarts is a pony. And of finally, .000001 quarts is a tablespoon. Most of these measurements are outdated, but you'll notice that you recognize several. What makes this fit so well with the binary system, is

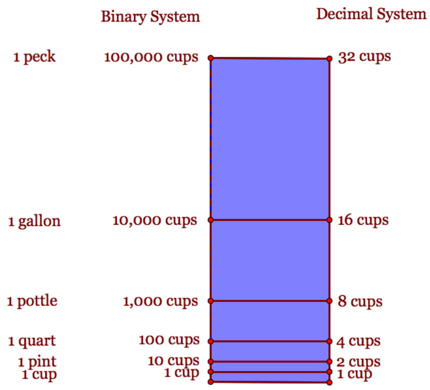

Figure 1: x=2y graphical representation.

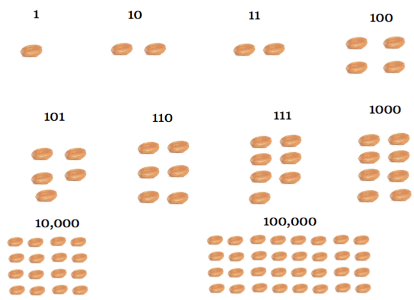

Figure 2: x=2y graphical representation.

that every measurement is a table spoon multiplied by two to some power. You might think that all these measurements are pretty small, since you start at such a small measurement, and only multiply by 2 each time. But it turns out that the sixteenth measurement, the butt, amounts to 32,768 tablespoons! That's 128 gallons! Yes, multiples of two can increase surprisingly fast!

There is a legend of the creator of chess. (A version of it can be found here: http://www.npr.org/blogs/krulwich/2012/09/15/160879929/that-old-rice-grains-on-the-chessboard-con-with-a-new-twist) It states that the king was so impressed with the creation of the game that he offered the creator, Sessa, a gift. He placed a grain of wheat on one of square of his chess board. He then placed a two grains on the next square. He placed four grains on the third, eight on the fourth, sixteen grains on the fifth square, and so on. He said to the king, "Double the grains for each consecutive square on the board. I will then take the chess board's collection of grain" The king said, "Is that all? Of course!" There are 64 squares on a chessboard, so this turns out to be one grain less than two to the power of 64 grains. Just how much is that? Well, one grain of wheat weighs approximately 64.69891 milligrams. (www.unitconversion.org/weight/grain-conversion.html) So that is $1.2 * 10^{2}1$ milligrams of grain. We use standard metric system conversion to find that this is about $1.2 * 10^{1}2$ metric tons of grain!

Today, the world generates about $6.5 * 10^{8}$ metric tons of grain per year.

That means that it would take well over a thousand years of modern wheat harvest to pay off Sessa! The king's counselor's calculated the worth of the grain, and determined that even after giving Sessa all of the possessions of the kingdom, he would still be owed money. And so, the story goes that the creator of chess became the newest king.

As referenced before, a major application of the binary number system is with computer coding, namely American Standard Code for Information Interchange (ASCII). ASCII has decimal numbers, upper and lower case letters, punctuation, and non-printing control codes. Each of these is an output for a seven digit binary number. More common than the seven digit binary code is the eight digit binary code for ASCII, increasing the length of each character by one, but doubling the number potential characters! Ever wonder what those ones and zeros refer to in computer programming? Well, this is one of the potential underlying codes that such images can refer to. The eight digit ASCII has 256 characters, due to the fact that two to the eight power is 256. Use this chart to see how you would write your name in binary ASCII. Notice that the lowercase alphabet starts at 01100001, or 97 in the decimal system. This is because control codes, numbers and capital letters precede the lower case letters in the numbering system. Once you've written out your code so that an old fashioned computer could read it, try out the online computer to check your work at http://www.branah.com/ascii-converter For more background on ASCII, see the website: http://www.tntbasic.com/learn/help/guides/asciicodesexplained.htm If you'd like a fun way to practice your binary, try out this site!

http://forums.cisco.com/CertCom/game/binary_game_page.htm