The University of Georgia

Mathematics Education
EMAT 4680/6680 Mathematics with Technology
Jim Wilson, Instructor

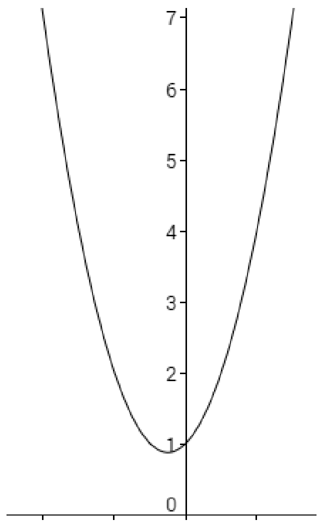
Jaime Maxey

Exploration 2

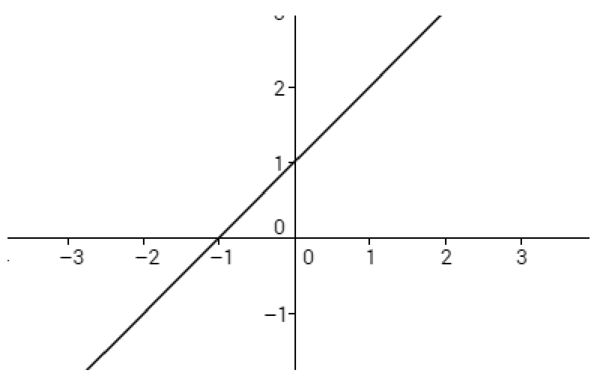
Examine the graphs for the parabola $y = ax^2 + bx + c$ for different values of a, b, and c.

Let us first examine the graph by changing the value of a while keeping b and c constant at 1.

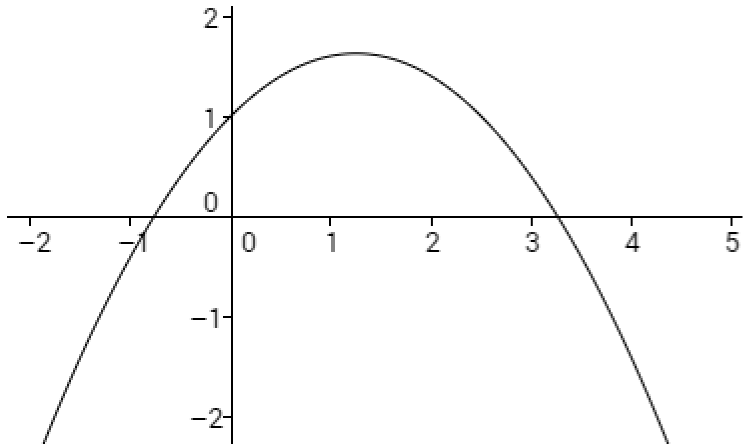
When a=2



When a =0



When a=-0.5

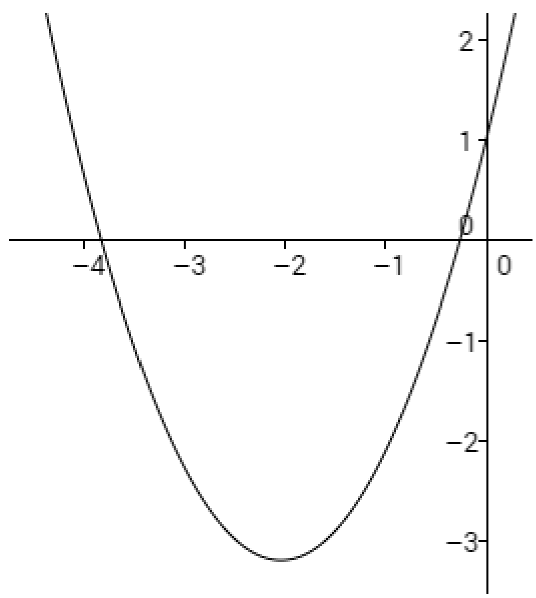


When a is negative, the graph reflects over the x axis. As a increases or decreases from zero, the parabola becomes skinnier. When a=0, the parabola becomes a straight line.

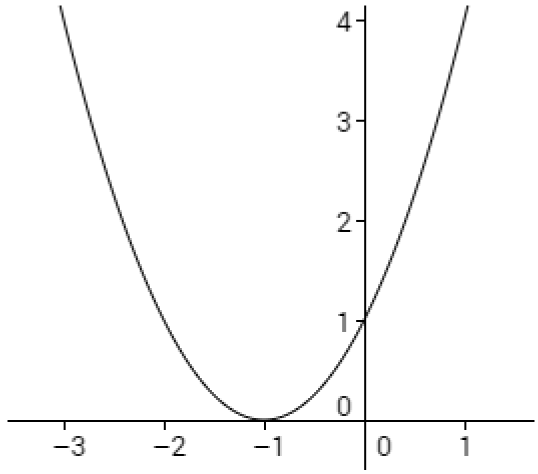
Our conclusions: If a<0, the parabola opens downward. If a > 0, the parabola opens upwards. If |a|>1, the parabola is thinner. If |a|<1, the parabola is wider. As |a| increases, the parabola shrinks.

Now let’s change the value of b and keep a and c constant at 1.

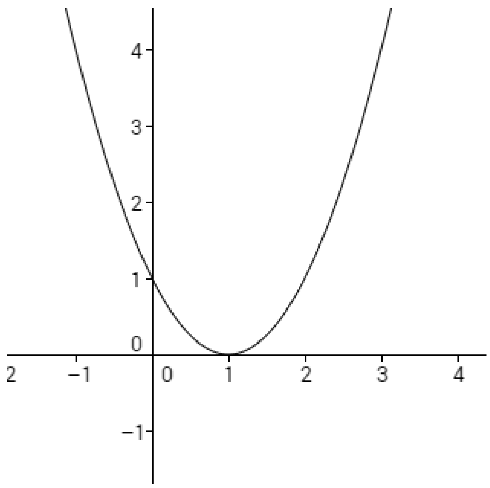
When b=4



When b=2



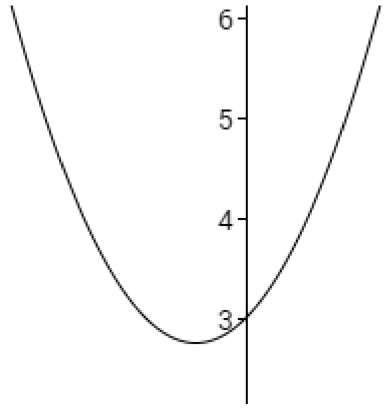
When b=-2



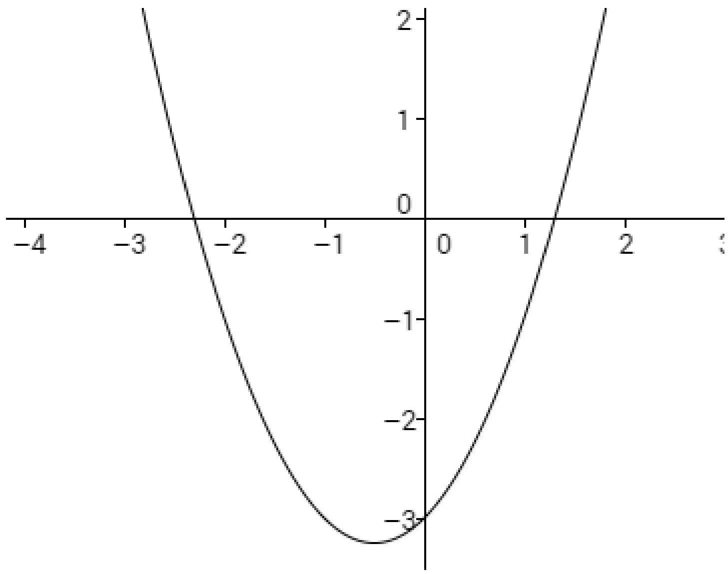
As we can see in the above diagrams, changing the value of b changes the vertex of the parabola. If b is positive, the vertex will shift towards the left and down the y axis. If b is negative, the vertex will shift towards the right and down the y-axis. The vertex will shift further away from the origin as |a| increases.

Now let us change the value of c and keep a and b constant at one.

C=3

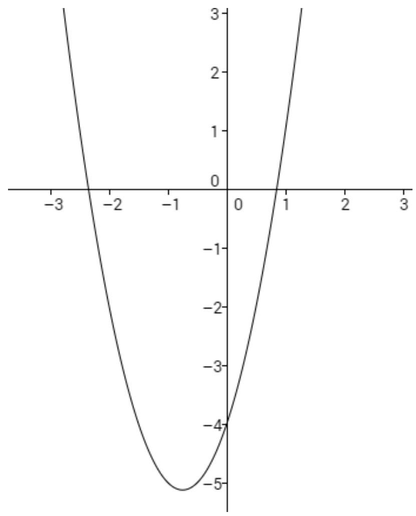


C=-3

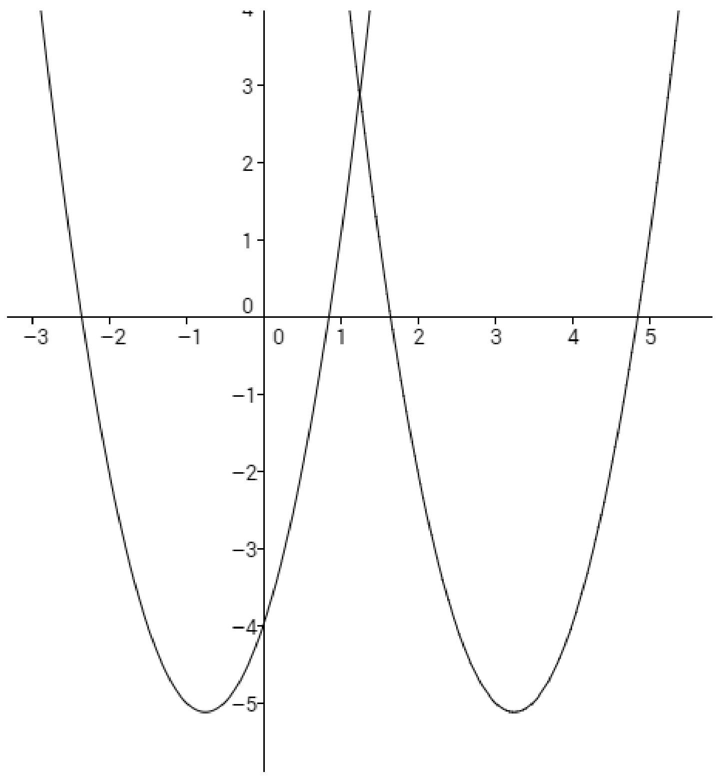


As we can see from the graphs above, altering c moves the parabola in a vertical direction. Increasing c will result in a vertical shift upwards and decreasing c will result in a vertical shift downwards.

We are now given the following parabola: $y = 2x^2 + 3x - 4$.

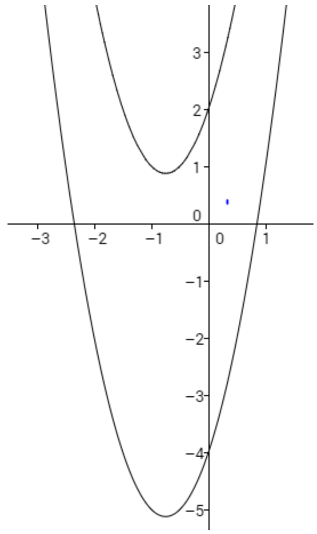


What happens when we replace each x with x-4?

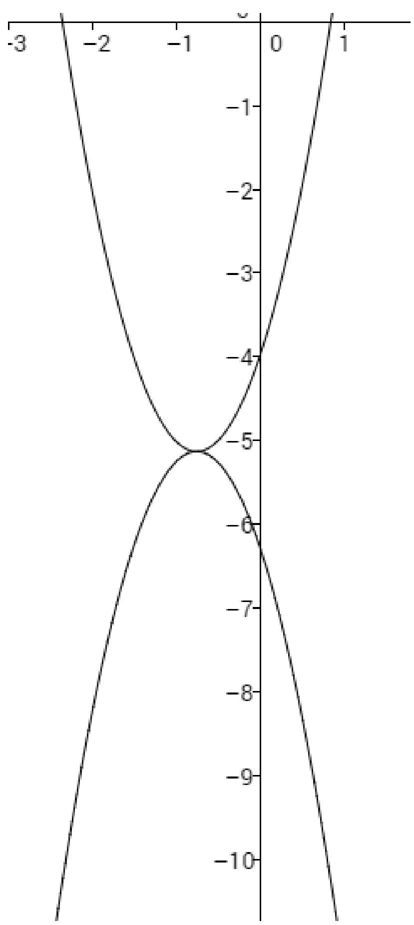


This moved our graph over 4 to the right.

Change the equation to move the vertex of the graph into the second quadrant. From our original equation I changed c=2 which resulted in a upwards shift of our graph into quadrant 2.



Change the equation to produce a graph concave down that shares the same vertex. How should we do this? Well we need to change the coefficient of our first term to be negative. Then we must shift our graph down and to the left in order for the graphs to share the same vertex.



The equation of our second graph would be $y = -2x^2 - 3x - 6.25$.

We can also write these quadratic equations in the standard form of $y=a(x-h)^2 + k$. We can re-arrange this equation to find h and k in terms of a, b, and c. Let us begin with our original equation: $y = ax^2 + bx + c$.

$$y = ax^2 + bx + c$$

Factor out an a from the first and second terms:

$$y = a(x^2 + (\frac{b}{a})x) + c$$

$$y = a(x^2 + \left(\frac{b}{a}\right)x + (\frac{b}{2a})^2 - \left(\frac{b}{2a}\right)^2) + c$$

$$y = a(x^1 + (\frac{b}{2a}))^2 - a\left(\frac{b}{2a}\right)^2 + c$$

$$y = a(x^1 + \left(\frac{b}{2a}\right))^2 - (b^2/4a) + c$$

$$y = a(x - h)^2 + k$$

So, we can see that

$$\mathrm{h} = -\frac{\mathrm{b}}{2\mathrm{a}}$$

$$k = (-b^2 + c)/4a.$$

These should be the coordinate points of our vertex.