Constructivism: Pedagogical Models, Conceptual Understanding, & Student Motivation

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Simon (1995) describes constructivism as,

The philosophical position that we as human beings have no access to an objective reality, that is, a reality independent of our way of knowing it. Rather, we construct our knowledge of our world from our perceptions and experiences, which are themselves mediated through our previous knowledge. (p. 115)

A mathematics course influenced by constructivism has students build on previously constructed knowledge. (Ward, 2001) Additionally, the course would be “student-centered and emphasize active learning, communication, reasoning, and the development of deep conceptual understanding of mathematics through a problem-solving curriculum” (Alsup, 2004, p.7). All of these characteristics “are consistent with the vision presented in the National Council of Teachers of Mathematics Principals and Standards for School Mathematics (NCTM, 2000)” (Alsup, 2004, p.3). Constructivism is an underlying theory of NCTM’s Standards and it also promoted as an alternative to traditional instruction (Schifter & Simon, 1993)(Ward, 2001). Additionally, Schifter and Simon (1993) conducted a study on the effects a pedagogy influenced by constructivism can have on students’ attitudes. They found that the “changes documented by [their] attitude surveys were consistent with NCTM’s 1989 Standards “Goals for Students”:

(1) Becoming a mathematical problem solver,

(2) Learning to communicate mathematically,

(3) Learning to reason mathematically,

(4) Learning to value mathematics, and

(5) Becoming confident in one’s ability to do mathematics” (Schifter & Simon, 1993, p. 337).

These results in addition to NCTM’s acknowledgment of constructivism can be interpreted as providing validity to pedagogy influenced by constructivism.

While “constructivism provides a useful framework for thinking about mathematics learning, it does not stipulate a particular model” (Simon, 1995, p.114). Smith (1999), Simon (1995), and Heinz et al. (2004) offer three different pedagogical models influenced by constructivism. Smith’s model is more of a framework for a general pedagogical model. His framework is divided into four categories: Presentation, pupil activity, reflection, and socialization (Smith, 1999). Simon’s pedagogical model, The Mathematics Teaching Cycle, is more complex than Smith’s model. Simon’s hypothetical learning trajectory makes up a large part of his model (Simon, 1995). Lastly, the model presented by Heinz et al. builds off of Simon’s hypothetical learning trajectory and the work of Piaget (Heinz, Kinzel, Simon, & Tzur, 2004). The results of Schifter’s and Simon’s study (1993), in addition to the three pedagogical models described above, show that pedagogy influenced by constructivism can promote conceptual understanding and student motivation.

In 1985, Schifter and Simon implemented a three-project. The project “provided experienced K-12 mathematics teachers with the support to develop a constructivist view of mathematics learning and instructional practices consistent with such a view” (Schifter & Simon, 1993, p. 331). They do not go into the details of the pedagogical model that they used, but one could assume it would be similar to Simon’s Mathematics Teaching Cycle since he is a co-author. The project allowed Schifter and Simon to study “the effects of the program on:

(1) Students’ attitudes towards mathematics,

(2) Students’ beliefs about mathematics learning,

(3) Students’ performance on standardized tests, and

(4) The nature and quality of the mathematical activity in the classroom”(Schifter & Simon, 1993, p, 332).

“Data was collected through surveys [completed by the students], standardized tests, and teacher reports of student change” (Schifter & Simon, 1993, p, 332). They found a significant increase, with p < 0.001, in the attitude scores of elementary school students (Schifter & Simon, 1993). They found no change in the attitude scores of secondary students. When it came to assessing students’ beliefs about mathematics, they had students “weigh [items] in response to the question: To do well in mathematics, how important are these? (Schifter & Simon, 1993, p.334)” Several items increased in importance level, p < 0.05, for elementary school students. Several items decreased as well. For secondary students, a couple of items increased, while a few items decreased. There was an overall positive change in student beliefs. However, there was only a slight change for secondary students (Schifter & Simon, 1993). Standardized tests were administered at all grade levels and no significant differences were observed in scores (Schifter & Simon, 1993). Teachers’ reports of student change assessed effects that can be classified as cognitive change, affective change, or social change. These reports along with data gathered from the surveys indicate a positive change in the nature and quality of the mathematical activity in the classroom, but results were not entirely conclusive (Schifter & Simon, 1993)

Overall, Schifter and Simon’s research “indicated a positive change in the attitudes, beliefs, and conceptual understanding” of elementary school students (Schifter & Simon, 1993, p.332). Since, students’ motivation is reflective of their attitudes towards and beliefs of mathematics, we can conclude that the constructivist practices implemented in this study promoted student motivation. Additionally, the lack of changes in standardized test scores reassures “concerns that greater attention to understanding and problem solving, particularly considering the additional time allotted to conceptual exploration, will lead to a decline in computational skill” (Schifter & Simon, 1993, p. 337). While additional time was allotted to conceptual explorations, conclusions concerning conceptual growth were not entirely conclusive. Schifter and Simon do not have any quantitative evidence of conceptual growth since the standardized tests they administered did not “adequately measure conceptual understanding” (Schifter & Simon, 1993, p. 334). However, teacher observations indicated a positive change in conceptual growth. Although additional research is advised, these observations suggest that the constructivist practices implemented in this study promoted conceptual understanding. Now, how is constructivism implemented in the classroom?

As mentioned above, constructivism “does not tell us how to teach mathematics [or] stipulate a particular model” (Simon, 1995, p.1) In the article, *Active Learning of Mathematics,* Smith describes a framework for a general pedagogical model based on constructivism. His model focuses on the presentation, pupil activity, reflection, and socialization involved in implementing successful learning activities. When it comes to the presentation, the teacher must present the activity in a challenging way that grabs the students’ attention (Smith, 1999). Smith suggests using surprise and cognitive conflict when creating the challenge. “Cognitive conflict [occurs when] learners’ experiences of an event do not fit with their current conceptions”(Heinz et al., 2004, p.307) When this disequilibrium occurs, the learning process is triggered. (Heinz et al, 2004) The presentation of the activity should also “indicate a need for the new or revised theories” (Smith, 199, p.109). Showing students the relevance of the material they are learning can serve as a motivation factor. Pupil activity refers to the learning activity students participate in. The learning activity should be a novel task that requires high cognitive demand (Smith, 1999). When it comes to conceptual understanding and growth, Smith quotes HMI (1985) stating, “progress in pupils mathematical understanding is more important than progress in the performance of skills” (Smith, 1999, p. 109). This implies that the conceptual understanding and growth allotted to students through the teacher’s presentation and use of novel tasks, is more important than student success at performing rote manipulations. Reflection occurs after the learning activity has been completed. For example, teachers could have their students complete a writing assignment or participate in a discussion (Smith, 1999). Reflection is important because it can help “learners integrate their new or revised theories and expectations with their other mental systems” (Smith, 1999, p. 109). Additionally, the modification of students’ theories and expectations can be seen as conceptual growth. Lastly, socialization refers to the sharing of knowledge. Student discussions are an example of how this could be accomplished in the classroom (Smith, 1999). According to Smith (1999), Socialization is “a means of escaping from our own individual perception of the world, with all their circumstances and boundaries into which we would otherwise be locked” (p. 110). So, the use of socialization opens students up to conceptions other than their own, promoting conceptual growth. Additionally, during these discussions students are “negotiating shared meanings” (Smith, 1999, p.110). The students’ individual conceptions contribute to the “shared meaning.” This contribution can show students the value of their conceptions and opinions, which in turn can increase students’ motivation. Smith’s model is illustrated in the diagram below.

Simon’s pedagogical model, The Mathematics Teaching Cycle, was based off of his pre-existing notations of social constructivism and the data he gathered from a three-year teaching experiment. The question, “How can mathematics teachers foster students’ construction of powerful mathematical ideas that took the community of mathematicians thousands of years to develop?” served as part of the underlying motivation for his model (Simon, 1995, p.118). Simon presents his pedagogical model as a solution. In his experiment, he examines, “the decision making of the teacher in [relation] to posing problems” (Simon, 1995, p. 122).

A key to one of his pre-existing theories involved the creation of cognitive conflict, or disequilibrium, by teachers (Simon, 1995). Like Smith, Simon believed that the creation of cognitive conflict, or disequilibrium, not only “engage[d] the learners full attention,” (Smith, 1999, p.109), but also triggered their adaptive learning process. However, in the experiment Simon found his attempts at creating disequilibrium were not always effective. The cognitive conflict was not eliciting the conceptual understanding that he wanted. As noted by Heinz et al., “Engendering cognitive conflict is a useful teaching approach when it works [and] attempts to provoke cognitive conflict do not necessarily result in the intended learning” (Heinz et al., 2004, p.308). Simon determined that the disconnection between the posed problem and the intended learning was the result of the teacher’s misconstrued understanding of the students’ conceptions. As students engaged in activities, their initial conceptions were changing. In response, the teacher had to constantly evaluate and change how they approached students based on their new conceptions. (Simon, 1995) These conclusions led to the development of Simon’s pedagogical model, The Mathematics Teaching Cycle. The Mathematics Teaching Cycle is illustrated in the picture below. Picture retrieved from (Simon, 1995, p. 136).

Simon's Model.tiff This model illustrates the relationships between a teacher’s knowledge, the hypothetical learning trajectory, and assessment of students’ knowledge. The hypothetical learning trajectory outlines a path from a teachers learning goals, to their planned learning activities, to their hypothesis of the learning process. Their hypothesis of the learning process is “a predication of how the student’s thinking and understanding will evolve in the context of the learning activities” (Simon, 1995, p. 136). Overall, this model shows that “the teacher’s knowledge evolves simultaneously with the growth in the student’s knowledge. Conceptual growth, for both teachers and students, is a result of these learning activities. As the students are learning mathematics, the teacher is learning about mathematics, learning, teaching, and about the mathematical thinking of his students” (Simon, 1995, p. 141). The key to this model is modification. As students’ conceptual understanding is changing, the teacher’s conceptual goals for their students should change as well.

Simon also agrees on the importance of “challenging the learner’s conceptions” (Simon, 1995, p.139) Like Smith, Simon does not think conceptual growth can be achieved through routine practice and rote manipulations (Smith, 1995). He believes the conceptual difficulties students face as a result of these challenges provide excellent learning opportunities. Conceptual growth is a by-product of these unexpected learning opportunities. Additionally, Schifter and Simon (1993) found that attitudes towards and beliefs of mathematics improved when “teachers tended to give more attention to problem solving and conceptual development, de-emphasizing computation and memorization” (p. 337). Student attitudes and beliefs of mathematics have a huge impact on students’ motivation.

Heinz et al. presents a pedagogical model that builds off the work of Piaget and Simon. Their model is an “elaboration of Piaget’s reflective abstraction and Simon’s hypothetical learning trajectory” (Heinz et al., 2004, p. 306). Similar to the question that served as motivation for Simon’s model, Heinz et al. (2004) asks, “How can learners construct mathematical conceptions beyond those that are currently available to them”(p. 307)? Heinz et al. also agrees that the creation of cognitive conflict does not necessarily play a role in the solution to this question. Notice how the question that Heinz et al. asked above, sounds very similar to the learning paradox. The learning paradox describes “the need to explain how learners get form a conceptually impoverished to a conceptually richer system by anything like a process of learning” (Heinz et al., 2004, p.309). The solution to this paradox is important to mathematics educators because it “would guide [them] in designing situations to foster specific conceptual changes,” allowing them to create conceptual growth (Heinz et al., 2004, p.308).

Piaget’s reflective abstraction, “the process by which new, more advanced conceptions develop out of existing conceptions,” offers a solution to the paradox (Heinz et al., 2004, p.312). However, Piaget notes how “his research was not done with any pedagogical purpose in mind” (Heinz et al., 2004, p.312). For educators to be able to effectively use reflective abstraction to address the question, “How can learners construct mathematical conceptions beyond those that are currently available to them?” additional elaboration is needed. Heinz et al. present their pedagogical model in response to this.

The model is divided into four steps. Although all of the models are influenced by students’ conceptions, this model seems to be more student focused than the previous two.

Step1 of the model deals with specifying student’s current knowledge. The information gathered in step 1 allows the teacher to determine students’ current conceptions and be able to set effective learning goals (Heinz et al., 2004).

Step 2 of the model deals with specifying the pedagogical goal. The conception we want students to develop is our goal. However, “specifying understanding involves articulating development distinctions as opposed to mathematical distinctions” (Heinz et al., 2004, p. 322). Therefore, “the current state and goal state [of student understanding], and the differences between them” must be included (Heinz et al., 2004, p. 322).

Step 3 of the model deals with identifying an activity sequence. In this step, the teacher wants to choose an activity that will “lead to an abstracted activity-effect relationship corresponding to the pedagogical goal” (Heinz et al., 2004, p. 322). The activity-effect relationship is Heinz et al.’s elaboration of Piaget’s reflective abstraction (Heinz et al., 2004). The activity-effect relationship describes the process in which students reflect on the results of their activity by distinguishing between positive and negative attempts. The students then “abstract a relationship between their activity and its effect” (Heinz et al., 2004, p. 320). It is this “reflection on a pattern in the activity-effect relationship that leads to the [creation] of new conceptions”(Heinz et al., 2004, p. 320). As students are distinguishing between positive and negative attempts and gathering more information, they are constantly making adjustments to their goal (Heinz et al., 2004). These adjustments represent conceptual growth. Note, that the students’ goals are different than the teacher’s pedagogical goal. It is their goal in relation to completing the activity (Heinz et al., 2004). Additionally, it is these goals that serve as motivation for students (Heinz et al., 2004, p. 320).

Step 4 of the model deals with selecting a task. The teacher wants to “select a task that will result in the students setting a goal and engaging in the intended activity sequence to accomplish that goal” (Heinz et al., 2004, p. 323). “Steps 2 through 4 are an elaboration of the steps found in Simon’s hypothetical learning trajectory” (Heinz et al., 2004, p. 323). Heinz et al.’s model is illustrated in the diagram below.

Smith (1999), Simon (1995), and Heinz et al. (2004) present three different pedagogical models influenced by constructivism. The creation of cognitive conflict and its relation to conceptual growth is discussed by all three authors, but is only present in Smith’s model. Simon and Heinz et al. discount cognitive conflict due to the fact that the intended conflict may not result in the learning that was intended by the teacher (Heinz et al., 2004, p. 307). All of these authors also show how their pedagogical models promote conceptual understanding and motivation. However, it is important to remember that the majority of their observations were qualitative. Conceptual growth and motivation are hard to measure quantitatively. The surveys used by Schifter and Simon (1993) showed a significant quantitative increase in motivation (attitudes and beliefs towards mathematics. However, the data on conceptual understanding was qualitative since the standardized tests they had in place did not adequately measure conceptual understanding (Schifter and Simon, 1993). When it comes to standardized tests today, this is still the case. However, with the recent release of the Common Core, tests that do a better job at measuring conceptual understanding are apparently in the work (EMAT 7050 discussion, 2013). It is also important to note that the increases Schifter and Simon found were for elementary school students. Results indicted slight increase or none at all for secondary school students. Schifter and Simon conclude “longer studies might inform us about the appropriate time scale for interventions” (Schifter and Simon, 1993, p. 337). Despite these discrepancies, Schifter and Simon’s study “indicated positive changes in the attitudes, beliefs, and conceptual understanding” overall (Schifter & Simon, 1993, p.332). These results, in addition to the three pedagogical models described above, show that pedagogy influenced by constructivism can promote conceptual understanding and student motivation.

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