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# "She's Always Been the Smart One. I've Always Been the Dumb One": Identities in the Mathematics Classroom

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The moment-to-moment dynamics of student discourse plays a large role in students' enacted mathematics identities. Discourse analysis was used to describe meaningful discursive patterns in the interactions of 2 students in a 7th-grade, technology-based, curricular unit (SimCalc MathWorlds®) and to show how mathematics identities are enacted at the microlevel. Frameworks were theoretically and empirically connected to identity to characterize the participants' relative positioning and the structural patterns in their discourse (e.g., who talks, who initiates sequences, whose ideas are taken up and publicly recognized). Data indicated that students' peer-to-peer discourse patterns explained the enactment of differing mathematics identities within the same local context. Thus, the ways people talk and interact are powerful influences on who they are, and can become, with respect to mathematics.

*Key words*: Affect; Classroom interaction; Discourse analysis; Social factors

*Interviewer*: What do you think a good math student looks like?

*Bonnie*: Not like me [she laughs]. . . . I'm nice to the teacher and stuff, I'm just not, I'm, I'm just not that good in math.

*Interviewer*: How do you think Teri [Bonnie's partner in mathematics class] views you as a math student?

*Bonnie*: I don't think she views me very well as a math student. . . . I view her as a really good math student . . . 'cuz she's just always been smart like that. . . . And that's how we've always been viewed. She's always been the smart one. I've always been the dumb one. (Student interview, May 2006)

Who we believe ourselves to be is a powerful influence on how we interact, engage, behave, and learn (Markus & Wurf, 1987; McCarthey & Moje, 2002; Wenger, 1998). Identities are important because they affect whether and how we engage in activities, both mathematical and otherwise, and also because they play a fundamental role in enhancing (or detracting from) our attitudes, dispositions,

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emotional development, and general sense of self. One goal of mathematics education is to help students develop positive dispositions toward mathematics—to become persistent, agentic, and confident (National Research Council, 2001). These traits are the cornerstone of powerful and productive mathematics identities that help learners handle frustration and struggles not only in mathematics but also in all areas of learning and, for that matter, life. For this reason and despite the difficulty of studying them, identities are an important area of research and deserve increased attention in our mathematics classrooms. My focus in this study was to identify and describe students' mathematics identities as they were enacted in daily classroom routines within a small-group, middle school setting. In the following sections, I frame this study by articulating a rationale for research on mathematics identities and exploring how other researchers have defined and studied this construct.

## WHY IDENTITY?

### *Identity as an Important Educational Outcome*

In the seminal book *Adding It Up*, the National Research Council recognized the affective component of learning mathematics, saying, "Students' disposition toward mathematics is a major factor in determining their educational success" (2001, p. 131). The authors defined *mathematical proficiency* as composed of five interrelated strands, one of which is related to the affective component of learning mathematics. They called this strand *productive disposition*, which, as they defined it, is broader than one's mathematics identity inasmuch as it encompasses beliefs about mathematics as a discipline and beliefs regarding how to be a successful learner of mathematics (in addition to beliefs about oneself as a learner and doer of mathematics). If students are to develop the other four strands of mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning), "they must believe that mathematics is understandable, not arbitrary; that, with diligent effort, it can be learned and used; and that they are capable of figuring it out" (p.131). Similarly, the National Council of Teachers of Mathematics described a major goal of school mathematics programs as creating autonomous learners who are "confident in their ability to tackle difficult problems, eager to figure things out on their own, flexible in exploring mathematical ideas and trying alternative solution paths, and willing to persevere" (NCTM, 2000, p. 21). One underlying goal of mathematics education, according to both documents, then, is to produce students who are willing to engage in challenging mathematics and see the value in doing so; in other words, to develop students with positive mathematics identities.

### *Identity and Persistence*

Additionally, mathematics identities and related affective constructs including attitudes, confidence, and beliefs (Fennema & Sherman, 1976) have been linked to students' persistence or willingness to continue studying mathematics (Armstrong

& Price, 1982; Boaler, 2002a; Boaler & Greeno, 2000; Hembree, 1990; Sherman & Fennema, 1977). The 1996 administration of the National Assessment of Educational Progress (NAEP) showed that as students progressed through school, more and more would choose to opt out of further mathematics study if given the choice (12% in 4th grade to 31% in 12th grade). These same students were also asked whether they liked mathematics. Although 69% of 4th graders responded affirmatively, only 50% of 12th graders did (Mitchell et al. 1999; see Wilkins & Ma, 2003 for similar findings). What NAEP results only alluded to, Armstrong and Price more clearly stated in their study of high school seniors: "Students who liked mathematics and believed they were good in it tended to take more high school mathematics than students with less positive attitudes" (1982, p. 102).

*Identity and Learning*

Although establishing an empirical relationship between identity and learning has been challenging, studies have linked the more general concept of *affect* to learning. These studies are largely correlational, using surveys and focusing on the construct of attitude. Ma and Kishor (1997), in a meta-analysis of 113 studies of attitude and achievement in mathematics, found a positive, statistically significant mean effect size of 0.12, indicating a positive relationship between attitude and mathematics achievement (see also Hembree, 1990, and Ma, 1999, for related findings regarding the negative relationship between mathematics anxiety and achievement). In Schoenfeld's (1989) study of high school students, he identified a strong correlation between one's grades in school mathematics and factors related to identity—expected mathematical performance and perceived ability. Similarly, Meece and colleagues (1990) used structural equation modeling to identify strong, positive links between a student's expectations for his or her mathematics performance and grades (a direct link) and between perceived ability and grades (an indirect link mediated by expectations).

From a theoretical standpoint, however, a stronger basis for linking identity and learning exists. Whether describing Discourses (Gee, 1990, 2001, 2005), figured worlds (Holland, Lachicotte, Skinner, & Cain, 1998), or communities of practice (Lave & Wenger, 1991; Wenger, 1998), many researchers posit situated views of learning in which they emphasize the importance of identity formation in learning (see also Sfard, 2008, and Sfard & Prusak, 2005). In these views, learning goes beyond constructing new and flexible understanding and entails becoming a different person with respect to the norms, practices, and modes of interaction determined by one's learning environment. What we learn in school is much more than the disciplines of mathematics, reading, history, and science. We learn who we are. And these identities, in turn, affect not only how we learn or fail to learn the subject matter at hand but also who we become—what we pursue, what makes us happy, and what we find meaningful. Markus and Nurius (1986) described this in terms of *possible selves*—the hoped for, potential, and (sometimes) feared people we can become. In the case of mathematics, new opportunities for classroom activity and participation can redefine what is meant by learning and doing mathematics. “Different pedagogies are not just vehicles for more or less knowledge, they shape the nature of the knowledge produced and define the identities students develop as mathematics learners through the practices in which they engage” (Boaler, 2002a, p. 132). And for some students, previously unimagined possible selves are now within the realm of possibility.

In summary, identity is important because it is an integral part of mathematical proficiency and because of its relationship to persistence in the field. Moreover, identity is a powerful and often overlooked factor in mathematics learning. Despite these reasons, research with a specific focus on identity within mathematics education has been relatively limited until recently (recent examples include Berry, 2008; Boaler & Greeno, 2000; Cobb, Gresalfi, & Hodge, 2009; Horn, 2008; Jackson, 2009; Martin, 2000, 2006; Nasir, 2002; Sfard, 2008; Sfard & Prusak, 2005; Solomon, 2009; Spencer, 2009; Walshaw, 2005, n.d.). One difficulty stems from the challenges of operationalizing identity in a tractable, observable, and measurable way. Sfard and Prusak (2005) observed that identity is often treated as a self-evident, experiential idea, which makes avoiding the difficult task of defining and operationalizing this construct more acceptable. However, interest in the study of identity within educational research has grown (e.g., Bloome, Power Carter, Christian, Otto, & Shuart-Faris, 2005; Gee, 2001; Lave & Wenger, 1991; McCarthey & Moje, 2002; Wenger, 1998; Wortham, 2004), increasing the need for scholars to explicitly articulate their methods and theoretical frameworks with as much transparence as possible. In the following section, I draw from literature both within and outside the field of mathematics education to synthesize key theoretical components of identity, situate my definition of identity within the larger body of research on mathematics identities, and highlight various approaches that researchers have taken to study this construct. I conclude the section by describing my methodological approach in this study to investigating students’ mathematics identities.

## A CLOSER LOOK AT RESEARCH ON IDENTITY

### *Identities as Multiple, Learned, and Negotiated in Community*

Prior researchers have categorized identities in a variety of ways. Identities can be conscious or subconscious; specific (I’m good at division) or general (I’m good at school); independent (self-defining traits, such as, I am hard-working) or interdependent (categorizations referencing others, such as, I am a member of the basketball team) (see Bloome et al., 2005; Fryberg & Markus, 2003). Identities can be based on self-perception and reflection or on what is learned about oneself through others (Davies & Harré, 2001; Markus & Wurf, 1987; McCarthey & Moje, 2002; Sfard & Prusak, 2005). Identities can be institutional (I am [diagnosed as] learning disabled) or based on one’s affiliation with a group (I am a University of Texas football fan) (Bloome et al., 2005; Gee, 2001). Identities can reference fixed characteristics (gender, race, SES) or arise on the basis of social relationships; they can also look toward the past, present, or future (Bloome et al., 2005; Markus & Nurius, 1986; Sfard & Prusak, 2005; Wenger, 1998).

Note that many of these characteristics are simultaneously present in a single enacted identity. Identity is no longer viewed as an attribute of a monolithic, static, true self, but is seen as a collection of self-representations, a "continually active, shifting array of . . . self-knowledge" (Markus & Wurf, 1987, p. 325) specific to given contexts and based on prevailing circumstances. I use the plural, *identities*, to indicate this multiplicity, flexibility, and fluidity. People can be both compassionate and indifferent, prideful and humble, or motivated and lazy depending on their contexts, the people with whom they interact, and past patterns of behavior (McCarthey & Moje, 2002).

Not only are identities invoked in specific contexts, they are dynamically negotiated and reinforced through social interaction within these contexts (Bloome et al., 2005; Davies & Harré, 2001; Johnston, 2004; McCarthey & Moje, 2002; Sfard & Prusak, 2005). Identity has as much to do with others as it does with self. All of us are aware that we can act in certain ways and take on certain roles in one group and behave, at times, quite differently in another. A large part of who we are is learned from how others interact and engage with us. This influence is especially strong for adolescents who take their cues for who they are becoming in large part on the basis of how others treat them (Erikson, 1968, p. 128).

Developing identities, whether imposed by others via social positioning or actively invoked, and whether accepted or rejected, are negotiated in the moment and depend on wider contexts of past history and life experiences (Gee, 2001; Wenger, 1998). Even comfortable, routinized, and seemingly innate identities were at one time malleable and co-constructed in social contexts. However, over time, repeated patterns of behavior solidify, resulting in relatively stable enactments of self that constrain how one interacts in given situations. Some researchers have described this solidification as the *thickening* of identity (Holland & Lave, 2001; Wortham, 2004), in which thickening is the process by which a person and a certain role or identity become increasingly associated over time.

### *Defining Identity*

Using these ideas as a basis, I define *identity* as a dynamic view of self, negotiated in a specific social context and informed by past history, events, personal narratives, experiences, routines, and ways of participating. An identity is who one is in a given community and, as such, is both individually and collectively defined. Although an identity is related to a role or ways of participating (e.g., the role of a *problem poser* in a mathematics classroom), I define *identity* in a broader sense. An identity also encompasses ways of being and talking; narratives; and affective components such as feelings, attitudes, and beliefs (Bruner, 1994; Gee, 2005; Wenger, 1998)—aspects not necessarily included in the term *role*. In Table 1, I present other researchers' definitions of identity, both within and outside of mathematics education, and identify key characteristics that are embedded in these definitions.

Similarities across the definitions include the ideas that identities are situated (i.e., context-specific) and related to activity within those contexts. Whereas some researchers focus on broader factors specific to groups of individuals including race, class, gender, school-level characteristics, and classroom/pedagogical characteristics, others focus on factors specific to individuals themselves such as beliefs about self/mathematics, goals, personal narratives, and moment-to-moment discourse. Also, some investigate and attempt to establish links between multiple levels of characteristics with differing grain sizes and how those factors interact to influence identity. Many of the researchers included in Table 1 might frame their work differently and assert that they draw from a broader list of characteristics than what I have indicated; however, for each definition, I identified a characteristic only if it was a particular focus (in contrast to a peripheral focus) of a researcher's conception of identity in the given reference.

Drawing heavily from Sfard's discursive approach and Martin's 2006 definition, I use the term *mathematics identity* to mean the ideas, often tacit, one has about who he or she is with respect to the subject of mathematics and its corresponding activities. Note that this definition includes a person's ways of talking, acting, and being and the ways in which others position one with respect to mathematics. Moreover, a mathematics identity is dependent on what it means do mathematics in a given community, classroom, or small group. As such, identity is situated; learned; stable and predictable, yet malleable; and is both individual and collective. To further develop the construct of identity, I now consider various approaches these researchers and others have taken to study and operationalize this construct.

### *Studying and Operationalizing Identity—A Synthesis of Previous Research*

Across a variety of fields, researchers have suggested that identity is critical to issues of learning and teaching (Boaler & Greeno, 2000; Gee, 1990; Holland et al., 1998; Markus & Wurf, 1987; McCarthey & Moje, 2002; Wenger, 1998); however, theorists have been less clear about the mechanisms influencing enacted identities. Researchers from some traditions (e.g., anthropology and critical theory) have emphasized the interplay between the local, personal construction of identities and broad social contexts that reach far beyond the walls of mathematics classrooms. These more global contexts include membership in certain groups and communities, cultural structures, institutional discourses, and historical events and struggles spanning long periods of time (Erickson, 2004; Gee, 1990; Holland & Lave, 2001; Horn, 2008; Martin, 2000). In particular, scholars within mathematics education whose work focuses on students of color or other underrepresented populations explore the enactment of identity with respect to tracking, racism and race, culture, gender, and class (Berry, 2008; Martin, 2000, 2006; Solomon, 2009; Spencer, 2009; Walshaw, 2005, n.d.). Although I acknowledge the importance of these broad social, political, institutional, and historical contexts, in this study and its corresponding analyses, I purposefully restrict my focus to smaller, more localized contexts to better

Table 1
*Characteristics of Identities—A Synthesis of Definitions*

| Author | Definition of *identity* and/or *mathematics identity* | Characteristics across definitions | | | | | |
|---|---|---|---|---|---|---|---|
| | | Activity/ practice-based | Narrative/ discourse emphasis | Positional/ context-specific | Multileveled factors w/ differing grain sizes | Beliefs/ disposition component | Emphasis on broad social, historical factors including race and class |
| Wenger (1998) | Wenger uses the constructs of engagement (participation within community), imagination (view of self within community), and alignment (degree to which communal activities are aligned with broader goals) to describe the process of identification. These three modes of belonging constitute one's identity. | X | | X | X | | |
| Boaler & Greeno (2000) | Learning is a process of identity formation wherein learners locate themselves within or outside of a particular community. Identities consist of relationships with the discipline of mathematics that emerge through classroom practices (see also Boaler, 2002a, 2002b). | X | | X | | | |
| Martin (2000, 2006) | An identity is composed of one's beliefs about mathematics ability, the importance of mathematics, constraints and opportunities afforded in local contexts, and motivations to obtain mathematical knowledge. Martin emphasizes sociohistorical, community, school, and intrapersonal forces (2000). More recently, Martin defines identities as dispositions and deeply held beliefs about one's ability to perform effectively in mathematical contexts and to use mathematics to change one's life. Identities are self-understandings built within context of doing mathematics (2006, p. 206). | X | | X | X | X | X |

Table 1 (*continued*)
*Characteristics of Identities—A Synthesis of Definitions*

| | | Characteristics across definitions | | | | | |
|---|---|---|---|---|---|---|---|
| Author | Definition of *identity* and/or *mathematics identity* | Activity/ practice-based | Narrative/ discourse emphasis | Positional/ context-specific | Multileveled factors w/ differing grain sizes | Beliefs/ disposition component | Emphasis on broad social, historical factors including race and class |
| Gee (2001) | An identity is the "kind of person" one is in a given context. Gee identifies four interrelated perspectives on identity (nature-based, institution-based, discourse-based, and affinity-based identities). | X | X | X | X | | |
| Sfard & Prusak (2005) | An identity is a collection of stories or narratives that are reifying, endorsable, and significant (p. 16). | | X | X | X | | |
| Solomon (2009) | An identity includes beliefs about one's self as a learner; beliefs about the nature of mathematics; engagement in mathematics (a la Wenger); and perception of self as a potential creator of, or participant in, mathematics (p. 27). | X | X | X | X | X | X |
| Bishop (see p. 38) | An identity is the set of beliefs that one has about who one is with respect to mathematics and its corresponding activities. An identity is dependent on what it means to do mathematics in a given context; as such, it is individually and collectively defined. Identities include ways of talking/acting/being as well as how others position one with respect to mathematics. | X | X | X | | X | |

understand the enactment[1] of differing identities among people sharing common contexts—namely, those in a single mathematics classroom.

Although not comprehensive, Table 2 lists factors related to school mathematics settings that researchers have suggested are related to identity. Moving down the first column, the factors decrease in grain size, shifting from macrolevel to microlevel factors. Within a given classroom, teachers and students select topics of study, plan curricular activities, and engage in academic tasks—all of which are macrolevel classroom factors influencing students' mathematics identities. Microlevel classroom factors include moment-to-moment discourse moves, self-narratives, and personal goals, which can vary widely across students in the same classroom.

*Identity and its relationship to curriculum, tasks, classroom norms, and teacher pedagogy.* Evidence for the effects of curricula, tasks, classroom norms, and teacher pedagogy on identity formation comes from studies across a number of disciplines.

Table 2
*School-based, Interpersonal, and Intrapersonal Factors Related to Identity Enactment*

| Factor | Supporting research |
|---|---|
| Tasks and Curricula | Boaler, 2002a; Empson, 2003[a]; Horn, 2008; Jackson, 2009; McCarthey, 2001; McCarthey & Moje, 2002; Nasir, 2002 |
| Classroom Norms, Participant Structures, Teacher Pedagogy, and/or Teacher Beliefs | Berry, 2008; Boaler & Greeno, 2000; Cobb et al., 2009; Empson, 2003[a]; Gee, 1990; Holland et al., 1998; Holland & Lave, 2001; Jackson, 2009; Lave & Wenger, 1991; Solomon, 2009; Spencer, 2009 |
| Discourse—Positioning, Narratives, and Footing | Bloome et al., 2005; Davies & Harré, 2001; Empson, 2003[a]; Gee, 1990, 2001, 2005; Goffman, 1981[a]; Solomon, 2009; Sfard, 2001[a], 2008; Sfard & Prusak, 2005; Walshaw, 2005, n.d.; Wortham, 2004 |
| Goals and Possible Selves | Cobb et al., 2009; Markus & Nurius, 1986; Markus & Wurf, 1987; Nasir, 2002; Polman, 2006; Wenger, 1998 |

[a]Indicates a study that does not posit a direct link to identity.

[1]I use the term *enactment* to indicate the fluid nature of identity (Gee, 2001; Sfard & Prusak, 2005; Wenger, 1998). An enacted identity, although existing in the present moment, is not temporally independent; it looks backward in that it reflects aspects of previously enacted identities and it looks forward in that it anticipates and influences future enactments. An enacted identity is a snapshot in time (to borrow a metaphor from Sfard & Prusak), in which the present moment is reflective of both the past and future. Thus, I focus on current enactments rather than questions of identity formation. Investigation of students' mathematics and life histories necessary to answer those questions is beyond the scope of this study.

Boaler, in studies of high school mathematics classes, found that differences in expectations, activities, and students' ways of participating (passive consumers of information or active participants in the negotiation of meaning) affected students' mathematics identities (Boaler, 2002a; Boaler & Greeno, 2000). Similarly, Cobb, Gresalfi, and Hodge (2009) found that students' personal mathematics identities were largely consistent with the microcultures established in particular mathematics classrooms. Cobb and colleagues developed an interpretive scheme to document the relationship between what they called *normative identities* (the mathematical activities, beliefs, roles, and expectations of a classroom) and *personal identities* (individual students' decisions to identify with, comply with, or resist identification with the larger classroom community). They found that students' expectations, obligations, forms of agency, and accountability in their mathematical activities were key mechanisms that explained differences in observed personal identities. In the field of literacy, McCarthey (2001) discovered that many of the fifth-grade students she studied, especially high achievers, had identities that were tied to their achievements and literacy practices in language arts. In a study of elementary and middle school social studies students, Polman (2006) found that tasks, norms for acceptable participation, and past experiences interacted with existing identities to positively shift one student's social studies identity while maintaining another student's negative social studies identity.

As these studies indicate, tasks, classroom norms, teacher pedagogy, and curricula are important factors that influence identity, but not in a consistent or predictable way. In fact, students in the same class, engaged in the same activity, abiding by the same classroom norms and participant structures can and do enact very different mathematics identities. These factors are not fine-grained enough to adequately explain the enactment of differing mathematics identities for students engaged in the same collective activities nor do they adequately account for the role others play in identity formation. However, the enactment of identity at the microlevel through discourse recognizes the role that others play in identity enactment and incorporates a potential explanation for the variation in discipline-specific identities across the same classroom. In the following section, I discuss research on the relationship between identity and discourse.

*Identity and its relationship to discourse.* Gee (2001, 2005), along with Sfard and Prusak (2005), has posited a view of identity as communicational and discursive. Gee (2001) described identity as an individual trait (i.e., specific to a person), yet created through the discourse of others so that one comes to be recognized in a certain way. Sfard and Prusak (2005) defined identity as a collection of stories or narratives, which, by their very natures, are discursive (see also Bruner, 1994). They took the perspective that one's identity is the narrative itself, in contrast to the more common claim that identity is simply represented or expressed in narratives. Although I do not go as far as defining identity as discourse (or narrative), I agree that discourse plays a critical role in enacting identities. According to Markus and Wurf, identity is an important part of any social interaction as each participant attempts "to shape a particular identity in the mind of his or her audience . . . [an] image of the self that one tries to convey to others" (1987, p. 325). What is the primary method for shaping and conveying these identities? Discourse. Every instance of communication affords an opportunity for participants to negotiate their respective identities and social positions (Davies & Harré, 2001; Gee, 2005; Sfard, 2001; Wetherell, 2001). Thus, the discursive work in small-group, school-based interactions extends beyond communicating to learn curriculum content to include the ways in which speakers and listeners present and accept certain views of themselves.

Sometimes the social positions taken up clearly reflect an enacted identity. For example, imagine a classroom in which a teacher consistently refers to her students as "mathematicians." This type of discourse move is an overt positioning act meant to clearly reflect and encourage students to enact a desired identity (see Johnston, 2004; O'Connor & Michaels, 1996; Wetherell, 2001). Often, though, one's relative social standing is not explicitly stated but implied by structural patterns in discourse. For example, initiation-response patterns, the distribution of turns of talk, the lengths of turns of talk, and whose ideas are taken up are all indicators of one's identity and status in a given community (Johnstone, 2002; Sfard, 2001). The consistent and recurrent repetition of these kinds of moment-to-moment discursive patterns can reinforce participants' identities in subtle ways that are often unnoticed.

On the basis of my synthesis of the literature, I chose a discursive approach to study identity. I made this decision because a discursive approach has (a) a strong, convincing, and coherent theoretical basis; (b) the means to account for the role others play in identity enactment; (c) the potential to explain the enactment of different identities within the same classroom; and (d) the capacity for a more transparent operationalization. Clearly, students' discursive interactions exist within and are influenced by the larger classroom community and norms and even broader social and historical factors; nevertheless, my primary objective was to consider the local, or microlevel, enactment of mathematics identities through discourse.

*Methodological Considerations*

*Defining discourse*. Because I take a discursive approach to identity, I briefly explain my use of the term *discourse* in this study. I define *discourse* to be the spoken and written words, semiotic systems, representations, and gestures of participants as they use language to communicate, interact, and act (Johnstone, 2002; see also Gee, 2005; Wetherell, 2001). My use of the term follows closely after Brown and Yule's (1983) and Gee's (2005) definitions of discourse as "language in use" (Brown & Yule, 1983, p. 1). Although primarily restricted to conversation in this study, discourse includes written text as well as nonverbal communication. Related to this are the implicit rules that order and govern how we interact discursively. I refer to these regularities, or rules, as *discursive routines* or patterns. My emphasis is on the work accomplished through discourse during human interaction and communication (Austin, 1999; Gee, 2005; Schiffrin, 1994;

Sfard, 2008), with a specific focus on the relationship between identities and discourse.[2] In addition, I propose a mutually constitutive relationship between identity and discourse. The words that participants use during interactions are both the means by which an identity is formed and the product of existing identities. Consequently, I do not distinguish between a given discursive move that constitutes identity and one that reflects identity. My position is that these actions co-occur. In other words, an identity is enacted through discourse, and, at the same time, it influences one's discourse.

*Methodological rationale*. The research traditions of ethnomethodology (Garfinkel, 1967), conversation analysis (Sacks, Schegloff, & Jefferson, 1974; Schiffrin, 1994; Sinclair & Coulthard, 1975), and interactional sociolinguistics (Goffman, 1981) provided a cohesive theoretical framework to consider the order and patterns of social interaction, social relations, and power dynamics and their influences on identity formation (Wetherell, 2001). I used the underlying theories of ethnomethodology and conversation analysis as my basis for examining the structure of students' everyday mathematical discourse to uncover seemingly hidden rules governing participation. We use routines to bring a sense of order and normalcy to our everyday conduct; these routines can reflect tacit beliefs about social ordering, power distribution, authority, who we are, what we believe is possible, and who we believe others to be. Thus, one can find evidence for identities in both the deeper structure of discourse—in routines of participation, in who has and maintains the floor during conversation, and in whose ideas are taken up—and through explicit positioning acts. I leveraged these theoretical perspectives to investigate the interactive and sometimes unpredictable enactment of students' mathematics identities. My research question was, What are students' moment-to-moment discourse patterns within a middle school mathematics classroom, and how might these patterns be related to one's enacted mathematics identity?

## RESEARCH DESIGN AND METHODS

### *Participants and Setting*

This study was conducted in a seventh-grade mathematics classroom in a city in south-central Texas as part of a larger program of research, Scaling Up SimCalc (Roschelle et al., 2010). The broader experimental study was an investigation of the effect on student learning of a technology-rich curriculum combined with professional development. Overall main effects were statistically and practically significant showing that students in SimCalc classrooms learned more than their

[2]Although issues of dominance and authority emerged as an important part of my study, I do not come from a critical discourse analysis perspective with a view of discourse as a political tool to reproduce ideologies and their underlying links to power and dominance within social systems.

counterparts in the control classrooms ($p < .0001$, student-level effect size of 0.63).

The classroom teacher had more than 25 years of teaching experience, the majority of which had been spent at this school. During this study, she implemented a new replacement curriculum unit addressing the concepts of rate and proportionality. The SimCalc MathWorlds technology and curriculum unit was designed to guide students to discover and connect various representations (graphical, tabular, verbal/written, and symbolic) of rate and proportionality in real-world problem contexts using technology-based simulations of motion (Shechtman, Roschelle, Haertel, & Knudsen, 2010). The simulations depict people, cars, or other icons moving across a small window while generating a position–time graph on the coordinate plane (see Figure 2). To facilitate independent and small-group work, the curriculum was designed around real-life scenarios that raised fundamental questions with respect to the concepts of rate, function, and proportionality.

Because both the classroom teacher and the new curriculum unit encouraged small-group work and pair work, the majority of student discourse in this class was peer to peer. Consequently, my focus was on students' moment-to-moment discourse patterns within small groups. Further, the rich, in-depth, microanalytic analyses necessary for this study led me to focus on a single case to better understand how moment-to-moment discourse moves related to identity enactment in a particular small-group setting. Thus, I selected one pair of students, Teri and Bonnie (pseudonyms), to observe during the 3-week unit. I selected these students after initial classroom observations and conversations with the teacher; they were primarily a sample of convenience, constrained by possibilities for camera placement in the computer lab and parental consent for video recording. After I explained my goal of investigating "typical" students' experiences with SimCalc, the teacher suggested I study Teri and Bonnie (who, I learned after unit completion, were best friends). Additionally, these two had predominantly positive attitudes toward school in general and wanted to be successful students.

*Classroom context.* Because the larger classroom context plays an important role in identity formation (Boaler & Greeno, 2000; Cobb et al., 2009), I describe the mathematics environment within which Teri and Bonnie functioned. Throughout the course of the unit, the curricular activities were structured in one of two ways: peer explorations/independent work or teacher demonstration/explanation. Two of the complete set of observation days were spent entirely in small-group, peer explorations in which students investigated problems and engaged in activities provided in the SimCalc curriculum. Three of the complete set of observation days consisted completely of teacher-led demonstrations and explanations, during which the teacher used the software for direct instruction with little student input. In the remaining observation days, these activity structures were combined (overall, roughly half of the total class time across the unit was spent in each of the two activity structures). The typical day began with students working on a problem from the workbook at the computer, followed by a teacher-led, whole-class conversation during which she explained the correct solution. This combination of independent paired work followed by teacher explanation of the task was repeated multiple times within most class periods. Student opportunities to explore during the SimCalc unit were typically limited to 10-minute intervals after which the classroom teacher provided correct answers.

### *Data Collection*

Sources of data included field notes and video recordings from 13 days of observations, semistructured group and individual interviews with the two focal students, and student artifacts (including student workbooks and unit posttests). The video camera was focused primarily on Teri and Bonnie to capture their discourse with each other and their interactions with the software. In addition, detailed field notes were taken each day during class and later expanded when viewing the video recording (before transcription). To facilitate a more focused analysis of the pair's discourse, all video recordings were transcribed. The interviews with Teri and Bonnie addressed (a) the pair's general impressions of the curricular unit and the SimCalc technology; (b) their developing knowledge of variation, rate, and the various representations of functions; and (c) their perceptions about mathematics, their own (and each other's) abilities in mathematics, and their mathematics identities.

### *Data Analysis*

I analyzed field notes, transcripts, and interviews through an open-coding process (Strauss & Corbin, 1998) to determine common themes and categories, focusing on Teri and Bonnie's discourse with each other and their mathematical activities during class. While I observed the students, took field notes, and did preliminary coding and analyses, a complex picture of the interactions among identities, discourse, and social relationships began to emerge. I then turned to microanalysis, using techniques from discourse analysis for a closer inspection of the data. This process was not linear, as my description might imply, but iterative and interconnected. Microanalysis led me to revise and expand categories and properties, to reconsider my interpretation of events, and to re-examine the data with revised coding schemes.

*Positioning acts*. On the basis of preliminary analyses and open coding, I coded the data for instances of positioning acts. In conversation, participants continually project and negotiate views of one another through discourse; this practice is commonly called *positioning*. Davies and Harré (2001) explain that "Positions are identified in part by extracting the autobiographical aspects of a conversation in which it becomes possible to find out how each conversant conceives of themselves and of the other participants by seeing what position they take up" (p. 264). I refer to these extracted autobiographical aspects as *positioning utterances*. Note that positioning is not unidirectional but relational and negotiated; one cannot be positioned unless one accepts that positioning (Davies & Harré, 2001). Five categories of positioning acts emerged from the data; these acts are further explained in the Findings section.

I assessed the reliability of my coding by determining the level of coding agreement between myself and an independent coder (a practicing teacher with whom I had previously done research) across the five categories of positioning acts. A subset consisting of 20% of the positioning acts identified in the data set was randomly selected; after discussing the five categories of positioning acts in detail and doing preliminary coding of instances of positioning acts, both the secondary coder and I coded the sample independently. Overall interrater agreement was 96.2%.

*Coding for discourse structure and function*. In addition to coding for positioning acts, I developed a coding scheme to code every turn of talk between Teri and Bonnie. My goal was to develop a microanalytic coding scheme to analyze both the structure of the discourse between Teri and Bonnie (i.e., patterns related to the distribution of initiations, responses, uptake of ideas, etc.) and the substance of their discourse (using what I call *function codes* to identify modes of participation and levels of intellectual work).

To identify underlying structural patterns in Teri and Bonnie's moment-to-moment discourse, I adapted a coding scheme developed by Wells (1996; Wells & Arauz, 2006), which is based on traditional Initiation-Response-Evaluation/Follow-up (IRE/F) patterns (see also Cazden, 2001; Mehan, 1979; Sinclair & Coulthard, 1975). The primary benefit of this coding scheme was that it facilitated the identification of underlying discursive structures in the pair's discourse (e.g., Initiation-Response patterns, routines of turn-taking, and normalized ways of participating).

Following Wells, I defined a *move* as the smallest building block in an interaction. It could be a statement, a question, or an answer; some researchers term this an *utterance* or a *turn* (Johnstone, 2002). In terms of the structure of conversation, each move can be categorized as an initiation (I), a response (R), or a follow-up (F). (Note that evaluation moves are subsumed into the follow-up category.) Typical triadic dialogue patterns in American classrooms use follow-up almost exclusively for evaluative purposes, signaling the end of a conversation. This kind of exchange did not accurately model the partner talk between Teri and Bonnie, because both continued to respond to one another well after the initial follow-up. Thus, instead of differentiating between two moves (responses and follow-up), which essentially fulfilled the same function, I decided to code moves only as Initiations and Responses (eliminating the follow-up category).

To provide a more complete picture of their discourse, I elaborated the Initiation and Response categories and classified each move as one of the following: an initiation, an initiation without response, a response to self, or a response to other. Initiation moves occurred only at the beginning of a new sequence of talk, which occurred when the pair transitioned to a new task or clearly shifted to a new topic of conversation. All other moves were coded as responses. If a participant's bid to begin a new topic was ignored, that move was coded as an initiation without response; if it was taken up, the move was coded as an initiation. Similarly, each response move was either coded as a response to the other or a response to oneself based on whether one's conversational partner ignored or responded to the previous comment. I used these codes, which were primarily theoretical but also data-based, to identify patterns in the structure of their discourse. The process of distinguishing these four types of moves was unambiguous; consequently, inter-rater agreement was not calculated for these codes.

Additionally, I coded each move with respect to its function: to explain, request information, evaluate, justify, acknowledge, and so on. Single conversational moves sometimes fulfilled more than one function and therefore received multiple codes. Drawing from speech act theory (Austin, 1999), my goal was to determine what was being accomplished with each talk move and to identify patterns in how each conversational partner participated (the full list of function codes is included in Figure 1). Each function code was further categorized as requiring a high or low level of intellectual work, in which intellectual work reflects the cognitive work set in motion and occurring in the discourse (Pierson, 2008). Intellectual work is similar to Webb's idea of cognitive processes and Stein and her colleagues' research on the cognitive demand of mathematical tasks (Stein, Grover, & Henningsen, 1996; Webb, Nemer, & Ing, 2006). I developed and used four intellectual work codes: *high-level give*, *low-level give*, *high-level request*, and *low-level request*. High levels of intellectual work extend thinking and include, for example, requests for justification, examples, interpretations of position-time graphs, or moves providing this type of information to one's conversational partner. Low levels of intellectual work include requesting or giving basic information (a recalled formula, definition, etc.), reading values from a chart or graph, making evaluative comments, or giving results from a calculation. Although my coding schemes were informed by theory and previous research, the intellectual work and function codes were based on and emerged from the data.

Once the coding scheme for intellectual work was finalized, I assessed its functionality and the reliability of my coding by determining the agreement between two independent coders. I randomly selected 10% of the sequences in the data set along with additional sequences I deemed problematic and potentially hard to code. After reviewing the definition of intellectual work, doing preliminary coding of sample sequences, and discussing the coding schemes in depth, the secondary coder and I independently coded the sample. Interrater agreement across the four categories of intellectual work in this sample was 87.9%. Figure 1 provides a visual representation of the coding scheme for discourse function and how the various levels of intellectual work were categorized with respect to the function codes.

In summary, each move was given three codes: an Initiation/Response code (i.e., a structure code), a function code, and an intellectual work code (which was essentially embedded into the function code). To illustrate this coding scheme, consider the excerpt in Table 3 from a discussion Teri and Bonnie had about unit prices and calculating a percent discount. The girls were determining the least expensive option for buying different numbers of uniforms (10, 20, and

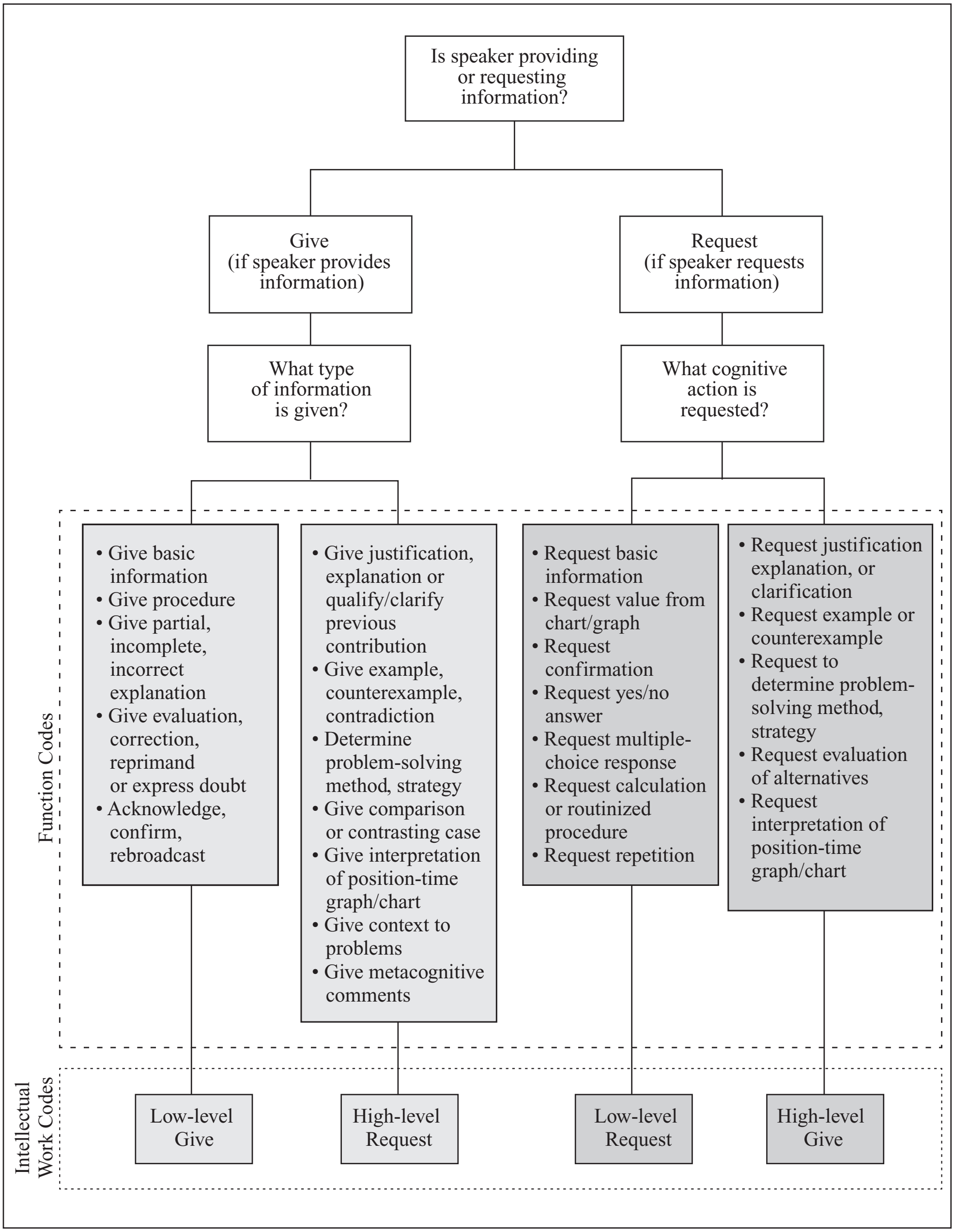



*Figure 1.* Coding flowchart for function codes categorized by intellectual work codes.

Table 3
*Coding Sample*

| Speaker | Text | Structure code | Function code | Intellectual work code |
|---|---|---|---|---|
| Teri | Okay. [Sighs and reads from workbook.] Soccerama also sells uniforms. They, their base price for one uniform is $50, but discounts for buying more are available. Which store, Soccer Universe or Soccerama, would be cheaper if they want to buy 10, 20, or 30 uniforms? Explain your reasoning. Oh, I hate this. I can never understand those things. | I | Give basic info | Low give |
| Bonnie | I can never understand the percent part of it. It's like adding tax and taking away tax. I never- | R to Other | Give basic info; Give context | High give |
| Teri | [Interrupting] Oh yeah it takes away tax like- | R to Other | Give basic info | Low give |
| Bonnie | [Interrupting] I know, I- | R to Other | Give confirmation | Low give |
| Teri | [Interrupting] how much? I know what like 15 percent is. It's like, 15 cents. But that's not really what it is. | R to Self | Give clarification | High give |

30 uniforms) at two fictional stores that offered different types of discounts.

Teri begins the sequence with an initiation (I) as she reads a problem from the workbook, which is coded as giving basic information and reflects a low level of intellectual work. Bonnie responds to Teri (R to Other) by agreeing and stating that she too struggles with percent problems (Give basic information) and then recontextualizes the problem by connecting it to a context she understands—calculating tax (Give context—a high-level move with respect to intellectual work). But before she can complete her thought, Teri interrupts, agreeing with Bonnie and restating Bonnie's initial example (R to Other), in which she compares a percent discount to taking away tax. Bonnie attempts to get the floor interrupting Teri to agree, but Teri continues her turn (R to Self), further refining the example, connecting 15% (presumably, of a dollar) to 15 cents (Give clarification). In the following sections, I use these coding schemes to examine how the girls' mathematics identities were enacted in their daily, microlevel interactions with each other.

## FINDINGS AND DISCUSSION

Analyses revealed the following salient features in Teri and Bonnie's discourse: (a) the joint positioning of Bonnie as the less capable, inferior mathematics student; and (b) asymmetrical structural and functional patterns within their discourse. Subsequently, I discuss these features and their relationship to identity, using representative classroom excerpts to highlight findings.

### *Positioning in Discourse*

Table 4 catalogues the ways that Teri and Bonnie jointly enacted their mathematics identities through their positioning of each other. During the 13 days of instruction, there were 123 total instances of positioning acts. The pair positioned each other through talk in the following ways:

- Using an authoritarian voice,
- Making statements of superiority or inferiority,
- Using face-saving moves,
- Building solidarity and providing encouragement, and
- Controlling problem-solving strategies.

*Authoritarian voice*. I defined an *authoritarian voice* to include adopting a critical and evaluative stance to what another is doing; constant monitoring and correcting of behavior and mathematical solutions; assuming responsibility for determining the correctness of solutions; giving directives (instead of suggestions or options); giving implied and explicit criticisms; and laying blame. There were 53 instances in which Teri adopted an authoritarian tone with Bonnie, and one instance in which Bonnie adopted such a tone with Teri.

By closely monitoring behavior ("You shouldn't be on the next one"), expressing disapproval of off-task behavior ("Quit messing around"), issuing directives ("You need to sharpen your pencil"), reprimanding, and correcting, Teri exercised authority over Bonnie. Teri consistently infused her statements with authority, and through these interactional moves, she appropriated a position of power such that she was in control. Interestingly, Bonnie's response was one of acceptance and alignment. By submitting to Teri, Bonnie surrendered her ability to make choices for herself and affirmed their relative positions of supervisor and subordinate.

*Statements of superiority and inferiority*. During the unit, Teri and Bonnie both made verbal statements of superiority and inferiority that reflected their underlying beliefs about who they were as mathematics students. Across the data set, 22 instances were coded as statements of intellectual inferiority or superiority with respect to mathematics. In each of these statements, either Bonnie adopted a position of inferiority (by self-identifying as "dumb" or taking up Teri's positioning of herself as "stupid") or Teri took up a position of superiority. These statements included making fun of and ridiculing each other as well as "I hate you" statements (which both girls apparently made in jest).

*Face-saving moves*. *Face*, as described by the sociologist Erving Goffman (1967), is related to conventions of politeness in conversation. Brown and Levinson (1978), in their discussion of politeness theory, described both positive and negative face. *Positive face* is the desire for approval, understanding, and others' good opinions, whereas *negative face* is the desire to act autonomously and in accordance with one's desires. Saving face can occur by avoiding situations that reveal a lack of understanding, such as asking for help or making mistakes or by co-opting problem-solving strategies to "get your way." Twelve instances in the data set were coded as face-saving moves.

On several occasions when the teacher asked the class who was not finished, Teri leaned over to Bonnie and instructed her to, "Raise your hand," instead of doing so herself. Table 4 displays evidence of Teri avoiding situations (e.g., raising her hand, asking the teacher for help, marking her homework 100 (indicating 100% correct) when she missed problems) that would reveal a lack of understanding and potentially compromise her mathematics identity. Needing help, making mistakes, and asking questions can be perceived as face-threatening acts if those actions are seen as inconsistent with mathematics identities of being smart, competent, and knowledgeable and in conflict with what doing mathematics entails. Independently, each of these actions might have alternative explanations (e.g., Teri and Bonnie might have agreed that Bonnie would be the one to ask for help that day), but the broader collection of discursive exchanges provides compelling evidence for their interpretation as examples of saving face.

Research from the field of psychology further supports the critical role that face plays in social environments. Although researchers often frame findings in terms of performance goals (goals and actions motivated by receiving positive, external evaluation of one's perceived competence) and learning goals (goals and actions motivated by an internal desire to learn and improve competence) (e.g., Dweck, 1986), they have identified a variety of face-saving behaviors characteristic of children holding performance goals. These characteristics include tendencies to (a) avoid challenges and difficult tasks; (b) avoid seeking needed help; (c) develop habits detrimental to the learning process, such as avoiding looking incompetent; and (d) make statements of a self-aggrandizing nature to shift attention away from the current (and, presumably, unsuccessful) academic activity (Dweck & Leggett, 1988; Gabriele & Montecinos, 2001). Although we do not know Teri's or Bonnie's goal orientations, Teri's face-saving moves seem consistent with a performance-goal motivation, enabling Teri to distance herself from failure and avoid feedback inconsistent with her mathematics identity of being competent and intellectually superior.

*Building solidarity and providing encouragement*. Positioning acts in the category of building solidarity and providing encouragement are characterized by anticipating and being responsive to what your conversational partner might need

Table 4
*Positioning Acts*

| Categories of positioning acts | Examples |
|---|---|
| Use of Authoritarian Voice | *Teri:* Quit messing around. Okay? For one second. (1/25/2006)<br><br>*Teri:* You need to sharpen your pencil.<br>[Bonnie immediately gets up and sharpens her pencil.] (1/26/2006)<br><br>*Teri:* What did you do last night?<br>*Bonnie:* I tried figuring it out. But—<br>*Teri:* [Interrupting] Show me what you didn't do last night [for homework].<br>Bonnie: This. [She holds booklet open to p. 52.] (1/31/2006)<br><br>*Teri:* I don't want to hear it, all right. Just move on and hurry up. We're so freaking far behind in this 'cuz you wouldn't do anything. Seriously you wouldn't do diddly squat. You just sat there and moaned.<br>*Bonnie:* I don't get it. That's why I asked you.<br>*Teri:* All right, just go. (1/27/2006) |
| Statements of Inferiority and Superiority | *Teri:* What is in that big head of yours? I mean, let me rephrase that, little head of yours?<br>*Bonnie:* Sam told me a penny—<br>*Teri:* —You don't have a penny in your freaking brain; you have nothing in there. . . . Bonnie, there's nothing in that brain of yours. (1/27/2006)<br><br>*Teri:* I don't know about this thing. I hate you.<br>*Bonnie:* I know. I hate you too. [Brief pause] *Hate* is such a, a strong word.<br>*Teri:* What? [She laughs.]<br>*Bonnie:* I don't know. (1/27/2006)<br><br>*Teri:* You looked at mine to see what you could figure out. Yeah, right, like you could do that [pointing to a problem in the workbook].<br>*Bonnie:* Hey, I'm not that stupid.<br>*Teri:* Um hum.<br>*Bonnie:* I know how to do fifth-grade math. (1/30/2006) |

Table 4 (*continued*)
*Positioning Acts*

| | |
|---|---|
| Face-saving Moves | [Students are grading their own homework at the beginning of class. Teacher's general pattern in checking the homework was to read the question, call on a student to answer, and then describe how the motion related to the graph.]<br>*Teacher:* [Reading #1] As Emily rides round and round on a Ferris wheel, her height off the ground is dependent on time. What answer is that? Raise your hand please. Yes?<br>*Student:* D.<br>[Bonnie and Teri write down correct answer when the student calls it out.]<br>*Teacher:* Do not change it. If you got it wrong, mark it wrong. Take a look at letter D. Can you see the Ferris wheel going up and down from that graph?<br>[At this point, Bonnie and Teri work ahead, answering the problems before the answers are given. At the end of grading, Bonnie and Teri have each missed two problems. Bonnie writes –2 on the top of her paper. Teri does not write anything on her paper (she missed the first two questions also).]<br>*Bonnie:* I didn't get the first two in time.<br>[Teri marks her homework paper 100 and hands it in.] (1/25/2006)<br><br>[Bonnie and Teri are working on a problem together and are stuck.]<br>*Teri:* I don't know. I can't think of anything. Can you go ask the teacher?<br>[Bonnie gets up and walks to teacher.] (1/27/2006)<br><br>*Teacher:* Okay, who's still working on Suiting Up [an activity in the workbook]?<br>*Teri:* [To Bonnie] Raise your hand.<br>[Bonnie raises her hand.] (1/31/2006) |

or say; interacting according to conventions of politeness (e.g., waiting for your partner to finish a problem); trying to help each other by explaining, demonstrating, or giving examples; making encouraging comments, such as "good job," "there you go," "I'm sorry," or "that's a good idea"; and expressing sympathy when a task is difficult. In the 21 instances of this code, 11 were cases in which Bonnie attempted to build solidarity with Teri and 10 were cases of Teri's attempts to build solidarity with Bonnie.

In 5 of Teri's 10 solidarity-building comments, she was checking to see whether

Table 4 (*continued*)
*Positioning Acts*

| | |
|---|---|
| Building Solidarity and Providing Encouragement | *Teri:* Don't you have to find like the unit rate or something?<br>*Bonnie:* [Nods yes.] I don't know how I finished that one last night. My grandpa was helping me with that so.<br>*Teri:* I don't know. I'm putting unit rate. I don't know.<br>*Bonnie:* Well, it's a good idea. (1/13/2006)<br><br>*Teri:* Uh oh, we got another one [a new problem]. You ready?<br>*Bonnie:* Yeah.<br>*Teri:* All right, I'm waiting. Go ahead, you can take your time.<br>*Bonnie:* Not all my time.<br>*Teri:* I know, but just try your best. We've got another one of those things to do. You can read this time, and we can take turns reading. (1/25/2006)<br><br>*Teri:* I have no clue, Bonnie.<br>*Bonnie:* I don't know nothing.<br>*Teri:* You do know stuff. (1/26/2006) |
| Direct and Control Problem Solving | *Teri:* Let's play a game. 'Cuz we're done.<br>*Bonnie:* We are?<br>*Teri:* We are done. We are done.<br><br>*Bonnie:* I'm gonna use this, okay? I'm gonna use this real quick.<br>[Bonnie points to screen when she says this and begins to play a simulation to answer a question in her workbook.]<br>*Teri:* It [the simulation] won't go any further. You already know though. I just showed you it.<br>[Bonnie is running the simulation using the step function when Teri takes control of the mouse.]<br>*Teri:* All right, hold on; I'll show you more in just a minute.<br>[She closes the simulation and opens the one she needs to do a new problem. Bonnie watches Teri resize the simulation and then flips her workbook to the problem Teri is working on. They never come back to Bonnie's problem.] |

Bonnie was ready to move on or if she understood (e.g., "All right, ready?" or "Do you need help?"). I coded these comments as building solidarity because they acknowledged Bonnie's needs and provided support. However, at the same time, they subtly reinforced Teri's role as the mathematical authority and positioned Bonnie in a role of dependence in that she finished tasks more slowly and needed assistance. In contrast, Bonnie's solidarity-building comments were composed of encouraging remarks (e.g., "That's a good idea"), attempts to explain or give clarifying examples, and agreement with Teri that a problem seemed difficult.

*Direct and control problem solving*. Finally, the last category of positioning acts was directing the pair's mathematical behavior and problem-solving activities. These acts included determining when the two were finished with the current task and could attempt a new problem as well as controlling their choice of strategy. In general, transitions to new problems occurred when Teri had completed the task with little consideration of where Bonnie was in her sense-making process. This practice reflected a self-focused learning trajectory for Teri and little attention to others' interests, needs, or questions.

Of the 15 instances coded as directing and controlling problem solving, in only 2 did Bonnie attempt to direct the problem-solving strategy. In one instance, Bonnie forcefully asserted her idea saying, "Let me show you an example. Sssh! Okay. This person would be going [draws a coordinate plane]. Really, okay, this is seconds." (Bonnie then drew the scale in seconds on the $x$-axis for a position-time graph.) Teri's response was, "You are so bad at graphing this," followed by, "Which class are you failing?" Whether intentional or not, Teri's response undermined Bonnie's attempt to direct the pair's mathematical actions. This is not to say that Bonnie's ideas or strategies were never taken up; in fact, they were. However, such instances appeared to occur not because Bonnie was a driving force pushing her idea but because Teri saw potential in the idea and made the decision to pursue the strategy. And even when Bonnie's ideas were taken up, she was rarely given credit for them.

The discursive exchanges between Bonnie and Teri revealed how they jointly enacted their mathematics identities through various positioning acts. Teri consistently positioned herself as more capable and mathematically competent than Bonnie throughout the SimCalc unit. She spoke with authority, controlled the majority of activities and uptake of ideas, and was the decision maker in the group. Inasmuch as Teri took up an identity of mathematical expert, Bonnie was simultaneously positioned as dependent, mathematically helpless, and, at times, unknowledgeable—an identity she did not resist and, in fact, helped to author.

### *Discourse Structure and Function*

To further investigate the relationship between identity and discourse, I turned to a microanalysis of the structure and function of Bonnie and Teri's mathematical conversations. Using the coding scheme described previously (see Figure 1 and Table 3), I analyzed patterns in the underlying structure of their discourse for evidence related to their mathematics identities. Specifically, I looked for robust and consistent patterns in Bonnie and Teri's dialogue, focusing on who had the floor, whose ideas were taken up, who was the primary initiator, modes of discursive participation, and average number of words per turn. As Sfard stated, "Meta-level intentions," including issues of power and identity, are likely to reside in the

Table 5
*Turn-By-Turn Analysis of Discourse* (*by Frequency and Percent of Column/Category Totals*)

| Speaker | Turns | Words | Words per turn | Initiations with response | Initiations without response | Response to self | Response to other |
|---|---|---|---|---|---|---|---|
| Bonnie | 531 | 4165 (34.5%) | 7.84 | 65 (31.4%) | 18 (51.4%) | 9 (19.6%) | 439 (53.5%) |
| Teri | 577 | 7905 (65.5%) | 13.70 | 142 (68.6%) | 17 (48.6%) | 37 (80.4%) | 381 (46.5%) |
| Total | 1108 | 12,070 | | 207 | 35 | 46 | 820 |

*Note*. Totals are across 13 days of instruction.

"mechanisms of interaction rather than in their explicit contents" (2001, p. 39). The data in Table 5 reveal differences in the structure of Bonnie and Teri's discourse at the microlevel.

In only 46 more turns of talk, Teri spoke almost twice as many words as Bonnie (3700 more), which is reflected in the disparity between the average numbers of words spoken per turn (13.7 and 7.8 for Teri and Bonnie, respectively). Of the 207 initiating moves that received a response, Bonnie made 31% and Teri made 69% of these moves. Of the initiations receiving no response from one's conversational partner, both Teri and Bonnie had approximately the same number of instances. But because Teri made more initiations, only 11% of her initiating moves were without a response (17 of 159 total initiations), whereas 22% of Bonnie's received no response (18 of 83). Of the 820 moves coded as a response to another, Bonnie made 54% of the moves and Teri, 46%. Additionally, 46 moves were coded as a response to oneself, either following an initiation without a response or continuing one's own line of thinking by ignoring or interrupting what one's partner has said; Teri made 80% of these moves and Bonnie, 20%.

I also coded the function of every move: Was the speaker evaluating, acknowledging the other's response, asking for the result of a calculation, offering an interpretation of a graph, giving a definition, asking for an explanation, and so on (see Figure 1 for function codes). Table 6 displays a summary of this coding, embedding function codes into their respective intellectual work categories. The table contains both frequency counts and percentages (percentages were based on the total number of discourse moves per student).

I highlight the following four observations from the data in Table 6:

1. The predominant mode of interaction between these two is to give information to each other (87% and 82% of the instances) instead of requesting it (13% and 24%). Both give information in roughly the same proportions, but Teri requests information almost twice as often as Bonnie (141 vs. 71 requests).

2. Neither girl asks high-level questions or provides high-level explanations, examples, counterexamples, or interpretations regularly. On average, fewer than 2 of every 100 moves are high-level questions and about 10 of every 100 moves provide high-level information.
3. If a question is posed, it will most likely be a low-level question. However, the frequencies with which Bonnie and Teri ask low-level questions differ. One of every 4.5 interactions for Teri involves making a low-level request of Bonnie; this ratio decreases to one of every 8.5 interactions for Bonnie.
4. The two students provide low-level and basic information in roughly the same proportion (with Bonnie doing slightly more of this), but they differ in their patterns of evaluating/correcting (7% for Bonnie and 12% for Teri) and acknowledging/confirming (14% for Bonnie and 5% for Teri).

Table 6
*Analysis of Discourse by Function and Intellectual Work* (*by Frequency and Percent of Total Moves*)

| Discourse move | Bonnie | Teri |
|---|---|---|
| Give information | 464 (87.4%) | 476 (82.4%) |
| High give | 45 (8%) | 67 (12%) |
| Low give | 419 (79%) | 409 (71%) |
| Give basic info | 285 (54%) | 291 (50%) |
| Evaluate/correct/reject/doubt other's response | 37 (7%) | 69 (12%) |
| Acknowledge/confirm | 72 (14%) | 26 (5%) |
| Request information | 71 (13.4%) | 141 (24.4%) |
| High request | 8 (1.5%) | 11 (1.9%) |
| Low request | 63 (11.9%) | 130 (22.5%) |
| Total moves | 531 | 577 |

*Note.* One move can fulfill multiple functions. Thus, column percentages sum to values larger than 100%. Not all function codes are included in the table. Totals are across 13 days of instruction.

According to the data from Tables 5 and 6, when interacting with Teri, Bonnie can expect the following:

- She (Bonnie) will rarely ask questions or initiate sequences;

- Almost 80% of her moves will be to provide low-level information of some sort, with 23% in response to Teri's requests for low-level information; and
- About one of every eight of her responses will be evaluated, corrected, rejected, reprimanded, or doubted, and 14% of her discursive activity will consist of acknowledging or accepting/confirming Teri's previous contribution.

Bonnie is quiet, listens, acquiesces, and provides trivial information.

Teri can expect the following when interacting with Bonnie:

- She (Teri) will be the primary initiator, talk twice as much, determine strategies for problems, pose questions every fourth move, and orchestrate the pair's mathematical conversations;
- Her primary mode of interacting is to provide low-level information, which she does in 71% of her moves; and
- She determines who gets the floor by responding to herself, selectively responding or not responding to Bonnie, and simply talking more.

Teri talks, controls, evaluates, and deals with trivial information.

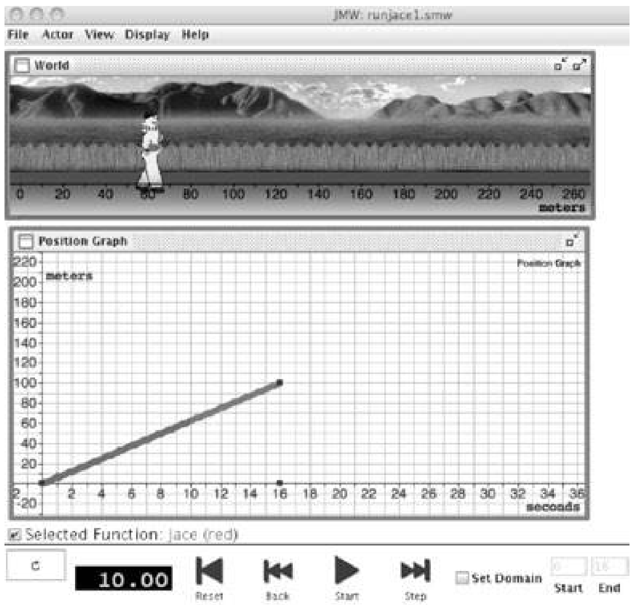
By controlling the pair's discourse, Teri exercised power. She constrained the ways in which Bonnie participated and controlled the uptake of both Bonnie's and her own ideas. Consequently, Bonnie's modes of discursive participation were primarily limited to responding to Teri's ideas—echoing, repeating, confirming—and providing the most basic kinds of information (e.g., reading points from graphs and calculating). Through repeated and systematic patterns in their discourse, Teri implicitly communicated that Bonnie had little of value to bring to their interactions beyond performing menial tasks that required minimal thinking, decision-making, or agency. These discursive routines enabled Bonnie and Teri to enact their respective mathematics identities of mathematically dumb and mathematically smart.

*Identities in action—an illustrative excerpt*. To better understand the discourse patterns displayed in Tables 5 and 6, consider the exchange in Table 7. This example illustrates how Bonnie's and Teri's identities were visible in the structure and substance of their small-group discourse. This interaction occurred when Bonnie and Teri began to integrate different representations of proportional relationships into their growing knowledge base of constant rates of change and the concept of function. In this sequence of talk, they are completing a table that corresponds to a simulation of a 100-meter race (see Figure 2). While the simulation runs, a position-time graph is generated on the coordinate plane. The graph starts at the origin and passes through the points (4, 25) and (8, 50) before the simulation ends. Note that the stated questions ask for only the first two rows of the table to be completed. These values can be read directly from the given position-time graph. However, Teri and Bonnie decided to complete the entire chart, including the time values at 100 meters and 200 meters, which they could not determine from the simulation by simply reading points from the graph.

When the sequence begins, Teri and Bonnie are jointly focused on a common,

# Run, Jace, Run

1. Open the file runjace1.mw. Watch the simulation and graph of Jace's 100-meter dash.
   a. Use the graph to answer: How many seconds has Jace run when he has gone 25 meters? How many seconds has he run when he has gone 50 meters?
   b. Now fill in the first two rows of the table.




| Seconds | Meters |
|---|---|
| | 25 |
| | 50 |
| | 100 |
| | 200 |
| 15 | |
| 30 | |



*Figure 2.* The SimCalc MathWorlds workbook activity and simulation for the doubling episode.

Table 7
*Bonnie and Teri's Doubling Episode*

| Turn | Speaker | Text | Structure code | Function code |
|---|---|---|---|---|
| Sequence 1 | | | | |
| 1 | *Bonnie:* | I don't get it. [Pause] | I w/o R | Give basic info |
| 2 | *Bonnie:* | I don't get it. [Bonnie stares at chart in workbook. She has written 4 seconds and 8 seconds in the first two rows, then writes 16 below 8.] | I | Give basic info (repetition) |
| 3 | *Teri:* | [Leaning over to look at Bonnie's workbook] That's what I'm trying to figure out too. | R to Other | Give basic info |
| 4 | *Bonnie:* | 'Cuz I thought it was doubling before. Wait. [Pause] Sixteen. | R to Other | Give explanation |
| 5 | *Teri:* | It's 8. [Teri points to graph on computer screen as she says 8. Pause as they work independently.] | R to Other | Give basic info |
| Sequence 2 | | | | |
| 6 | *Teri:* | Four seconds? [Teri uses cursor to point to the approximate point (4, 25) on the graph of the simulation.] | I | Request value off graph |
| 7 | *Bonnie:* | It was at 25. | R to Other | Give basic info |
| 8 | *Teri:* | Right. | R to Other | Acknowledge/ Confirm |
| 9 | *Bonnie:* | Eight seconds. | R to Other | Request value off graph |
| 10 | *Teri:* | It went to the—'cuz see the line was there. [Teri points to the approximate point (8, 50) on the computer screen.] | R to Other | Give basic info |
| 11 | *Bonnie:* | Yeah. | R to Other | Acknowledge/ Confirm |
| 12 | *Teri:* | Okay, keep going. [Teri advances to the next "step" of the simulation using the step-by-step replay feature.] | I w/o R | Request value off graph |
| 13 | *Bonnie:* | Eight seconds it was at? [Pause] 8? | I w/o R | Request value off graph (repetition) |
| 14 | *Teri:* | No. We're not— | R to Other | Eval/Correct/ Reject |
| 15 | *Bonnie:* | [Interrupting Teri] Fifty. | R to Self | Give basic info |
| 16 | *Teri:* | It was there. [On the computer screen Teri points to the approximate point (16, 100) on the graph.] On the hundred. 100 meters. | R to Self | Give basic info |
| 17 | *Bonnie:* | Yeah. | R to Other | Acknowledge/ Confirm |

Table 7 (*continued*)
*Bonnie and Teri's Doubling Episode*

| | | | | |
|---|---|---|---|---|
| 18 | *Teri:* | Two hundred [Teri means 100] would about be 14 or 15, I'm thinking. There. [Teri points at the approximate point (15, 100), which is not a point on the graph.] 'Cuz 15 and that line. [Teri traces over horizontally from 100 on *y*-axis to approximately (15, 100).] | R to Self | Interpret graph (makes prediction) |
| 19 | *Bonnie:* | [Interrupting Teri with enthusiasm] This is like doubling that line! [Bonnie holds thumb and forefinger about 2 inches apart moving them up and down, gesturing toward the screen that the vertical distance from the *x*-axis to (8, 50) is double the distance from the *x*-axis to (4, 25).] | I w/o R | Give explanation |
| 20 | *Teri:* | Maybe it's 16, cuz here's 15. [Points to graph again and pauses.] 100 [traces horizontally from *y*-value of 100 again]. Probably about 16. | R to Self | Interpret graph (use to predict) |
| 21 | *Bonnie:* | I thought it was like that 'cuz it was doubling it. But if you look at this. [Bonnie points toward the value of 15 in the time column.] | R to Other | Acknowledge/ Confirm; Give Justification; Give counterexample |
| 22 | *Teri:* | Yeah, I just realized that. | R to Other | Acknowledge/ Confirm |
| 23 | *Bonnie:* | 'Cuz, but, 15 plus 15 is 30. Maybe that has something to. I'll put a star by this. | R to Other | Give explanation |

mutually agreed upon task—completing the chart. Despite her claims to the contrary, Bonnie did at least partially "get it" as indicated by the data she entered in the table (Turn 2) and her description of the doubling pattern she observed (Turn 4). Initially, the pair alternated turns, prompting one another with *y*-values corresponding to the missing *x*-values to enter into the chart. After Teri successfully filled in the first two rows of the table, she was ready to find the missing *x*-value for 100 meters (see Turn 12). The rest of the exchange focuses on identifying this value.

First, consider whose ideas were given priority and how the two responded to each other. Notice the instances in which Teri did not meaningfully respond to Bonnie's initiations (see Turns 13 and 14, Turn 16, and Turns 19 and 20). In each case, Teri appeared to be more concerned with following her own line of reasoning (six responses to herself in Tables 7 and 8) instead of entertaining alternative ideas or questions. She exhibited little sensitivity to Bonnie's needs (e.g., did not respond

Table 8
*Bonnie and Teri's Doubling Episode* (*continued*)

| Turn | Speaker | Text | Structure code | Function code |
|---|---|---|---|---|
| Sequence 3 | | | | |
| 24 | *Teri:* | [Quick and excited] I see what it's doing. This is, it's times by 2. It's doubling it. You see that? | I | Explain; Request confirmation |
| 25 | *Bonnie:* | Yeah. I, I know this. | R to Other | Evaluation |
| 26 | *Teri:* | But how can we get 15? [Teri points to 15 in the chart.] | I | Request explanation |
| 27 | *Bonnie:* | Yeah. Maybe it's– | | |
| 28 | *Teri:* | [Interrupting] 'Cuz see, that's also doubling it right there too. [She points to 15 and 30 in workbook chart.] | R to Self | Interpret chart |
| 29 | *Bonnie:* | [Continuing her previous turn (see Turn 27)] Maybe there's a pattern like. It's sort of a weird pattern. Maybe they're running back? I have no idea; maybe they're like running backwards. [Pause] | R to Other | Explain; Contextualize |
| Sequence 4 | | | | |
| 30 | *Teri:* | Let me see what's at 200. 200 meters. [Teri replays simulation, which ends at 50 meters, not 200 meters.] | I | Give basic info |
| 31 | *Bonnie:* | 'Cuz when I looked at the 15 and the 30, I thought it would either be by 5s, right. [Bonnie could be referring to scale on the graph itself or a pattern for the *x*-values.] | R to Other (see Turn 26) | Partial/unclear explanation |
| 32 | *Teri:* | Do you think that's going to be it right there? [Teri moves cursor to approximately 25 on *x*-axis.] | R to Self | Give interpretation; Request confirmation |
| 33 | *Bonnie:* | I think it would be like at 29. [Pause] Round 29 to 30, I would say. | R to Other | Give basic info |
| 34 | *Teri:* | It's right on about the 25 [pointing to computer]. | R to Other | Give basic info |
| 35 | *Bonnie:* | Yeah. | R to Other | Acknowledge/ Confirm |
| 36 | *Teri:* | Okay. That's what I don't get then, 'cuz 16, wait. Maybe it's 32. [Teri traces horizontally from a *y*-value of 200 across the monitor estimating where it would intersect the graph of the line if the line continued.] | R to Self | Give interpretation of graph/chart |
| 37 | *Bonnie:* | Yeah, that does make sense. 16 and 16 is 32. [Bonnie writes 32 in the chart for 200 meters.] | R to Other | Explain |
| 38 | *Teri:* | Uh huh. | R to Other | Acknowledge/ Confirm |

to Bonnie's request for the distance at 8 seconds) and appeared to, at best, only superficially acknowledge Bonnie's insight (i.e., the doubling pattern) in turn 22. In fact, exactly what Teri "realized" at that point is unclear. Was her realization that the time-value of 15 was seemingly out of place because it did not fit the vertical pattern of increasing time values? Or did she recognize the same doubling pattern that Bonnie had observed previously? The latter seems unlikely, given Teri's first turn of talk in Table 8. Here we see her excitedly identify the doubling pattern, seemingly noticing it for the first time.

In contrast, Teri never initiated a comment that did not receive a response. In fact, her initiations often had multiple responses because both she and Bonnie responded. Consistent with patterns in Tables 5 and 6, here Bonnie responds to Teri's ideas, obligingly acknowledging, echoing, and confirming. This exchange concludes in Table 8.

Although both students initiated dialogue roughly the same number of times in this episode (Bonnie, 4 initiations; Teri, 5 initiations), the difference is in the uptake of ideas. Teri responded to herself 6 times and to Bonnie 8 times; Bonnie responded to herself once and to Teri 13 times. All of Teri's requests and ideas were publicly discussed and responded to, whereas 5 of Bonnie's moves in this episode were never taken up or responded to by her partner. This is normal for Bonnie. When asked to describe how they worked together Bonnie explained, "She (Teri) knows it first, and then I finally get it" (May 2006).

Notice that Bonnie gives voice to "doubling" four times (see turns 4, 19, 21, and 23), yet Teri does not appear to explicitly acknowledge Bonnie's idea as Bonnie's. In fact, she seemingly claims it as her own (turn 24 in Table 8) stating, "I see what it's doing [quick and excited]. This is—it's times by 2. It's doubling it. You see that?" Bonnie replied, "Yeah. I, I know this." Either Teri did not hear Bonnie's earlier comments (perhaps because she was unable to understand them), she chose to ignore them, or it was only now that she decided Bonnie's "doubling" explanation was a viable solution strategy. Regardless, Bonnie's reaction (or lack thereof) in this sequence indicates that unacknowledged ideas and inequitable interaction were the norm for her. Although the pair eventually mentions the doubling pattern, Bonnie never receives credit for her idea, and her contribution is not publicly recognized. Moreover, Bonnie adopts Teri's estimation technique when trying to calculate the time value at 200 meters (see turn 33 in Table 8). Bonnie, the student with less academic power and a negative mathematics identity, acquiesced to Teri's less sophisticated problem-solving strategy. In this exchange, Bonnie learned more than the time values at 100 and 200 meters. Every time her ideas were ignored and her thinking was not valued, her mathematics identity as the "dumb one" was subtly reinforced.

### *Bonnie and Teri—Using Discourse to Enact Their Identities*

Teri and Bonnie's discourse reflects and constitutes normalcy in who they are as mathematics students. Bonnie said, "She's [Teri] just always been smart like that

. . . and that's how we've always been viewed. She's always been the smart one; I've always been the dumb one" (May 2006). Teri enacted her identity of the mathematically "smart one" by leveraging subtle discursive moves and positioning herself as mathematically knowledgeable. This included frequently ignoring and dismissing Bonnie's ideas, drawing attention to her own ideas, controlling Initiation-Response patterns and thereby determining modes of participation in mathematics activities, and inviting virtually no reciprocity. Bonnie enacted her identity of the mathematically "dumb one" through the familiar actions of waiting for more knowledgeable others to act and make decisions, participating primarily in low-level ways, frequently surrendering her freedom to direct mathematical action, and having her ideas ignored. This pattern is normal in their world. Because they talk and act accordingly, neither girl thinks that Bonnie is good at mathematics. And because neither girl thinks that Bonnie is good at mathematics, they talk and act accordingly.

## CONCLUSIONS AND IMPLICATIONS

In this study, I developed a structured way to characterize meaningful discourse patterns in small-group interactions. Specifically, I devised frameworks that were theoretically and empirically connected to identity and that characterized the participants' positioning of each other as well as the microlevel structural patterns in their discourse. On the basis of the analyses from the previous sections, I conclude that through Teri's control of discourse at the microlevel and their repeated and joint positioning of Bonnie as mathematically incapable with little to contribute, the girls enacted their respective identities of "smart" and "dumb." In other words, discourse played a critical role in the mathematics identities that Bonnie and Teri enacted.

### *Limitations*

Throughout this article, my focus has been the microlevel of classroom discourse, with the goal of understanding and describing the enactment of differing identities within the same local context. Clearly, Bonnie and Teri operated within multiple, and often broader, contexts, including their current mathematics classroom, school, and community; past mathematics classrooms and experiences; other content areas; family units; gendered (and racial) expectations; larger life histories; and so on. This additional information, although necessary for describing a complete and holistic account of their identities, was beyond the scope of this study. Consequently, I can say little about related contexts and other mediating factors (e.g., curriculum, institutional structures, classroom norms, race, and gender) and whether or how they influenced the girls' identities; such was not the purpose of this study.

For example, I did not have access to Bonnie and Teri's discourse and interactions with each other outside of school or in other academic subjects. Perhaps the pair would have enacted identities of "smart" and "dumb" not only in mathematics but also in history, science, and English. Their mathematics identities could, in fact, be artifacts of Teri and Bonnie's normal interactional routines outside of school and their relationship as best friends. Additionally, I do not know how other students viewed them as mathematics students. Pairing with different partners during this unit might have led them to enact different identities. The fact that they were being video recorded might have altered their discourse and behavior in such a way that was not truly reflective of their mathematics identities. Or, alternatively, perhaps Bonnie's discourse and actions were not reflective (or constitutive) of an identity of mathematically dumb, but occurred simply because she was not invested in or motivated by the mathematics, and was not inclined to speak and act as if she were. It could also have been that the two were engaged in role-playing, wherein their entire interaction was merely an act. It is possible, then, that the pair's discourse might not reflect either girl's identity, but instead be an artifact of an observer effect, their friendship, or a lack of motivation.

However, the absence of data in support of any of these explanations suggests that these alternative interpretations are unlikely. In fact, data indicate just the opposite, because there were multiple instances in which Teri and Bonnie stated that they forgot they were being recorded, implying that changes in their interactions due to being recorded were unlikely. Additionally, there was one activity during the SimCalc unit that required students to write a memo to the school principal regarding fuel costs for out-of-town athletic events. Teri asked Bonnie for help completing this task, stating that she did so because she (Bonnie) was a good writer, indicating that Bonnie was not automatically the "dumb one" in all situations. Moreover, in individual interviews—one involving stimulated recall—neither girl indicated that her discourse and activities were disingenuous or an act. Both Bonnie and Teri expressed a desire to do well in school and, specifically, in mathematics and, at least outwardly, appeared to be engaged. Thus, in this context and setting, the enacted identities of mathematically smart and mathematically dumb appeared to be robust for Teri and Bonnie.

What I have done in this study is provide a fine-grained, detailed analysis of discourse and identity that is situated and relative to Bonnie and Teri's interactions with each other. By limiting the analysis to the microlevel, I was able to identify key explanatory mechanisms with the potential to account for differing individual mathematics identities for students sharing similarities in broader contexts. In Bonnie and Teri's case, we see that they had much in common: participation in a reform-based, technology-rich environment; a mathematically competent teacher; and a curriculum with authentic problems situated in meaningful, everyday contexts. Despite these similarities, they enacted vastly different mathematics identities. By focusing on the microlevel, I was able to study how this difference came to be; as a result, I was able to identify discourse as a key mechanism for identity enactment.

### *Implications*

In closing, I see three broad categories of implications arising from this study. First, I consider implications for further research on identity. Second, I discuss why seemingly trivial differences in discursive patterns are really not trivial inasmuch as they have implications for equitable participation. Finally, I suggest recommendations for instruction.

*Implications for research*. Bonnie and Teri have shown us the importance of discourse, specifically peer-to-peer discourse, in identity enactment. In particular, through this study I have identified the discursive constructs of *positioning* and *underlying structural patterns* as useful tools to investigate mathematics identities; these constructs are useful because they provide a tractable, data-based way of identifying students' enacted mathematics identities. Thus, one approach to research on mathematics identities should continue to be a discursive approach, using microlevel routines and positioning as potential starting points. Research in this vein can draw from the initial categorizations and coding schemes I provide, refine and broaden other already existing discursive constructs (e.g., the central role of narratives in Sfard, 2008, and Sfard & Prusak, 2005), and identify new discursive constructs that are sure to emerge.

More broadly, scholars who do research on identity use a variety of analytic tools and conduct analyses displaying a range of grain sizes (see Tables 1 and 2). Some focus on the individual and microlevel of identity enactment using the constructs of moment-to-moment discourse; narratives; personal goals; and rich, ethnographic accounts of student's mathematics experiences. Others focus on group-level characteristics (e.g., classroom-level characteristics), considering the interaction of curriculum, classroom norms, and teacher pedagogy/beliefs with identity. Moving forward, research needs to explore ways to coherently integrate the macro- and microperspectives. Both Martin (2000) and Solomon (2009) have taken steps in this direction, devising frameworks and conducting analyses that look across multiple layers to incorporate broad sociohistorical and political factors as well as factors specific to individuals in their accounts of identity. However, an important next step for research on identity will be to create new models and methodological approaches that will help us understand and operationalize these dynamic, nested, complex environments that function at both individual and collective levels in the enactment of students' identities (perhaps drawing from research on complex, adaptive systems—see Lemke, 2000; Senge, 1990).

*Accumulation of disadvantage and issues of equity*. Differences in discursive details may appear trivial at first glance; however, small differences in subtle details accrue over time. According to Johnstone (2002), struggles over whose words are used and who gets to speak are indicative of power and status and, I would add, one's identity. For instance, initiation-response patterns, the distribution of turns of talk, how one is positioned, and whose ideas are taken up are all indicators of identity. Teri leveraged these discursive moves to enact an identity of mathematically smart; Bonnie did not, and as a result, she enacted an identity of mathematically dumb and incompetent. Sociologists have described this idea in using the terms *accumulation of advantage* and *accumulation of disadvantage* (Valian, 1998;

Zuckerman, 1998). Their premise is that small differences (in discourse, status, resources, rewards, achievements, etc.) accumulate over time, much like interest in a bank account. What seem to be small differences when viewed in isolation eventually result in large disparities and inequities. From this point of view, molehills really do become mountains (Valian, 1998).

In her book *Why So Slow?* Valian (1998) discussed how advantage and disadvantage can occur in social settings and how this construct can be a mechanism for systemic inequities. She gave a hypothetical example of a professional meeting and described the effects of whose comments are taken seriously and whose are ignored: "By the time the meeting ends, people who were equal in my eyes when it began are unequal" (p. 4). The people whose remarks were ignored lost prestige when their contributions were implicitly considered to be low-value because they were ignored by the group. She continued, "Because they now have less prestige, they will be listened to less in the future. . . . The gap between them and people who are gaining attention for their remarks will widen as their small initial failures accrue and make future failures more likely" (p. 4).

Whether the difference is in whose ideas are valued, the relative positions participants take up, or microlevel discursive structures, small inequities now can have large consequences later. In Bonnie and Teri's case, it was the consistent and recurrent repetition of these discursive patterns that enabled them to enact undesirable mathematics identities. The positioning acts and microlevel interactions did not occur once or twice but characterized the nature of their interactions throughout the unit. And, over time, advantage accumulated for Teri, and disadvantage accumulated for Bonnie. Inevitably, this process raises questions of equity.

Although issues of equity are typically considered in terms of group disparities rather than individual ones, one fundamental goal in mathematics education is to ensure that all children have sufficient access and opportunities to be successful in mathematics (NCTM, 2000). If discourse is, indeed, viewed as a resource (see Gee, 2005), Bonnie and Teri provided a pattern of consistent and pervasive differences in discursive opportunities that advantage one and disadvantage another. Because of the routinized nature of the students' interactions, Bonnie's opportunity to enact an identity other than the dumb one appears to be limited, at best.

*Instructional implications*. The fact that Bonnie and Teri's classroom teacher was unaware that the two were enacting inequitable identities leads to the first instructional implication: the importance of listening to student-to-student discourse. Monitoring student talk enables one to identify who participates and how, who does not participate and why, and what kinds of mathematics identities students are enacting. The five types of positioning acts—using an authoritarian voice, making statements of inferiority or superiority, engaging in face-saving moves, building solidarity and encouraging one another, and controlling problem-solving strategies—are categories for which to listen to identify inequitable and unproductive ways of talking. In addition, monitoring interactions to determine whether one student dominates conversation or if another's ideas are rarely, if ever, heard, taken up, or credited to them can help to identify inequitable discourse patterns.

Second, this study has instructional implications related to the design and implementation of collaborative learning environments. The situated nature of this analysis indicates that the ways in which students are paired can significantly influence the mathematics identities they enact. Moreover, this study is an existence proof that students do not always know how to, or choose to, interact equitably, productively, and positively. It is critical, then, for the class explicitly to discuss and model appropriate ways of interacting in small groups (Sfard & Kieran, 2001, Swing & Peterson, 1982). This might include collectively establishing discursive norms and consistently enacting and enforcing those norms. In short, careful attention should be given to how students are paired and placed in small groups and the ways in which they are (or are not) prepared to work together in these collaborative learning environments.

And finally, there are instructional implications related to teachers' discursive moves. Through this study, I identified discourse as a key mechanism for identity enactment. Although my analysis was restricted to students' discourse with one another, I suggest that teachers can also leverage these discursive constructs as a tool to nudge students toward positive mathematics identities and to model appropriate ways of interacting. Cohen and colleagues' work on *complex instruction* emphasizes the importance of the teacher, highlighting in particular her role in positioning students as competent (Cohen & Lotan, 1995). Similarly, other researchers found that positioning and animating students by attributing specific mathematical claims and ideas to individuals provides opportunities for students to view themselves as viable, agentic doers of mathematics (Empson, 2003; Goffman, 1981; O'Connor & Michaels, 1996). When someone (in this case, a teacher) publicly values what one has to say and thinks that one's ideas are good, one learns to respect one's own thinking and can begin to enact a positive mathematics identity.

This study revealed that regularities in discourse are related to how we identify ourselves with respect to mathematics. The words we use can help others, and ourselves, to enact certain identities. Thus, the ways in which we talk and interact with each other are powerful because they affect who we are and who we can become with respect to mathematics.

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