The Motivation of Secondary School Students in Mathematical Word Problem Solving

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Abstract

Introduction. Motivation is an important factor in the learning of mathematics. Within this area of education, word problem solving is central in most mathematics curricula of Secondary School. The objective of this research is to detect the differences in motivation in terms of the strategies used to solve word problems.

Method. It is analyzed the resolution procedures of three word problems. Furthermore, motivation data were collected through a questionnaire based on expectancy-value model of Eccles and her colleagues and adapted to the application in math learning. The sample consists of 598 students in 8th, 9th and 10th grade of Secondary Education in the Basque Autonomous Community.

Results. The results indicate that the group of students with algebraic resolution profile obtained higher scores in both task value and self-efficacy. In addition, the group without a define profile of resolution declares that the study of mathematics means greater loss of opportunity to do other activities.

Discussion. On the one hand, the students who solve problems algebraically shows a higher motivation comparing with the mixed resolution group; therefore, it appears that the alleged flexibility in resolution means less motivation. On the other hand, it is discussed the low motivation of students without a resolution profile in relation to their performance.

Keywords. math education, word problem solving, motivational model of expectancy-value, secondary education.

La Motivación en la Resolución de Problemas Aritmético-algebraicos. Un Estudio con Alumnado de Educación Secundaria

Resumen

Introducción. La motivación constituye un factor importante en el aprendizaje de las matemáticas. Dentro de este ámbito educativo, la resolución de problemas ocupa un lugar central en la mayoría de currículos de matemáticas en Educación Secundaria. El objetivo de esta investigación es detectar las diferencias en motivación en función de las estrategias empleadas para la resolución de problemas aritmético-algebraicos.

Método. Se han analizado los procedimientos de resolución de tres problemas verbales aritmético-algebraicos y se han recopilado datos sobre la motivación mediante un cuestionario basado en el modelo de expectativa-valor de Eccles y sus colaboradores adaptado para la aplicación en el aprendizaje de las matemáticas. La muestra está compuesta por 598 estudiantes de 2º, 3º y 4º de Educación Secundaria Obligatoria (ESO) de la Comunidad Autónoma Vasca.

Resultados. Los resultados obtenidos indican que el grupo de alumnos con perfil de resolución algebraico obtiene puntuaciones superiores tanto en valor de la tarea como en la autoeficacia percibida. Además, el grupo de resolución sin perfil definido declara que el estudio de las matemáticas supone una mayor pérdida de oportunidades para realizar otras actividades.

Discusión. Por una parte, el alumnado que resuelve los problemas algebraicamente muestra una motivación superior al grupo que alterna el álgebra y la aritmética como estrategias de resolución; por tanto, parece ser que la supuesta flexibilidad en la resolución implica menor motivación. Por otra parte, se discute la baja motivación del alumnado sin perfil de resolución definido en relación a su bajo rendimiento.

Palabras clave. Educación matemática, resolución de problemas, modelo motivacional de expectativa-valor, educación secundaria.

Introduction

Educational research in mathematics recognises the relevance of word problem solving. The influence of motivation in all areas of instruction, including the teaching and learning of mathematics, is also well-known. What follows is a review of previous research from both these areas.

Word problem solving

In the information technology society of today, learning mathematics is related with finding solutions in complex situations and also with the conceptual competence which requires students to develop verbal ability (Sciarra and Seirup, 2008). These abilities, acquired principally by learning to solve problems, promote verbal reasoning and constitute the basis of a multitude of curriculums and programmes of study in mathematics throughout the world.

Word problems are defined as the set of problems which in educational contexts are solved through the application of various elementary arithmetic operations successively combined with one another until a result is found (arithmetic procedure) or through the formulation of equations which are later solved to obtain a result (algebraic procedure) (Cerdán, 2008). It follows, therefore, that in order to solve these problems and obtain a result, it is necessary to know how to calculate arithmetic expressions or how to solve equations.

The dichotomy which arises when evaluating a problem as either arithmetic or algebraic represents one of the major controversies in educational contexts in algebra. Wagner and Kieran (1988) propose a series of questions which allows us to delimit the nature of problems and define the difficulties which may appear when solving them arithmetically or algebraically. Bednarz and Janvier (1996) define arithmetic problems as connected since a relationship can be easily established between two known data, while algebraic problems, on the other hand, are regarded as disconnected, since no connection can be established between the known data. Cerdán (2008) claims that it is not the structure of the problem which determines whether it is arithmetic or algebraic but rather the process undertaken when translating its verbal formulation into arithmetic or algebraic expressions.

Mathematical word problem solving is spread widely across the different educational cycles; the discipline is first introduced in Primary Education with arithmetic operations. The transition to Secondary Education is marked by the introduction of a new technique for problem solving: algebra, a much more effective technique since it can be applied to problems of all kinds.

The transition from arithmetic to algebra is a complex one for secondary students and the fact that both techniques share symbols such as addition, subtraction, multiplication, division and equality only complicates the process (Kieran, 2007). However, many of the problems which are presented in Secondary are not noticeably different from those presented in Primary, thus facilitating the recently-acquired algebraic process of resolution (Stacey and MacGregor, 2000).

Different studies have detected a number of difficulties which secondary students encounter when they are first introduced to algebra:

1. The difficulty of operating with unknowns (Herscovics and Linchevski, 1994; Filloy, Rojano, and Puig, 2008).

2. The tendency to interpret the equality sign as an indicator of a result (operational sense) as opposed to a relation between two quantities (relational sense) (Knuth, Alibali, Mcneil, Weinberg, and Stephens, 2005)

3. The difficulty of understanding the use of letters to indicate unknowns (Booth, 1984).

4. The complexity involved in transforming the verbal formulation into an equation. This problem has been associated with the syntax of the verbal formulation (Clement, 1982; Clement, Lochhead and Soloway, 1979; Fisher, Borchert, and Bassok, 2011).

The motivational model of expectancy-value

Motivation is one of the most thoroughly researched topics in the educational context. A wide variety of theoretical positions have been employed with a view to gaining insight into academic motivation. They each focus on key factors in motivation, such as self-efficacy (Bandura, 1997), expectancy and task value (Wigfield and Eccles,

2000), goal-orientation (Ames, 1992), self-concept (Guay, Marsh, and Boivin, 2003) and attributions (Skinner, Wellborn, and Connell, 1990).

Motivation related to the learning of mathematics has been studied on lots of occasions, both in secondary education (Ahmed, Van der Werf, Kuyper, and Minnaert, 2013; Phan, 2012) and at university level (Olani, Hoekstra, Harskamp, and van der Werf, 2011; Peters, 2013; Sciarra and Seirup, 2008). Motivation, nevertheless, is a complex psycho-instructional variable, both for the multitude of viewpoints and measurements which are associated with it and for the variations which it undergoes from one educational discipline to another (Bong, 2001). In this sense, some researchers claim that certain motivational indicators may be more specific in certain domains than they are in others. For example, Green, Martin, and Marsh (2007) point out that the value assigned by students to tasks in mathematics shows a high degree of specificity, whereas others such as anxiety show a more general tendency among different disciplines or subjects. This suggests that the evaluation of motivation in mathematics, especially in areas such as value assignment, can be better assessed when it is directed at a specific domain rather than to general motivation.

Academic achievement in mathematics is greatly influenced by a student's motivation and involvement in the learning process. Although each of the theories analysed provides us with a valuable perspective into academic motivation, focussing on one of them to the detriment of the others can limit our research (Bong, 1996).

It has not, therefore, been resolved whether any one given model which measures general motivation is capable of reflecting students' involvement in mathematics, or whether it is necessary to obtain a number of indicators which might provide a more reliable motivational analysis. One of the most complete models which has studied motivation in mathematics is expectancy-value theory (Wigfield and Eccles, 2000); this model has shown that beliefs regarding value and the expectation of success are related to the effort expended in learning mathematics, and show a higher relevance in this discipline.

Expectancy-value theory has been very influential in research on motivation, giving rise to a large quantity of empirical studies which corroborate both its utility and its validity (Eccles, 2005a; Wigfield and Eccles, 2000). This theory of motivation argues that the choice, constancy and performance of individuals can be explained through

self-efficacy (or the perception of one's own capacity to carry out the task) and the value that individuals assign to their own activity (Wigfield and Eccles, 2000). The expectation of success, therefore, reflects individuals' beliefs regarding their own capacity in a given domain. This component is similar to the construction of self-efficacy in social cognitive theory (Bandura, 1997). Students with a weak belief in their own self-efficacy may be affected by doubts and uncertainty, while a high self-efficacy belief promotes confidence and positive sentiments towards one's own abilities.

Value is concerned with the perceived qualities of a task and how these perceptions influence students' attitudes when the task is carried out. In the same way, this factor explores the level of interest and utility which a person associates with a given task. In this model, value is broken down into four components: importance, interest, utility and cost. Importance analyses the relevance of doing the task well. Interest or intrinsic motivation is defined as the degree to which a person enjoys performing the task or the interest felt in the task's content (Wigfield and Eccles, 1992). The third element, utility, refers to the degree to which a task may be instrumental in achieving a future goal (Pintrich and Schunk, 2002). The final component is cost, which is conceptualized as the negative aspects associated with a task, such as the effort involved or the loss of opportunities to perform other tasks.

Previous studies uphold the soundness of this motivational theory and show that both expectations and values are directly related to academic performance and with the choice of areas of study in the specific domain of mathematics (Spinath, Spinath, Harlaar, and Plomin, 2006). More specifically, performance is more directly related to expectancy than to value (Steinmayr and Spinath 2009), while value is a more efficient indicator for constancy and students' choices regarding future fields of study (Eccles, 2005b; Wigfield and Eccles, 1992).

It should be pointed out that even though numerous research studies link this motivational model to the study of mathematics no evidence has so far been found which might clarify the relationship between the problem solving strategies employed and motivation. Given the scarcity of studies in this respect and the educational value of these variables, the goal of this present study is to analyse differences in motivation in mathematics as a function of the problem solving strategies employed in arithmetic and algebra. The study is centred on students in Obligatory Secondary Education (ESO).

Aims

Finally, the general goal of this study is to contribute to the clarification of the link between the method of instruction in mathematics and motivation; a greater understanding of this connection would permit to plan educational activities both in the school context and in the family, with a view to improving education and learning in a field which is so important.

Method

Participants

The sample is composed of 631 students in 2nd grade ESO (13-14 years,), 3rd grade ESO (14-15 years) and 4th grade ESO (15-16 years). In the sample, 292 were female students, and 242 were male. Due to errors or omissions in the questionnaires, the final sample was reduced to 598 subjects. Data were collected from 8 educational centres in the Basque Autonomous Community, of which 5 formed part of the public network and 3 belonged to the private, government-supported network.

Measuring Instruments

Mathematical motivation

For the purpose of studying motivation, the expectancy-value model was adopted. On the one hand, the scale of expectation in self-efficacy is measured. For this purpose, items were selected from the scale of self-efficacy in the MSLQ questionnaire (Pintrich et al., 1991). On the other hand, the scale of task-value is evaluated with its four categories. The components of the task-value are defined and evaluated by Eccles and his collaborators (Eccles et al., 1983; Eccles and Wigfield, 1995). All these items have been adapted by Berger and Karabenick (2011) so that they can be applied to mathematics. A translation of this questionnaire into Spanish was used (Appendix 1).

As is recommended by the original authors, the first three value components (interest, utility and importance) were grouped together and cost was measured separately. In this way, three motivational indicators were studied: value, cost, and self-efficacy expectancy.

The questionnaire is structured as follows:

- Self-efficacy (3 items): This measures the students' beliefs in their own self-efficacy, as regards mathematics. For example: "I'm certain I can understand the most difficult material presented in math ".

- Task-value:

  ➢ *Interest* (3 items): This refers to the interest or liking for mathematics. For example: "I like maths".

  ➢ *Utility* (3 items): This contains items relative to the use of mathematics in everyday life. For example: "I believe that math is valuable because it will help me in the future".

  ➢ *Importance (3 items)*: This measures the importance which mathematics may have for each one's personality. For example: "It is important for me to be a person who who can reason using math formulas and operations".

  ➢ *Cost* (2 items): This takes into account the effort which is involved in doing mathematics when it comes to measuring mathematics against other activities. For example: "I have to give up a lot to do well in math".

The students is expected to reply using a Likert-style scale with 5 points (where 1= completely disagree and 5 = completely agree).

Problem solving

The three problems proposed are introduced by Stacey and MacGregor (2000) and follow a progression from lesser to greater complexity with regard to the possibility of finding a solution with non-algebraic methods (Appendix 2).

In order to meet the objective of measuring the strategies that were adapted in trying to solve each problem, a codification was followed using four categories proposed by Khng and Lee (2009):

1. Algebraic: If the strategy is defined by one or more unknowns and is solved by using one or more equations, it is considered algebraic.

2. Arithmetic: If the strategy and the process followed in finding a solution is based on an arithmetic technique, that is, without making use of unknowns or equations, it is treated as arithmetic.

3. Mixed: This is a category into which problems are placed in which a variable is used in some part of the process but the technique is predominantly arithmetic. These procedures are considered to be arithmetic.

4. No strategy/no reply: Into this category, those procedures are placed which cannot be identified or which leave the problems unsolved.

A solution was classified as arithmetic when it fell into the arithmetic or the mixed categories. As already indicated the measuring instrument used to evaluate problem solving techniques consisted of three word problems of an arithmetic-algebraic nature. These problems were given to students to solve. Each participant was placed in one or other of the following three categories:

- Group G3 (or algebraic profile group): This category corresponds to those subjects who correctly solved all the problems algebraically, or who, alternatively, solved two problems algebraically, and adapted an algebraic strategy in the third (by identifying the unknowns and the equation), though they fail to find the correct solution due to an error in calculation while following the correct strategy.

- Group G2 (or mixed profile group): In this group are gathered those participants who use both algebraic and arithmetic strategies, depending on the problem to be solved. This includes therefore those who correctly solved two problems algebraically and one arithmetically or vice versa (two arithmetically and one algebraically)

- Group G1 (or undefined group): This is the group of the leftovers, those who did not solve the problems completely, using one strategy or the other, or by alternating between them. In this group are included those

participants who obtained in one, two, or three problems the classification *no strategy/no reply*.

Group G3 is composed of participants who solve the problems algebraically; Group G2 is composed of those who solve the problems in a mixed manner, sometimes arithmetically and other times algebraically. Group G1 participants make little or no use of algebra and show no mastery of arithmetic techniques either. This categorization, which prioritizes algebraic strategies as opposed to arithmetic, was chosen because the goal in ESO from the outset is to promote the use of algebraic strategies in the resolution of word problems. Arithmetic (or heuristic) techniques, when used after Primary Education are a tool to link up with students' previous experience (Khng and Lee, 2009) and should be complementary to acquiring mastery in algebra as a problem solving technique.

The relevance of the G2 group is outlined in a previous study (Gasco and Villarroel, 2012) which indicates that a large number of students use arithmetic in problems which lend themselves to this approach but nevertheless use algebra when an arithmetic approach becomes more difficult.

Statistic analysis

For the purposes of studying the differences between variables, the test chosen was the Kruskal-Wallis nonparametric test for k independent samples. To carry out the post-hoc tests we used the Mann-Whitney nonparametric U test. The use of nonparametric tests was deemed to be necessary because the data collected were neither homoscedastic nor normal according to established criteria.

In the tests for the analysis of two independent samples, for the Mann-Whitney U, the effect size was calculated from the parameter r (Field, 2009; Rosenthal, 1991). The interpretation of coefficient r is as follows: $r=.10$, weak effect size; $r=.30$, moderate effect size; and from $r=.50$ onward strong effect size (the values of r lie between 0 and 1).

Results

Table 1 represents the descriptive data of the sample distribution:

Table 1. Sample distribution by grade and sex (absolute frequencies and percentages)

		SEX		
		Female	Male	Total
GRADE	2ºESO	104	88	192 (32.1%)
	3ºESO	116	95	211 (35.3%)
	4ºESO	103	92	195 (32.6%)
	Total	323 (54%)	275 (46%)	N=598

Table 2 represents the distribution of the resolution groups by grade:

Table 2. Resolution groups by grade

		GRADE			
		2º ESO	3º ESO	4º ESO	Total
GROUPS	G1	105 (54.7%)	70 (33.2%)	62 (31.8%)	237 (39.6%)
	G2	30 (15.6%)	52 (24.6%)	17 (8.7%)	99 (16.6%)
	G3	57 (29.7%)	89 (42.2%)	116 (59.5%)	262 (43.8%)
	Total	192 (32.1%)	211 (35.3%)	195 (32.6%)	N=598

G1=The undefined group; G2=The mixed profile group; G3=The algebraic profile group

Table 3 represents the analysis of the differences in motivation scales (value, cost, and self-efficacy) among the three resolution groups (G1, G2 and G3):

***Table 3.* Differences in motivation scales among resolution groups**

Scale	Group	Mean	Standard deviation	Kruskal-Wallis test		
				χ^2	gl	p
Value	G1	1.90	.67			
	G2	3.43	.71	364.78	2	.000
	G3	3.74	.62			
Cost	G1	2.65	1.01			
	G2	2.22	.99	40.22	2	.000
	G3	2.11	.80			
Self-efficacy	G1	1.65	.75			
	G2	3.56	.71	369.25	2	.000
	G3	3.85	.72			

G1=The undefined group; G2=The mixed profile group; G3=The algebraic profile group

There are statistically significant intergroup differences in all three scales analysed (p<.001). Value refers to the interest, utility and importance assigned by students to learning mathematics. Cost refers to the effort involved in studying mathematics as compared with other activities. Self-efficacy measures students' belief in their own personal capacity in so far as the study of mathematics is concerned.

For the purposes of obtaining a more exact picture of differences, a two by two comparison was then carried out through a post-hoc test, specifically the Mann-Whitney U test (parameters z and p) with their respective effect sizes (parameter r):

1) *Differences between G1 and G2*: Statistically significant differences were found in the three motivational scales (*Value* (z=-12.55, p<.001, r=.68); *Cost* (z=-3.48, p<.001, r=.19); *Self-efficacy* (z=-13.13, p<.001, r=.71). The students in the mixed group, G2, who solved problems by alternating between algebra and arithmetic, obtained mean results higher than those obtained by students in the non-defined group, G1, in *value and self-efficacy*. However, the G1 group obtained a higher mean result in the *cost* scale. The effect size, r, is strong in the case of value and self-efficacy (r>.50) and between weak and moderate in the case of cost (.10<r<.30); that is to say the variation has a greater weight in the first two scales.

2) *Differences between G2 and G3.* Statistically significant differences were found in the *value* and *self-efficacy* scales but not in the scale of *cost*: *Value* (z=-4.08, p<.001, r=.22); *Cost* (z=-.70, p>.05); *Self-efficacy* (z=-3.63, p<.001, r=.19). The solution group belonging to the algebraic category, G3, obtained higher mean scores than the mixed category group, G2, in both scales. The effect size in the differences, both in value and in self-sufficiency, is between weak and moderate (.10<r<.30).

3) *Differences between G1 and G3*: Statistically significant differences were found in the three motivational scales (*Value* (z=-17.94, p<.001, r=.80); *Cost* (z=-6.31, p<.001, r=.28); *Self-efficacy* (z=-17.93, p<.001, r=.80). The group belonging to the algebraic category, G3, obtained higher scores than the undefined group, G1, in value and self-efficacy. On the other hand, group GI obtained higher scores in the cost scale. The effect size, represented by the parameter r, is strong in the case of value and self-efficacy and moderate in the case of cost, the index being r =.80 in the first two cases and r=.28 in the case of cost.

Discussion

The results obtained in this research regarding motivation in the study of mathematics can be divided into two blocks: one concerning the differences between the algebraic and mixed category groups and the another concerning these two groups and the non-defined group.

Students in the algebraic group obtained higher scores than the mixed group in the value and self-efficacy scales. Students who solve problems algebraically attach a higher value to the mathematical task in the sense that they find it more interesting and assign it a greater utility and importance. In addition they consider themselves to have a higher capacity to tackle problems. This is an interesting and significant result in that previous research has not related the procedure adopted in problem solving with motivation. The cost involved is the only scale in which there is no significant difference between the algebraic and the mixed category groups. It follows that the group which uses both algebraic and arithmetic procedures does not perceive a greater effort involved in the learning of mathematics than the group which solves problems using exclusively algebraic strategies.

In light of these results, the data presented here support the thesis that students who use the algebraic method in problem solving are more motivated in learning mathematics, at least in so far as the value- expectancy motivational model is concerned. Reciprocally, it is possible that a higher motivation in mathematics provides students with an incentive to use more exhaustive and abstract mathematical strategies. This implies that the practice and mastery of algebraic techniques in Secondary Education may have wide-spreading and relevant implications for education that go far beyond mere improvement in the capacity to solve problems.

The second result obtained is that related to the differences between the undefined group and the algebraic and mixed groups. These latter groups obtain relevantly higher scores than the non-defined group in value and self-efficacy. In addition, the students of both these groups assign an inferior cost to learning mathematics than do their counterparts in the non-defined group. There is a clear contrast between those who solve or approach correctly all the problems (the algebraic and mixed groups) and those who fail to do so. It seems likely then that the principal reason for the difference between them lies precisely in their academic achievement in mathematics.

In today's information technology society, performance in mathematics conditions the ability to solve complex situations and the conceptual abilities which demand verbal ability (Sciarra and Seirup, 2008). As was pointed out in the introduction, this ability, which is acquired mainly through competence in problem solving, promotes verbal

reasoning and constitutes the basis of a multitude of curriculums and programmes of study throughout the world. In this sense, various reasons have been found for focussing on mathematical achievement in educational research. Various studies point out that, as distinct from other areas of study, the learning of mathematics takes place in school and for this reason is particularly sensitive to forms of instruction (Burkam, Ready, Lee and LoGerfo, 2004; Porter, 1989).

To return to questions of motivation, positive correlations were found between self-efficacy and higher levels of academic achievement in the area of mathematics (Schunk and Pajares, 2002; Zimmerman and Martínez-Pons, 1990). Specifically, Pajares (1996) associates a greater perceived competence in mathematical problem solving with higher performance in this area. In addition, students work harder and are more motivated to try out more and more difficult tasks when they believe that they have the ability to do so successfully (Bandura, 1994; Covington, 1984; Weiner, 1985).

Work carried out by Eccles, Wigfield and their collaborators, among others, have emphasised the specific contribution of self-efficacy expectations and task-value to success in various academic subjects, including mathematics. They also point out the predictive capacity of this motivational model. (Eccles, 1987, Eccles, Adler and Meece, 1984; Eccles, Wigfield, Harold and Blumenfeld, 1993; Wigfield, Eccles, MacIver, Reuman and Midgley, 1991).

The cost scale, though it has been little studied, represents a significant variable in students' academic performance. The belief that the cost associated with a given behaviour may be very high can often lead to its not being carried out. On the other hand, the more the subjective cost of a given task increases, the more its net value diminishes (Wigfield and Eccles, 1992, 2000; Eccles and Wigfield, 2002).

In light of the results obtained, it can be affirmed that students who solve arithmetic-algebraic word problems through the algebraic method stand out for their high degree of motivation in mathematics as an academic subject, specifically in their task-valuation and in their expectations of self-efficacy.

Finally, the limitations of the present study cannot be obviated. Given that this is the first time that a Spanish version of this questionnaire has been applied, it would be of great interest to repeat this trial and test it out on a more extensive sample of students.

This would help to ascertain how it applies in educational systems which employ Spanish as the language of instruction. It should not be forgotten that the application of questionnaires can in itself partly distort the data obtained. The results concerning motivation in mathematics were obtained from data in self-reports, with the risks that this implies as to their credibility. Further research would need to make use of personal interviews and/or specific activities carried out in the classroom so as to measure more exactly the influence of this psycho-instructional variable.

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Appendices

Appendix 1:

Motivation questionnaire (Berger, & Karabenick, 2011):

Task Value

Interest

1. I like math.

2. I enjoy doing math.

3. Math is exciting to me.

Importance

1. It is important to me to be the kind of person who is good at math.

2. I believe that being good at math is an important part of who I am.

3. It is important to me to be a person who can reason using math formulas and operations.

Utility

1. I believe that math is valuable because it will help me in the future.

2. I believe that math will be useful for me later in life.

3. I believe that being good at math will be useful when I get a job or go to college.

Cost

1. I have to give up a lot to do well in math.

2. I believe that success in math requires that I give up other activities that I enjoy.

Expectancy

Self-efficacy

1. I believe I will receive an excellent grade in math.

2. I'm certain I can understand the most difficult material presented in math.

3. I'm confident I can learn the basic concepts taught in math.

Appendix 2:

Mathematical word problems (Stacey, & MacGregor, 2000)

1. Some money is shared between Mark and Jan so that Mark gets $5 more than Jane gets. The money to be shared is $47. How much money do get Mark and Jane?

2. A bus took students on a 3-day tour. The distance traveled on Day 2 was 85 Km farther than on Day 1. The distance traveled on Day 3 was 125 Km farther than on Day 1. The total distance was 1410 Km. How much distance does travel each day?

3. I think of a number, multiply it by 8, subtract 3, and then divide by 3. The result is twice the number I first thought of. What was the number?