The ultimate goal of high school mathematics teachers is to create a meaningful learning environment that is conducive to teaching students the necessary concepts for academic achievement. Unfortunately, evidence suggests that many secondary educators still teach in a rote lecture style that focuses on the teacher providing information to passive, uninvolved students. Current mathematics reform movements endorse inquiry-based, “guide on the side” instruction grounded in constructivist pedagogy. The authors’ research examines the effects of constructivist teaching and learning in pre-service secondary mathematics courses. The applicability of constructivism to teach secondary mathematical concepts, using practical instructional ideas, will conclude the article.

Introduction
Typical high school mathematics’ students often express their frustration through comments such as “Why do I need to know this stuff,” or “You do not even use this math in real life.” These comments resonate in many high school mathematics classrooms today. Many high school students feel disconnected from their math instruction and perceive it as irrelevant to their lives, impacting their levels of interest and mathematics achievement. While some may blame the content of the curriculum, the instructional approach implemented in the classroom also determines the students’ motivation to learning new mathematical concepts. Teachers’ beliefs, behaviors and attitudes are invaluable variables to student learning.

Teachers’ belief systems are considered to be a factor that affects their teaching style and their use of instructional approaches. Teachers’ beliefs about mathematics, and the teaching and learning of mathematics play a significant role in shaping their instructional practice (Thompson, 1992). Current mathematics research and reform movements endorse inquiry-based, “guide on the side” instruction grounded in constructivist pedagogy (Gibson & Van Strat, 2001). Most notably, the National Council of Teachers of Mathematics (NCTM,
1989; 1991; 1995) recommends using a constructivist method of teaching, in which learners develop meaning based on experience and inquiry.

“Guide on the side”
Secondary mathematics teachers’ beliefs about learning and teaching mathematics are plotted along the continuum of instructional approaches. Direct instruction is an alternative approach to constructivism. It is agreed that, while these two pedagogies are not diametrically opposed, constructivism has been found to be essential in generating greater success in mathematics classrooms.

Direct instruction is frequently referred to as teacher-centered, as the content of the lesson is transmitted directly from the teacher to the student (Slavin, 2004). The traditional teacher in this setting could be seen as the “sage on the stage.” Techniques that are typically identified as direct instruction include lectures, textbook usage, choral responses, and the completion of worksheets.

At the other end of the continuum, constructivism, or student-centered learning, places greater responsibility of discovering and learning information on the students. The teacher, as “the guide on the side,” facilitates the students’ construction of meaning and the understanding of the content. Constructivism, as defined by Draper, “is the philosophy, or belief, that learners create their own knowledge based on interactions with their environment including their interactions with other people” (2002, p. 522). Research confirms that constructivist math instruction improves secondary math learning (Grant, 1998). Constructivist math instruction, a research-based practice, encourages “content to self” connections and enhances student learning. Therefore, students better understand the relevance of mathematical concepts and become more motivated and interested in their math courses, thereby improving math performance and meeting the standards.

Meeting the standards
Studies have found that math performance declines as high school students progress through the secondary grades and begin to lag behind their international peers. The 2003 Program for International Student Assessment (PISA) determined that 15 year-old students from the United States ranked 24th out of 29 countries in math literacy and problem-solving. From a national perspective, the 2005 Nation’s Report Card indicated that only 69% of eighth graders performed at or above the Basic level. Unfortunately, 31% of our nation’s students are not acquiring minimal math mastery. Difficulty levels and other educational factors may contribute to this problem but student disconnect and disinterest also may be contributing factors to the United States students’ low achievement in secondary mathematics. In response to the poor math performance of U.S. high school students, the National Mathematics Advisory Panel’s recent report emphasized that the “instructional practices, programs and materials that have proven effective in improving math learning,” and “research needs in support of math education” must be addressed “to strengthen math education in order to give our students the skills to succeed in the 21st century” (U. S. Department of Education, 2007, p. 2). Clearly, educators must take into account the students’ attitudes toward math and the method of their math instruction for students to be successful.

Current Classroom Practices
Research has found that teachers tend to teach in the manner that they were taught during their own educational experiences. Many teachers hold the predominant idea that the role of the teacher was to give information to students, an idea stemming from their own traditional school experiences (Civil, 1992). Our recent study of pre-service math teachers confirmed that both in-service secondary math teachers and college math professors continue to traditionally teach in a didactic, teacher-directed, “sage on the stage” approach in today’s high school and undergraduate classrooms. After surveying 49 pre-service math teachers (15 methods students and 34 student teachers) pertaining to pedagogical approaches in college mathematics courses and fieldwork observation classrooms, our students’ fieldwork reports described teacher-centered classrooms with few constructivist principles in action.
Interestingly, when considering the experience of college mathematics courses, 100% of the methods students identified lecture as an approach used to a great extent. Independent practice and textbook driven lessons were somewhat commonly used, as seen by 53% of the methods students. In a similar fashion, 94% of the student teachers believed that lecture was used to a great extent, and 62% believed that textbook driven lessons also were used to a great extent in their college mathematics courses.

During the fieldwork experiences, the techniques employed to the greatest extent fell into the direct instruction arena, lecture (80% of the methods students, 68% of the student teachers), and independent practice (67% of the methods students and 82% of the student teachers). The third direct instruction category, textbook-driven instruction, was seen with some frequency by 60% of the methods students and 53% of the student teachers.

The results of this simple survey indicate that there is a disconnect between reality and necessity. A greater emphasis on direct instruction techniques in both college mathematics courses and fieldwork observations was reported, yet the belief about the ways lesson plans should be written and the ways that students best learn express a need for constructivist approaches.

Mathematics instructional strategies that work!

Mathematics educators must be exposed to alternative forms of instruction to help secondary students achieve in mathematics and to better meet the standards. “Good teaching,” that is constructivist-centered instruction, must become a priority for educators to ensure that students obtain more authentic and critical learning experiences. Cooperative learning, hands-on activities, discovery learning, differentiated instruction, technology, distributed practice, critical thinking, and manipulatives are elements that embrace the constructivist educational philosophy. In these instances, the teacher could be described as the “guide on the side” to better educate students and meet the standards. Student-centered lesson plans are essential for anyone embracing constructivism in the mathematics classroom. In addition, secondary teachers can formulate a repertoire of instructional strategies that work in the mathematics classroom. Though the implementation may differ from year to year, based upon the ideas of the students and the mathematics topics covered, certain patterns have emerged. The following is a selection of authentic pedagogical ideas.

Teaching Idea #1: Use rulers and protractors generously in your geometry lessons!

If you want your students to know that the sum of the interior angles of a triangle equals 180, create a student-centered motivational activity that allows them to discover this. Create a handout with three different triangles, and on the handout direct them to work with a partner and use a protractor to measure the interior angles. At the bottom of the page, ask them to find the sums of the three angles in each triangle. While not all of the pairs will get exactly 180, students will see that the answers will converge around 180.

If you want your students to remember the properties of a rectangle, allow them five minutes at the beginning of class to measure the sides, angles, and diagonals of rectangles. Create a handout with a few different rectangles, and on the handout direct the students to work with a partner and use a ruler and protractor to measure the interior angles. At the bottom of the page, ask students to find the sums of the three angles in each triangle. While not all of the pairs will get exactly 180, students will see that the answers will converge around 180.

If you want your students to be convinced that the longest side of a triangle is opposite the largest angle and the shortest side is opposite the shortest angle, allow them five minutes at the beginning of class to discover the theorem. Create a different handout for each small group of three students. Each group should receive a paper containing a triangle with directions to measure the three angles and the three sides. At
the bottom of the page, ask the students to list the sides and angles in ascending order. Groups can then report their results.

What’s important to remember about these activities is that rounding errors need to be discussed, good classroom management skills are essential, and these activities can be used as a five to ten minute motivation, leaving plenty of time for guided and independent practice.

Teaching Idea #2: Utilize those graphing calculators!
Since it takes almost no time to type an equation into the graphing calculator, students should do so regularly. This allows them to see what various functions look like on a regular basis, not simply when they reach the chapter on graphing functions. With enough exposure to the visual representations, students will be able to predict whether a graph will be linear, parabolic, or cubic before even being formally taught this content.

The graphing calculator also provides a great check to otherwise mundane equations. After an equation has been solved for the variable, students can enter each side of the equation into the graphing calculator, graph both sides of the equation, and use the proper tools to search for points of intersection. This again exposes students to content before its time but allows students to see the connections between a solution and what the word “solution” means in terms of the graph.

Teaching Idea #3: Incorporate manipulatives when appropriate!
Many secondary mathematics teachers agree that there simply is not enough time to be creative every day in the mathematics classroom or that some lessons simply do not lend themselves well to hands-on activities. While this may be true, there are some activities that are worth the time and should not be passed up.

One such crowd pleaser is the discovery of $\pi$. Prior to the class, the teacher should prepare trays or boxes with rulers, string, various cylindrical objects such as household and, and a direction sheet. Direct the students to wrap the string around a can. Next, remove the string and, with a ruler, measure the amount of string that was needed. Third, students use a ruler to measure the diameter of the circles. Finally, students divide the circumference by the diameter and report their results to the class. While most students do not get 3.14, the answers tend to converge towards that number.

Another crowd favorite is the tactile representation of the Pythagorean theorem. Using pre-drawn right triangles, with each of the three sides forming its own square, students fill up the two smaller squares, the squares of the legs $a$ and $b$, with square tiles or paper strips. Next, students use those tiles to fill the square of $c$, the square formed by the hypotenuse of the right triangle. Lo and behold, just the right amount fits! There are also great animated versions of the Pythagorean theorem on-line.

Next is an additional hands-on activity that is useful for the trigonometry lesson that introduces students to the concept of the radian. After experimenting with the group set-up and the pair set-up, a majority of the students liked this activity best when it was completed in a whole-class setting. The teacher starts with a large circle that is either drawn on the board or, if using poster paper, taped to the board. The center of the circle must be visible, and it helps if a diameter is drawn. Two student volunteers use string to create a radius and cut the string so that it is the exact length of the radius. Next, starting from one endpoint of the diameter, they use the string to “trace” the circle. The goal is to see how many radii fit around a semi-circle. To show students that this number is constant, different volunteers should be used to repeat the experiment on a few different circles. Once students see that three radii fit around the semi-circle, with a little bit left over, they are joyful to learn that the left-over portion is approximately 14.

Final Thoughts
Constructivism, a research-based learning approach that has been proven effective, can help secondary mathematics teachers meet the U. S. challenge of improving student achievement. Assuming the role as “guide on the side” requires teachers to step off the stage, relinquish some of their power, and release the textbooks to allow their students to be actively engaged and take some responsibility of their own learning. Although taking such a leap may bring
some teachers out of their comfort zones, the students’ statements equating mathematics with fun may be well worth the effort.

References