The purpose of this paper is to provide teachers with best practices advice for teaching fractions, decimals, and percents to students with learning disabilities (LD). Although it would be wonderful to be able to state that within this limited teaching sphere, educators will find an extensive and comprehensive litany of empirically validated instructional practices, interpretation of this body of research suggests that the most that can be concluded are that: (a) a small number of instructional practices have been investigated, (b) very few skill-specific teaching strategies have been validated for students with LD, and (c) a small set of general teaching practices have demonstrated promising and often significant effects when applied to instructional problems. Many more strategies and practices have been validated for typical students than have been validated for students with LD. Nevertheless, these practices suggest approaches to instruction of fractions, decimals, and percents that are consonant with best practices in special education.

The specific content covered in this paper (i.e., fractions, decimals, and percents) was selected because these problems share a common mathematical function base. The unifying theme across these areas is division. As we shall discuss later, using this theme as a base for instruction across fractions, decimals, and percents in mathematics has both utility and parsimony.

The instructional practices suggested in this paper are organized into the following categories: general guidelines for instruction, problems in representation of fractional and decimal numbers, problems in comparison of fractions and decimal numbers, problems in computation with fractional and decimal numbers, and problems in renaming and reducing fractions and decimals numbers. We have also included applications of technology-based instruction where appropriate. Following a brief discussion of the underlying theoretical base, recent research, and future trends in each of these areas, specific “best practices” derived from the instructional literature are presented.

ACHIEVEMENT IN FRACTIONS, DECIMALS, AND PERCENTS

Achievement in mathematics is quite limited for many students with LD (Cawley & Miller, 1989) and even lower for students with developmental disabilities (Parmar, Cawley & Miller, 1994). While it is likely that curriculum may account for some of the achievement deficits found in this population, Cawley and Miller suggest that the deficits of some students may actually be due to factors intrinsic to the learner and are not solely caused by poor teaching or curricula.

Fractions are a consistent and recurring area of concern for classroom teachers of students with LD. The areas of skill deficits most consistently reported by middle school and high school teachers of students with LD are related to fractions, decimals, and percents (McLeod & Armstrong, 1982). These deficits included both terminology related to fractions and operations with fractions. Studies of the performance of students with LD on secondary competency tests also found significant skill deficits in fractions, decimals, and percents (Algozzine, O'Shea, Crews, & Stoddard, 1987).
It is not surprising that many students with LD experience difficulty in reasoning and computing with fractions, decimals, and percents. The development of these abilities in typical students appears to be quite slow and complex (Tourniaire & Pulos, 1985). In fact, college students in teacher education programs demonstrated considerable difficulty both in explaining the meaning and in generating appropriate representations of fractional problems (Ball, 1990). Often students will be able to employ correct strategies for one problem but fall back on less appropriate strategies when faced with more difficult problems. As students acquire experience in solving these types of problems, their strategies become increasingly correct and applicable to more complex problem types. However, Tourniaire and Pulos (1985) concluded that students demonstrate great variability in their response to instruction in fractions. Given the low achievement status of students with LD, even when compared to other low achieving peers (Kavale, Fuchs, & Scruggs, 1994), it is likely that many instructional programs carried out in general education lack the clarity and intensity required to promote acceptable outcomes for students with LD.

REMEDIAL PROGRAMS

Tourniaire and Pulos (1985) noted that while studies of individual interventions indicate that reasoning related to fractions, decimals, and percents can be taught, “the methods are probably too costly to be of practical importance” (p. 198). While this is quite likely to be true in regard to general education settings, the cornerstone of special education is the provision of intensive, often individually administered treatments to students. Evidence from the perspective of general education suggests that treatments with sufficient intensity may not be practical in that setting (Tourniaire & Pulos, 1985) and that attempts to provide special education services for students with LD in general education settings (i.e., full inclusion) have yielded less than acceptable outcomes for students with LD (Zigmond & Baker, 1994; Zigmond et al., 1995). The logical conclusion from this research is that instruction in fractions, decimals, and percents for students with LD requires specific instructional grouping as an important component of effective programming.

Options such as peer tutoring and cooperative learning may be helpful in some cases (e.g., for review and practice activities); however, the error filled and tentative nature of competence demonstrated by typical students in this area of achievement (Lankford Jr., 1974; Tourniaire & Pulos, 1985) suggests that carefully designed intensive instruction delivered, at least initially, by competent teachers will be most profitable for students with LD. Additionally, students with mild disabilities require more intensive instruction to promote mathematical competence than is available given the spiral nature of many general education curricula (Parmar et al., 1994). While such instruction is time-consuming and expensive, it is nevertheless important because the proportional reasoning required for mastery of fractions, decimals, and percents is “both the capstone of middle school mathematics and the cornerstone of all that is to follow” (Heller, Post, Behr & Lesh, 1990; p. 400). In the next section, we discuss the curricular and instructional issues related to fractions, decimals, and percents.

FOUNDATION FOR INSTRUCTION: THE “BIG IDEA”

Because school curricula are increasing in size and complexity and students with LD experience difficulty mastering and integrating the curriculum, teachers must identify the
most important concepts and skills for instruction. One condition of many of the most important instructional concepts is the presence of a “Big Idea” (Carnine, 1994). Big Ideas in an instructional program are the important concepts that provide the anchors and connections for the to-be-learned information (Baker, Simmons & Kameenui, 1994). According to Carnine, teaching the “Big Idea” that underlies numerous, but related, instructional objectives, should replace teaching for coverage of a broad curriculum area (where instructional objectives are frequently taught as a series of unrelated skills) with teaching a few inter-connected “Big Ideas” which will be taught thoroughly.

Probably the most important concept related to and underlying fractions, decimals and percents is ratio or, in simpler terms, division (Carnine, Jones & Dixon, 1994) or partitioning (Mack, 1990). The term proportion (Tourniaire, 1986) also describes the conceptual domain containing fractions, decimals, and percents. Interestingly, the most frequently reported area of skill deficit for students with LD found by McLeod and Armstrong (1982) was division of whole numbers followed closely by mathematical skills related to fractions and decimals. This finding supports the proposed relationship between the development of skills in division and those in fractions, decimals, and percents.

By structuring instruction regarding fractions around the “Big Idea” of division, teachers can provide clearer explanations of fractions (e.g., by demonstrating that 3/4 is a fraction that represents 3 parts of a whole which was originally divided into 4 equal parts). By stressing the underlying unifying concept of division, students learn that there are explicit links between fractions and decimals and percents, and that mathematical curriculum objectives that are typically taught as discrete, unconnected skills, can, and should, be connected to mathematical skills already mastered. Not only does this particular “Big Idea” make conceptual sense, but practical aspects of instruction (e.g., manipulation and computation of such problems with calculators) are enhanced. Finally, mixed and decimal numbers may be introduced as the outcomes of division problems that have fractional and decimal remainders respectively.

TEACHING FRACTIONS

This section contains both general and specific instructional strategies for teaching fractions. Following the general teaching guidelines, most of which are applicable to many areas of mathematics instruction, teaching techniques are described that enhance mastery of fraction skills related to representation, comparison, computation, and renaming.

General Guidelines

As with other areas of mathematics instruction, students “need to be explicitly taught that (a) it is smart to ask questions when they do not understand; (b) errors are a natural part of learning; and (c) mathematical knowledge gleaned from daily living experiences is relevant to understanding the formal mathematics taught in school” (Scheid, 1994, p. 10). Teachers of students with LD are advised to incorporate the following general instructional techniques into their programs of instruction regarding fractions, decimals, and percents.

* ensure that students have mastered the prerequisite skills for the tasks to be learned
as part of your advance organizer presentation, demonstrate the “Big Idea” and interconnected relationships between division, fractions, decimals, and percents (e.g., physically divide a whole into five equal pieces)

introduce skill instruction with succinct and unambiguous demonstration of the task to be learned (e.g., solve several problems while the student observes)

introduce instruction using concrete materials (i.e., manipulatives) before proceeding to semi-concrete materials (e.g., pictorial representations) before proceeding to abstract problems (e.g., numerical representation)

ensure that your teaching examples include sufficient practice opportunities to produce task mastery

ensure that your teaching examples include variations of all problem types to avoid students making incorrect generalizations (e.g., that all fractions represent parts of a whole)

provide systematic instruction on discriminating among different problem types that is designed to enable students to know which solution to employ (e.g., teach students to attend to the operation sign, perhaps by circling it, so that they will select the correct computational algorithm)

provide guided practice (following teacher-led demonstrations) before assigning independent work (i.e., have teacher and student work several problems together)

assign homework and independent practice problems only after the teacher has ensured (by observing or testing after demonstration and guided practice) that students have achieved a significant degree of correct performance (e.g., over 70% correct) in order to avoid students practicing error patterns in their responses

use the clinical ‘math interview’ technique (suggested by Bryant in this issue) to identify and remediate error patterns (i.e., have the student work missed problems aloud which permits the teacher to detect accurately defective algorithms)

link instruction directly to life skills and question the utility of teaching tasks that have little value for either future learning or outside-the-classroom performance

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Teach and test for transfer of classroom learning to life skills

Representation

Many students bring a great deal of informal understanding of fractions to their instruction in mathematics; however, it is often difficult for students to integrate formal instruction with their informal knowledge (Mack, 1990). Among the problems that Mack noted were a tendency to consider fractions as whole numbers rather than proportions or rational numbers, and the inability to solve problems expressed symbolically even when the students were able to solve similar problems expressed in the context of real-world situations. Additional problems in representation of fractional numbers include lack of understanding that fractions can represent a part of a set as well as a part of a whole unit, and that fractions represent a certain number of equal sized parts (Baroody & Hume, 1991). Teachers should also bear in mind that representation of fractions can be a very abstract and difficult task for students that is sometimes beyond the ability of even their teachers (Ball, 1990).

Fair-sharing activities. In general, when students demonstrate difficulty in acquisition of relevant concepts and understanding in mathematics instruction related to fractions, teachers are advised to create representations that are more concrete and meaningful (Mastropieri & Scruggs, 1994). One concrete and meaningful way of representing fractions with both parts of whole and of sets is through fair-sharing
activities (Baroody & Hume, 1991). In fair-sharing activities, students must distribute commodities equally among a group of students. Given eight students and a pie, each student would receive one eighth of the pie. Given eight students and 24 erasers, each student would receive three erasers. In fair-sharing activities, teachers should include both whole units (e.g., pies, pizzas) and sets of units (e.g., erasers, paper clips, pencils, raisins) to be divided among the students. Care should be taken to reinforce that the basic activity is one of dividing or partitioning the original amount into equal subgroups (Mack, 1990).

Fair-sharing activities represent a highly informal and intuitive basis for understanding fractions. According to Mack (1990) many students who are able to deal with fractions at this level are unable to deal successfully with fractions presented symbolically. Introduction of the formal symbolism of fractions can begin when students have a clear understanding of fair-sharing activities. Teachers can represent fair-sharing activities numerically, emphasizing the relationship between the numerals and the real life situation. For example, teachers can ask students to divide a sandwich equally among three students. Each student would get one of three equal parts. When students understand this basic concept, the notation of 1/3 may be introduced as a commonly accepted way of writing this relationship (Baroody & Hume, 1991). Again, both whole units and sets of units should be used to teach students fractional notation to avoid the common but mistaken belief that fractions are always parts of a whole.

Concrete objects. Among the materials that can be used to make representation of fractions more concrete are real objects such as erasers, paper clips, pencils, and pies. However, instruction should encourage students to move from the concrete level of representation to more abstract representations. Among the materials which can be used to encourage more abstract representation of fractions are tangrams, Cuisenaire rods, pattern blocks, fraction strips, and number sticks (Baroody, 1993; Baroody & Hume, 1991).

Cuisenaire rods are bars that use length and color to represent numbers. Teachers with limited budgets may wish to create “fraction strips” (Van de Walle & Thompson, 1984) which are posterboard alternatives to the commercially-produced Cuisenaire rods. Because these objects have consistent proportional relationships and do not carry specific numeric values, they are well-suited to promote general logical thinking and observation of proportional relationships. For example, students can be instructed to select a relatively long rod or strip. This unit is then designated as the “whole.” [Van de Walle and Thompson (1984) suggested that the word “whole” be used rather than “one” because one is also used in fraction values such as one-fourth, resulting in confusion for some students.] Students are then requested to identify strips that are one-half, one-third, and so on of the whole. To promote understanding that fractions are equal proportions of a whole rather than a cardinal value, the unit representing the whole can be changed, resulting in different lengths for the resulting unit fractions (1/2, 1/4, etc.). Teachers can refer to the rods by color names to describe the relationships between the strips. Mixed numbers and improper fractions can be introduced by having students group objects beyond the whole unit. For example, “If the dark green strip is the whole, the red strip is one-third. It takes five units to make 5/3.” Teachers can create student problem sheets by replacing the underlined words in the example above with blanks which the students complete on their own.
Comparison

Comparison of fractions is sometimes difficult for students who regard fractions as discrete whole numbers rather than as proportions (Mack, 1990). Baroody and Hume (1991) suggested that students often compare whole numbers by using a strategy which indicates that the number which comes later in a counting series is the larger. When applying this strategy to fractions such as 1/3 and 1/4, students might compare the denominators and erroneously conclude that the fourth is larger than the third because four comes after three in the counting series. Students committing this type of error are probably applying knowledge of whole numbers to fractions. By relating the formal symbols to realistic situations and manipulative representations of fractional amounts, students may be less likely to consider the fourth as larger than the third, because they can see that 1/4 of the pie (or 1/4 of the set of erasers) is smaller than 1/3 of the pie.

Another source of difficulty in comparison of fractions relates to the size of the original unit. While one-half is a larger proportion than one-third, one-third of 90 pounds is a larger absolute quantity than one-half of 50 pounds. Students should always be encouraged to name the whole unit when comparing fractions to gain a sense of both the absolute and relative size of the units (Baroody & Hume, 1991). Teachers must be prepared to incorporate both sufficient numbers of instructional examples to guarantee mastery and sufficient variety of examples to ensure understanding of these complex problems.

Concrete objects. A number of activities with Cuisenaire rods and fraction strips can provide a concrete foundation for comparison activities. For example, students can compare the relative sizes of unit fractions given different wholes. In this activity, students can be asked, “If the dark green strip is the whole, which strip is one-half, one-fourth, etc. If the brown strip is the whole?” and so on. From these activities it may be demonstrated that, as the size of the whole changes, the size of the fractions change proportionally. Also, that one-half of a dark green strip is physically smaller than one-half of an orange strip.

Computation

Effective teaching of fraction computational skills can include teacher use of most if not all of the general guidelines presented earlier. Several of these practices were incorporated into a direct instruction program by Perkins and Cullinan (1985) who found that student errors decreased and task mastery increased directly as a function of an instructional program that included proven mastery of prerequisite skills, daily probes, extensive and periodic review, guided practice, verbal prompts, and corrective feedback. Perhaps the most important recommendations for teaching fraction computation are: (a) Ensure that numerical computation (e.g., addition of fractions) is always preceded by student understanding of the meaning of the arithmetic operation, and (b) Ensure that students can describe a representation of the computational problem (i.e., a real life problem) before they are required to master the mechanics of computation. There is no doubt that students can master calculations without being able to provide an example from their experience (e.g., college students could divide fractions correctly, but could not describe a real life problem that conformed to a problem similar to 2 3/4 x 4 2/3 [Ball, 1990]). (c) Provide adequate guided practice to ensure that students do not invent error patterns to reach solutions, and (d) provide sufficient practice opportunities to ensure mastery and fluency.
Renaming and Reducing

Fractions and decimals represent proportional relations that can be expressed in a variety of terms. Mastropieri and Scruggs (1994) suggested that reducing fractions after calculations presents a difficulty for students because it is difficult to specify procedures that can be used in every instance. However, there are some general questions that can be asked in regard to every fraction which can assist students in reducing tasks. Table 1 presents these questions.

TEACHING DECIMALS AND PERCENTS

The decimal number system provides a more powerful method of representing quantity than the systems of whole number and common fractions (Wearne & Hiebert, 1988). Competence with decimal numbers is necessary for calculations involving money as well as using calculators. Many students find learning decimal numbers to be an easier task than mastering fractions (Bley & Thornton, 1995). However, typical students often have difficulty linking conceptual understanding of decimals to the rules for manipulating symbols and solving decimal problems (Heibert & Wearne, 1986). Teachers working with students with LD should, therefore, be prepared to provide high-quality and explicit instruction in the decimal system. Many of the general guidelines for instruction of fractions also apply to teaching decimal numbers. The next sections discuss areas that are specific to the decimal system.

Representation of Decimal Numbers

It is important to anchor students’ understanding of decimal numbers in some concrete activity. Without explicit connections to link concrete and familiar conceptual bases, students often perceive the decimal system to be a new symbol system representing new concepts rather than an extension of an already partially mastered system of numeration (Heibert & Wearne, 1986). Decimal numbers are proportions incremented in units of ten. Certain manipulatives such as Deines blocks or base ten blocks lend themselves quite easily to representation of decimal numbers. These blocks are similar to the more familiar Cuisenaire rods but are segmented to show individual units and are incremented by multiples of ten. Therefore, ten singles make a row, ten rows make a flat, and ten flats make a cube (see Figure 1). Using tens blocks, teachers can create activities similar to those described for fraction strips and Cuisenaire rods, (e.g., “If the flat is the whole, what is the value of the single?”). Practice with concrete materials, such as tens blocks should begin well before introduction of symbolic notation to establish sound meanings for the symbols used to represent decimal numbers (Wearne & Hiebert, 1988).

Comparison

Many students have difficulty comparing the magnitude of different decimal numbers when they have different numbers of digits to the right of the decimal point. Hiebert and Wearne (1986) suggested that this was a result of ignoring the decimal point and treating the digits as if they were whole numbers. Children making such errors would judge 1.29 to be larger than 1.4. Another error identified by Hiebert and Wearne in comparison of decimal numbers was over generalization of the role of zero in numeral configuration. In
whole numbers, adding a zero to the right of a numeral increases its value tenfold but
adding a zero to the right of a decimal number has no effect on the number. Teaching
children to estimate values and round numbers can help to establish a sense of relative
values for decimal numbers. In the example given, the child would be taught to round
1.29 and say “1.29 is about 1.3. 1.3 is less than 1.4.” This is a fairly sophisticated skill;
children require frequent practice to acquire and maintain the ability to compare decimal
numbers. Estimation activities also can serve as a foundation for checking the
reasonableness of calculations.

Computation

Early activities involving computations with decimal numbers should be represented with
base tens blocks, or, for students capable of greater abstraction, graph paper may be used
to represent the calculations. Children can sometimes verbalize the rule for lining up
decimal points without understanding that this procedure enables addition and subtraction
of like units. Sufficient representation and comparison activities should be undertaken to
support this conceptual understanding before computation instruction begins. These
activities also lead to representation of calculation using area as a representational
metaphor. Students can represent multiplication and division problems involving
decimals on 10 by 10 graph units. Using a graph to show the action of a multiplication
problem enables students to see concretely that multiplying .7 by .3 yields a product of 21
of 100 squares as in [Figure 2].

Finally, calculators, because they operate in the decimal system, are a natural tool
for computations involving decimals. Students should receive explicit instruction in the
use of calculators for complex computations and proportions. Students must have a firm
understanding of the type of problem they are executing as well as the magnitude of the
numbers involved. Estimation and verification activities should therefore accompany all
calculation activities. Because calculators possess a great deal of power, it is important
that students learn to gauge the reasonableness of the calculator’s output. Without this
metacognitive awareness, calculators produce little more than meaningless rote activities.

SUMMARY

The purpose of this paper was to describe best practice activities in the mathematical
areas of fractions, decimals, and percents. Unfortunately, teachers will not find a
comprehensive set of empirically validated instructional practices for students with LD
which is sufficiently developed to guide all aspects of instructional decision-making.
However, there is a body of research sufficient to provide teachers with sound advice
regarding general approaches for these areas of instruction.

Conceptually, fractions, decimals, and percents should be grounded in the “Big
Idea” of division which will enable students to understand the interconnectedness of ideas
across problems, learning activities, and teachers. Following a brief overview of recent
research, we presented both general and specific strategies designed to enhance student
performance in fractions, decimals, and percents. We also recommended that for students
with LD, instruction may need to be structured around two time-honored special
education traditions, individualization (where instructed groups are determined by
readiness to learn as demonstrated by mastery of prerequisite skills), and intensity (where
extended and sufficient instruction, guided practice, and review are provided to ensure
task mastery). Fractions, decimals, and percents are important elements of mathematical
education, representing both the capstone of elementary mathematics and the gateway to higher mathematical learning. Fractions, decimals, and percents also represent areas of particular difficulty for many students with LD. Nevertheless, when given appropriate support and intensity of instruction at the proper levels of concreteness, students with LD may attain more acceptable treatment outcomes than have been evident in the past.

REFERENCES


**UPDATED REFERENCES**


## Table 1. General Questions for Reducing Fractions

1. Can the denominator be divided by the numerator? If so, divide the numerator and denominator by the numerator.
   \[ \frac{4}{8} = \frac{1}{2} \]

2. Do the numerator and denominator both end in 0? If so, divide by 10.
   \[ \frac{20}{30} = \frac{2}{3} \]

3. Do the numerator and denominator end in 0 or 5? If so, divide by 5.
   \[ \frac{15}{20} = \frac{3}{4} \]

4. Can the numerator and denominator be divided by 3? If so, divide by 3.
   \[ \frac{3}{9} = \frac{1}{3} \]

Adapted from Mastropieri and Scruggs (1994). Adapted with permission.
Figure 2. Using Graph Paper to Represent Multiplication with Decimals*

* Adapted from Bley & Thornton (1995)