Exploring graphs in the xb-plane

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Objective: Graphs in the xb-plane and patterns for finding roots.

Consider the equation $x^2 + bx + 1 = 0$
When we graph this relation in the xb plane we get the following graph.
We can change the constant parameter of $c$ by inputting different positive and negative values, which will vary the graph in the $xb$ plane. Below is a graph of different positive values of the parameter $c$. On this graph we can also see that when $c=0$ we have a straight line $x=-b$ and a straight line $x=0$. 
Below is a graph of positive values of c along with negative values of the parameter c. When we put all of these varying values of parameter c on one graph together we have a family of hyperbolas.
Now, we can use these graphs in order to find roots without common procedural methods. In the xb plane we can plot horizontal lines, b=c where c is some constant, and find intersection points. So, we can take any value of b, let's say b=3, and overlay this equation on the graph of \(x^2 + bx + 1 = 0\). If it intersects the curve in the xb plane, then the intersection points correspond to the roots of the original equation for that value of b. We can do this for any value of c. The number of times that the horizontal line \(b=c\) intersects the quadratic will give us the number of roots. Below we have the following graph.

From this graph, we can see that there are no real solutions whenever b is between -2 and 2, one real solution whenever b is -2 or 2, and two real solutions whenever b is greater than 2 or less than -2. We can check this pattern by analyzing the quadratic formula. When b is between -2 and 2, the discriminant value will be negative, which gives no real solutions. When b=2 or b=-2, the quadratic formula gives one root of \(-b/2\). When b is greater than 2 or less than -2, the discriminant value will be positive, which will give two real solutions.