The University of Georgia

Mathematics Education
EMAT 4680/6680 Mathematics with Technology
Jim Wilson, Instructor

Exploration 10: Parametric Curves

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Graph the following:

$$x = \cos(t)$$
$$y = \sin(t)$$
$$\text{for } 0 \leq t \leq 2\pi$$

For various a and b investigate the following:

$$x = a\ \mathrm{cost}(t)$$
$$y = b \sin(t)$$
$$\text{for } 0 \leq t \leq 2\pi$$

A parametric curve in the plane is a pair of functions $x = f(t)$ and $y = g(t)$ where the two continuous functions define ordered paird (x,y). The two equations are called parametric equations of the curve and are functions of our parameter.

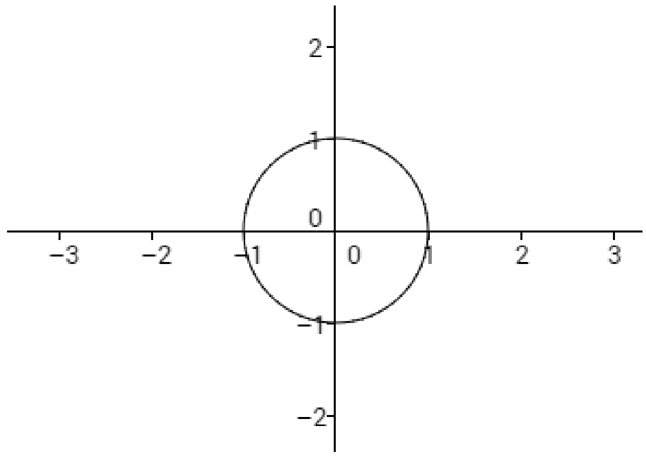
For example:

$$x = \cos(t)$$
$$y = \sin(t)$$
$$\text{for } 0 \leq t \leq 2\pi$$

are parametric equations for the unit circle, with the variable t acting as the parameter.

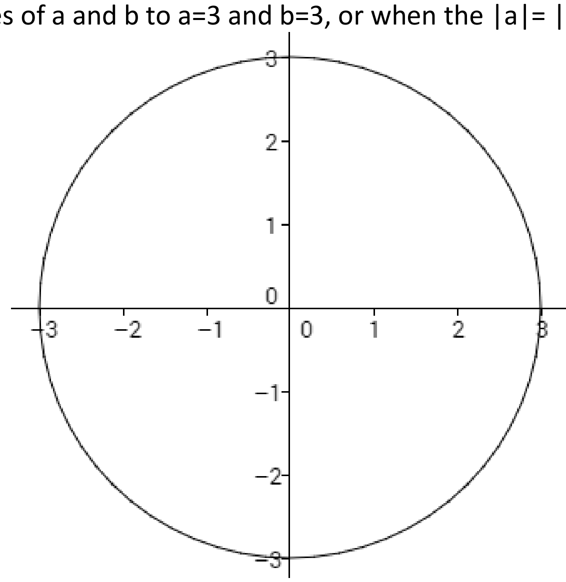
Let's examine the graph of the following when a=1 and b=1:

$$x = a\ \mathrm{cost}(t)$$
$$y = b \sin(t)$$
$$\text{for } 0 \leq t \leq 2\pi$$



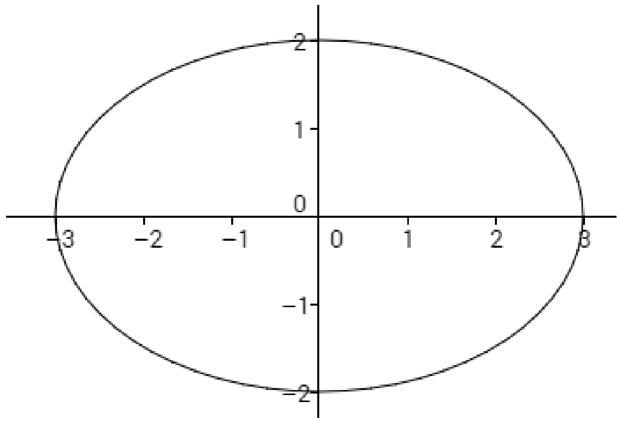
As explained before, this is these are the parametric equations for the unit circle centered at (0,0) with a radius equal to 1.

Now let us change the values of a and b to a=3 and b=3, or when the |a|= |b|.



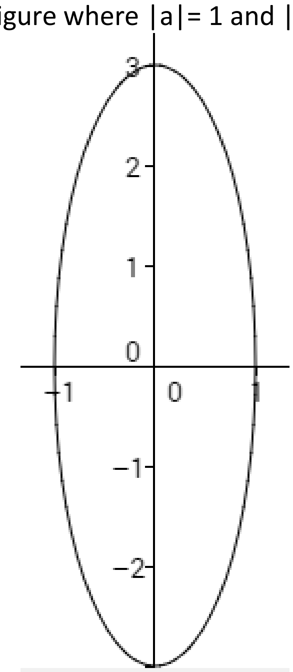
We can see from this change that the radius has now change to 3.

What happens when |a|> |b|? Let us define |a|=3 and |b|=2.



We can see from our graph above that we now have an ellipse. We can also determine that the value of |a| determines the horizontal length of our figure and the value of |b| determines the vertical length of our figure.

We can see this again in the following figure where |a|= 1 and |b|=3.



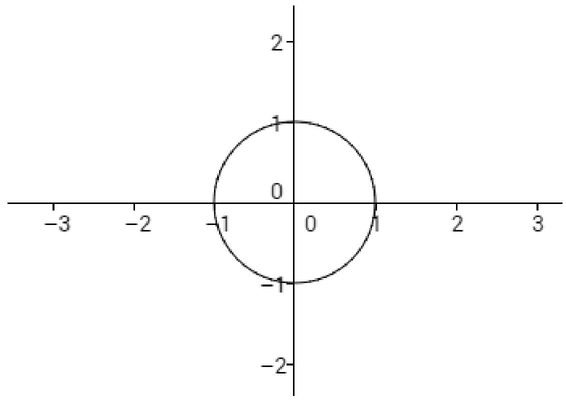
Let us now investigate the following:

$x = \cos(at)$

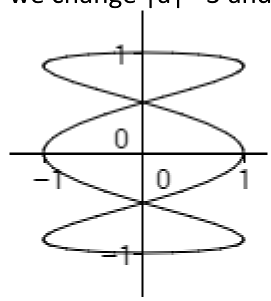
$y = \sin(bt)$

for $0 \le t \le 2\pi$

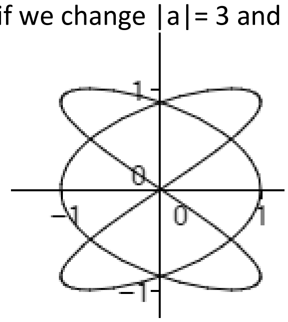
Here is the graph when a=1 and b=1.



We have our unit circle, as expected.
What if we change |a|= 3 and |b|=1?



What if we change |a|= 3 and |b|=2?



Some interesting patterns emerge here.