The University of Georgia

Mathematics Education
EMAT 4680/6680 Mathematics with Technology
Jim Wilson, Instructor

Exploring Parametric Curves

By: Mallory Thomas

Objective: Review the meaning of a parametric curve. Look at an example of the sine and cosine functions and explore how changing values will affect the resulting parametric curve.

In these problems we are using parametric equations. A parametric equation is when we have introduced a parameter, t, and define the curve as follows: x=f(t) and y=g(t). A parametric curve is a collection of points of the form (x,y)=(f(t),g(t)), where t can vary.

Now we can explore the following parametric equations for different values of a and b:

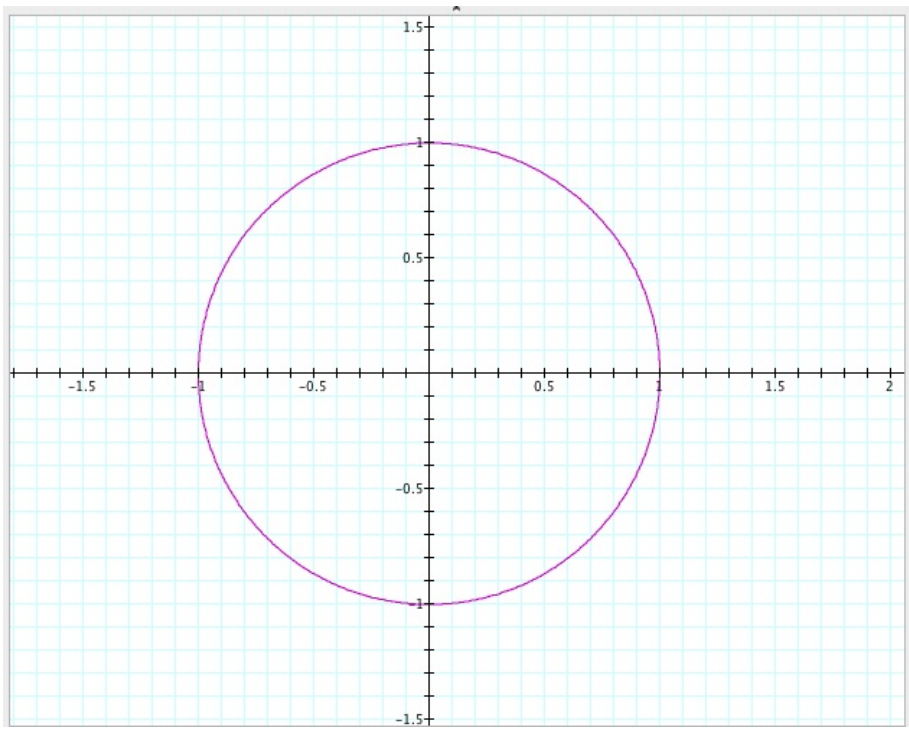
$$x = \text{a}\cos(t)$$
$$y = b\sin(t)$$
$$for\ 0 \leq t \leq 2\pi$$

We can first note that if we look at the case where a=0 and b=0, we will simply have the origin because x=0 and y=0.

Next we can look at the case where both a and b equal 1. (Below is the parametric curve.)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix}$$

From the graph we see that when a=1 and b=1 we get a circle of radius 1.

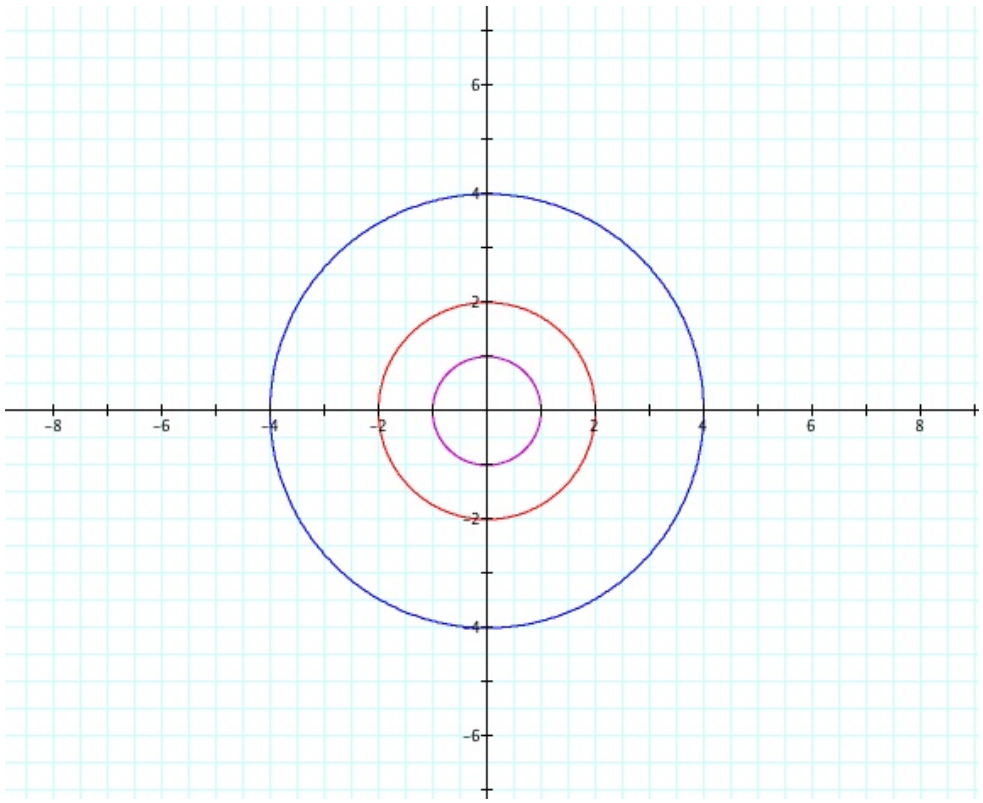


Now if we vary a and b and keep them equivalent, we get the graphs below.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\,(t) \\ \sin\,(t) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2\cos\,(t) \\ 2\sin\,(t) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4\cos\,(t) \\ 4\sin\,(t) \end{bmatrix}$$



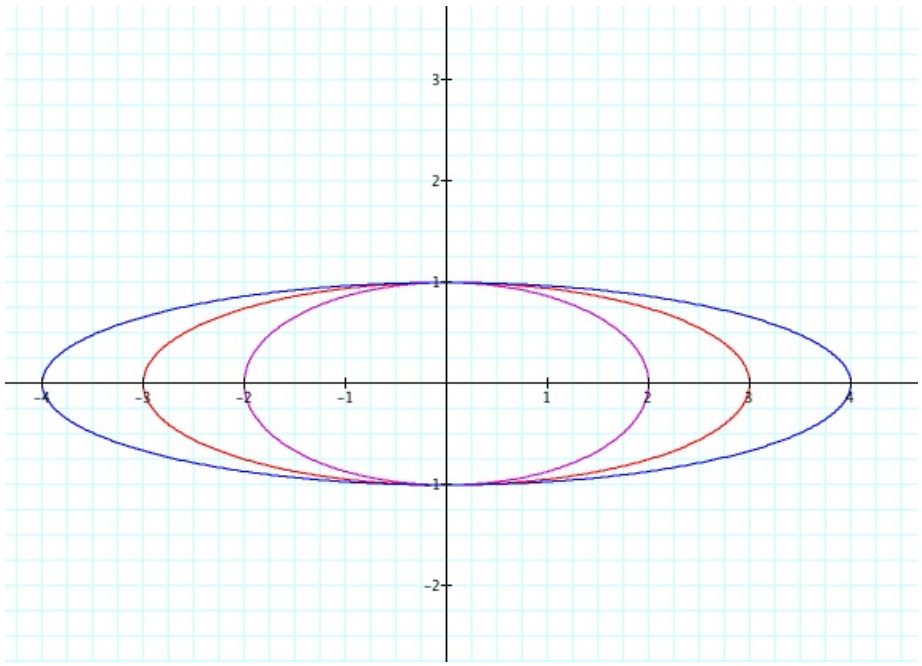
From these curves we see that if a and b are equivalent, then the value of a and b represents the radius length of the circle created by the parametric equation.

Now, what if a is greater than b? Below is a graph of several parametric equations where $b < a$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2\cos(t) \\ \sin(t) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3\cos(t) \\ \sin(t) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4\cos(t) \\ \sin(t) \end{bmatrix}$$



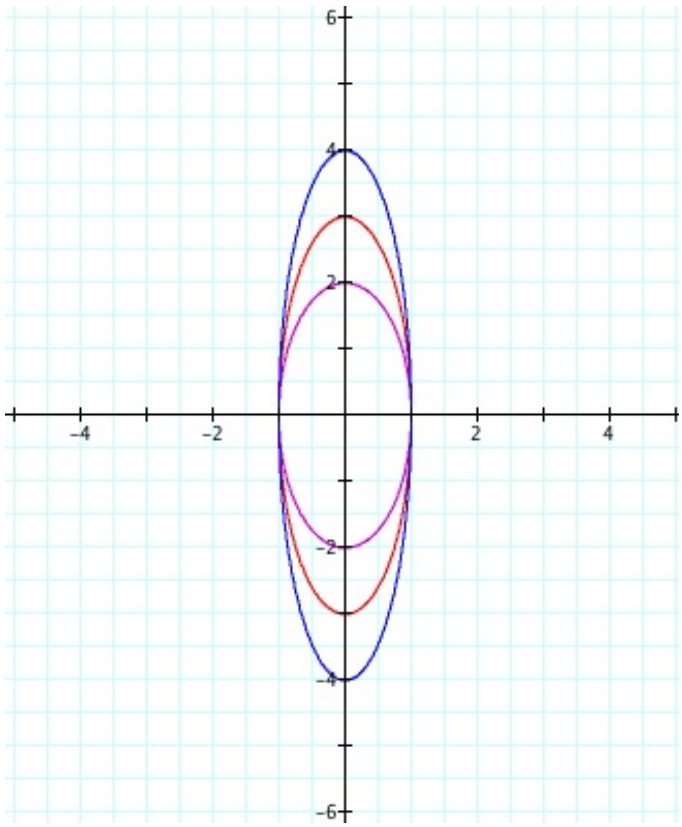
From the graph we can see that instead of a circle, we get an ellipse. Since sine is our vertical or component, the curves for every parametric equation intercept the y-axis at b and -b. Also, since cosine is our horizontal component, the curves for every parametric equation intercept the x-axis at a and –a. Therefore, we can conclude that the horizontal length of the ellipse is $2|a|$ and the vertical length of the ellipse is $2|b|$.

What is b is greater than a? Below is a graph of several parametric equations where $a < b$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(t) \\ 2\sin(t) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(t) \\ 3\sin(t) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(t) \\ 4\sin(t) \end{bmatrix}$$



Again we find an ellipse rather than a circle. We can also make the same conclusion as above: the horizontal length of the ellipse is $2|a|$ and the vertical length of the ellipse is $2|b|$. We can now conclude that when $b < a$ we have a horizontal ellipse, when $a < b$ we have a vertical ellipse, and when $a = b$ we have a circle with a radius equivalent to both a and b.