Early Childhood Educ J (2013) 41:355–364
DOI 10.1007/s10643-013-0580-y

Text

Analysis of Place Value Instruction and Development in Pre-Kindergarten Mathematics

Patrick McGuire · Mable B. Kinzie

Published online: 14 February 2013

Abstract Development of two-digit place value understanding in the elementary grades has been the subject of some study; however, research at the pre-kindergarten (Pre-K) level is limited. This two-part paper begins by providing an overview of two-digit place value instruction in Pre-K and describes the component parts of a research-based math curriculum, *MyTeachingPartner Math (MTP Math)*. Part two presents the results of a video analysis of classroom interactions across four *MTP Math* place value activities facilitated by two high quality teachers. Particular attention is given to the primary conceptual hurdles faced by students, as well as the scaffolding strategies employed by teachers. Results indicate that students possess a conceptual understanding of the ones place prior to the tens place and initially struggle the concept of unitizing groups of ten. Considerations are discussed for improving the quality of teacher-child interactions in pre-kindergarten that can best support children's thinking and learning.

Keywords Early childhood education · Place value · Teaching and learning

P. McGuire (✉)
College of Education, University of Colorado at Colorado Springs, 1420 Austin Bluffs Parkway, Columbine Hall 3039, Colorado Springs, CO 80918, USA
e-mail: mcguire577@gmail.com; pmcguire@uccs.edu

M. B. Kinzie
Curry School of Education, University of Virginia, Ruffner Hall 238, 405 Emmet Street, Charlottesville, VA 22904-4261, USA
e-mail: kinzie@virginia.edu

Introduction

Experts are in collective agreement that place value instruction should be integrated in the elementary grades (National Council of Teachers of Mathematics 2000; National Research Council 2009) and that students' place value understanding serves as a prerequisite skill for later mathematics success. Although numerous studies have investigated the development of students' place value understanding in later elementary grades (Fuson and Briars 1990; Hiebert and Wearne 1992; Ross 1986), fewer studies have specifically investigated the development of two-digit place value skills in Pre-Kindergarten (Pre-K). The lack of place-value specific research at the earliest grade levels comes despite the call from the National Council of Teachers of Mathematics (NCTM 2000, in their Number and Operations Standards for Grades Pre-K-2) for students to "use multiple models to develop initial understandings of place value and the base-ten number system".

Most likely the lack of attention given to place value in Pre-K stems from concerns about the developmental appropriateness of targeting two-digit place value instruction in early mathematics. For example, Fuson (1990) has suggested that place value instruction should be integrated with multi-digit addition or subtraction operations and postponed until at least second grade. Others have suggested that a complete understanding of place value requires a working knowledge of part-whole number relationships (Cobb and Wheatley 1988; Fischer 1990; Ross 1986). Consequently, foundational place value activities and instruction are rarely provided or studied in Pre-Kindergarten classrooms. A possible consequence of the delayed exposure to even the most basic place value instruction can perhaps be seen in the misunderstandings evident for early and middle elementary students, particularly when students attempt multi-digit addition and subtraction (Fuson 1990) and multiplication and division of two-digit numbers (Murata 2004). These difficulties have been shown to persist late as fifth or sixth grade for some students (Kamii 1986) and are exacerbated for low-achieving populations (Ho and Cheng 1997). Consequently, a better understanding of how children develop two-digit place value concepts at the earliest levels (prior to first grade), as well as the difficulties and misconceptions they encounter when learning these concepts, is of substantial potential value to the mathematics education community, though it is relatively unexplored to date. This emergent understanding and development of place value also has direct ramifications related to the newly adopted Common Core State Standards for Mathematics, standards that target place value development starting in Kindergarten.

Despite the fact that this paper investigates a topic that is traditionally reserved for later elementary grades, we are not advocating for this difficult concept to be pushed onto young children. Instead, we explore how children's understanding of place value begins to emerge in Pre-K and how it can be best supported by developmentally appropriate instructional techniques. This emergent understanding is best described by Carpenter and Lehrer's (1999) definition, one that suggests "understanding is not an all-or-none phenomenon… virtually all complex ideas or processes can be understood at a number of levels and in quite different ways" (p. 20). Consequently this paper does not explore or advocate for the implementation of more advanced place value concepts (i.e., understanding of computational algorithms or multi-digit addition and subtraction operations, scientific notation, or decimals), concepts that should be reserved for later elementary mathematics. Rather, this paper explores Pre-K students' *emergent* place value understanding, analyzing the basic knowledge of the component parts of two-digit numerals (tens vs. ones) and the ability to operate with groups of ten and remainders, or "leftovers".

Two research questions drove this study:

1. What are the different barriers associated with Pre-K students' initial understanding of place value AND
2. How do effective Pre-K teachers scaffold students' emergent place value understanding?

Method

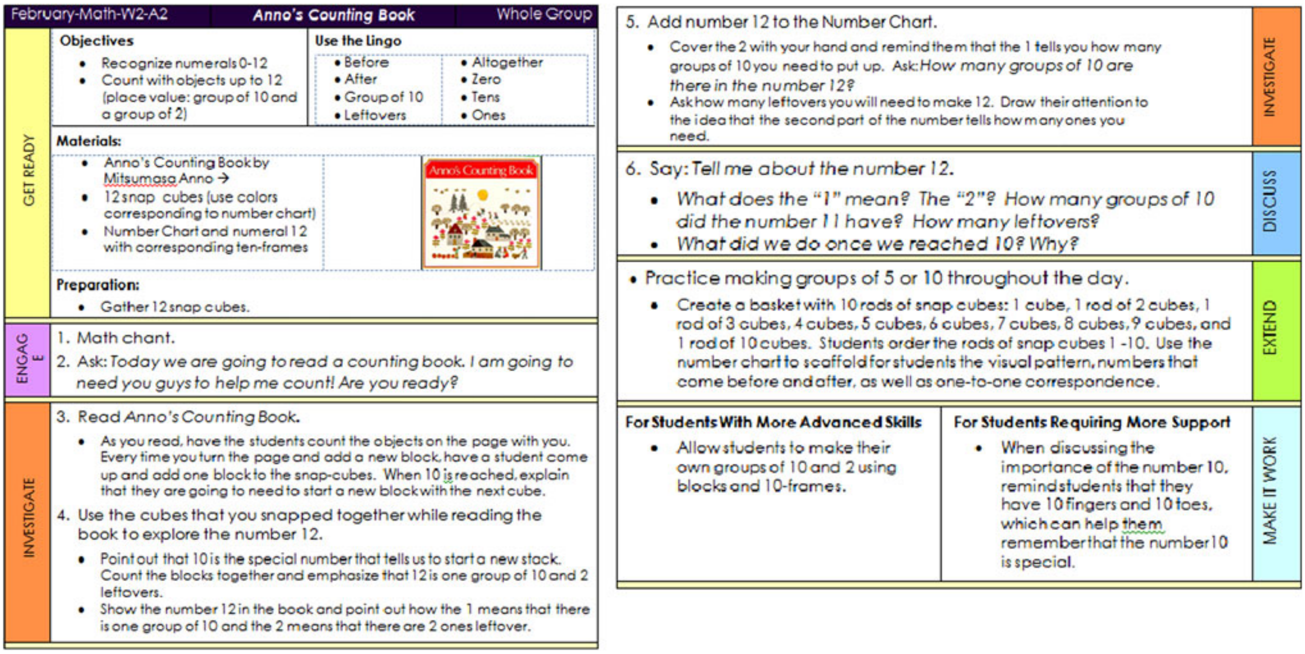
Setting

This study involved an analysis of teacher-child interactions in two Pre-K classrooms participating in a state-wide initiative serving children who may be at-risk for difficulty in school. Both classrooms were part of public school districts located near a medium-sized mid-Atlantic city and were chosen from eight classrooms participating in pilot testing of the *MyTeachingPartner Math-Science* curricula. Classroom One included 16 students (seven boys and nine girls); nine were Black/African American students, four were Hispanic, one was white, and two were students of mixed race. All children were between the ages of four and five. Two students in this classroom had Individualized Education Plans (IEPs). This classroom included areas for computer use, block play, reading, listening, and dramatic play; however, it did not contain an area specifically designed for students to explore mathematics with math-related manipulatives. Classroom Two included 16 students (six boys and ten girls). Of these, eleven were white, four were Black/African American, and one was of mixed race. All children were between the ages of four and five. This classroom also contained two students with IEPs. The teacher of Classroom Two was also a Caucasian female in her late 30's with a bachelor's degree in education and an additional year of post-graduate work in education.

MTP Math Curricular Overview

The *MyTeachingPartner Math* (*MTP Math*) curriculum is a subset of a larger Pre-K curriculum, *MyTeachingPartner Math-Science*. For the purposes of this paper, however, only the components of the curriculum directly related to place value development are discussed. Of the 65 total *MTP Math* activities, five activities include learning objectives directly related to two-digit place value (see Fig. 1 for a sample activity and Appendix for a full description of all five *MTP-Math* place value activities). The learning objectives for these activities are based on the multiple representations framework, and as previous researchers have suggested, rest heavily on children's ability to make connections between the quantification of objects in groups of ten and treating these groups of ten as single units (Fosnot and Dolk 2001; Fuson 1990). As suggested by the National Research Council (NRC 2009), "to begin to understand the base-ten place value system, children must be able to view ten ones as forming a single unit of ten" (p. 45), in other words, children must be able to "unitize" (Fosnot and Dolk 2001). Therefore all *MTP Math* place value-related instruction is systematically anchored around the unit of ten, a benchmark number in children's mathematical development (Copley 2000; Van de Walle 2003) using the *MTP Math Number Chart,* which is described in detail below.

The *MTP Math* two-digit place value instructional framework is predicated upon the first two stages of the concrete-representational-abstract instructional sequence of mathematics, an instructional strategy that is considered to



February-Math-W2-A2 **Anno's Counting Book** Whole Group

GET READY

Objectives
- Recognize numerals 0-12
- Count with objects up to 12 (place value: group of 10 and a group of 2)

Use the Lingo
- Before
- After
- Group of 10
- Leftovers
- Altogether
- Zero
- Tens
- Ones

Materials:
- Anno's Counting Book by Mitsumasa Anno →
- 12 snap cubes (use colors corresponding to number chart)
- Number Chart and numeral 12 with corresponding ten-frames

Anno's Counting Book

Preparation:
- Gather 12 snap cubes.

ENGAGE

1. Math chant.
2. Ask: *Today we are going to read a counting book. I am going to need you guys to help me count! Are you ready?*

INVESTIGATE

3. Read *Anno's Counting Book.*
   - As you read, have the students count the objects on the page with you. Every time you turn the page and add a new block, have a student come up and add one block to the snap-cubes. When 10 is reached, explain that they are going to need to start a new block with the next cube.
4. Use the cubes that you snapped together while reading the book to explore the number 12.
   - Point out that 10 is the special number that tells us to start a new stack. Count the blocks together and emphasize that 12 is one group of 10 and 2 leftovers.
   - Show the number 12 in the book and point out how the 1 means that there is one group of 10 and the 2 means that there are 2 ones leftover.

INVESTIGATE

5. Add number 12 to the Number Chart.
   - Cover the 2 with your hand and remind them that the 1 tells you how many groups of 10 you need to put up. Ask: *How many groups of 10 are there in the number 12?*
   - Ask how many leftovers you will need to make 12. Draw their attention to the idea that the second part of the number tells how many ones you need.

DISCUSS

6. Say: *Tell me about the number 12.*
   - *What does the "1" mean? The "2"? How many groups of 10 did the number 11 have? How many leftovers?*
   - *What did we do once we reached 10? Why?*

EXTEND

- Practice making groups of 5 or 10 throughout the day.
  - Create a basket with 10 rods of snap cubes: 1 cube, 1 rod of 2 cubes, 1 rod of 3 cubes, 4 cubes, 5 cubes, 6 cubes, 7 cubes, 8 cubes, 9 cubes, and 1 rod of 10 cubes. Students order the rods of snap cubes 1 -10. Use the number chart to scaffold for students the visual pattern, numbers that come before and after, as well as one-to-one correspondence.

MAKE IT WORK

For Students With More Advanced Skills	**For Students Requiring More Support**
• Allow students to make their own groups of 10 and 2 using blocks and 10-frames.	• When discussing the importance of the number 10, remind students that they have 10 fingers and 10 toes, which can help them remember that the number 10 is special.

Fig. 1 Sample MTP-math place value activity: Anno's counting book

be developmentally appropriate for all grade levels and has achieved the Access Center Research Continuum's highest ranking of "evidence-based research" (Access Center 2004). For example, this instructional sequence has been shown to be effective for mathematics instruction involving place value with students with learning disabilities (Peterson et al. 1988) and to remediate deficits involving subtraction and regrouping for at-risk elementary student populations (Flores 2010). This instructional strategy is also heavily influenced by the NCTM process standard which suggests that students should be exposed to multiple representations (NCTM 2000).

MTP Math Number Chart

The *MTP Math Number Chart* (Fig. 2) is based on a similar chart used in the *Big Math for Little Kids* preschool mathematics curriculum (Ginsburg et al. 2003) in that 0 and related numerals (10, 20, and 30) occupy the left-most column in the table, and 9 and numbers related to 9 (9, 19, 29, 39) appear in the right-most column. To this design, color-coded ten-frames were added. These frames were adapted from physical base-ten blocks developed by Dienes (1960) to include a base-ten representation of each number for the numbers 0 through 39. Both numerals and the corresponding ten-frame representations contained on the chart are systematically color-coded (blue for the tens place, orange for the ones place) to help students make connections between the relative magnitude of numbers (quantitative value) to the written symbols (positional digits) that they represent as well as support children's ability to accept the different equivalences of numerical representations (Ainsworth 1999). Further, the color-coded ten-frame representations in the *MTP Math Number Chart* systematically assist students in establishing connections between Arabic numerals (e.g., "15"), verbal number names (e.g., spoken word "fifteen") and concrete representations (See Fig. 3). This exercise in connecting the three numerical representations is consistent with an extensive literature suggesting that early mathematics instruction should involve opportunities to build connections among numbers and representations (Baroody 1990; Clement 2004; Fuson 1990; Hiebert and Wearne 1992; NCTM 2000) as well as research suggesting that students as young as five or six have a high competence for representing mathematical concepts if supported by a high quality classroom community (diSessa et la. 1991).

In addition to helping students make connections among different representations of numbers (e.g., verbal, Arabic numeral, and pictorial), the *MTP Math Number Chart* is specifically designed to scaffold students in their understanding of the part-to-whole relationships within numbers (see Fig. 4), relationships that have been demonstrated to be foundational to place value understanding (Fischer 1990; Hunting 2003) as well as underlie the mathematical operations of addition and subtraction (Copley 2000). As part of its numeric and pictorial representations, the *MTP Math Number Chart* displays the component parts of each number in terms of groups of tens and ones or "leftovers" (e.g., 24 is 2 groups of 10 and 4 "leftovers").

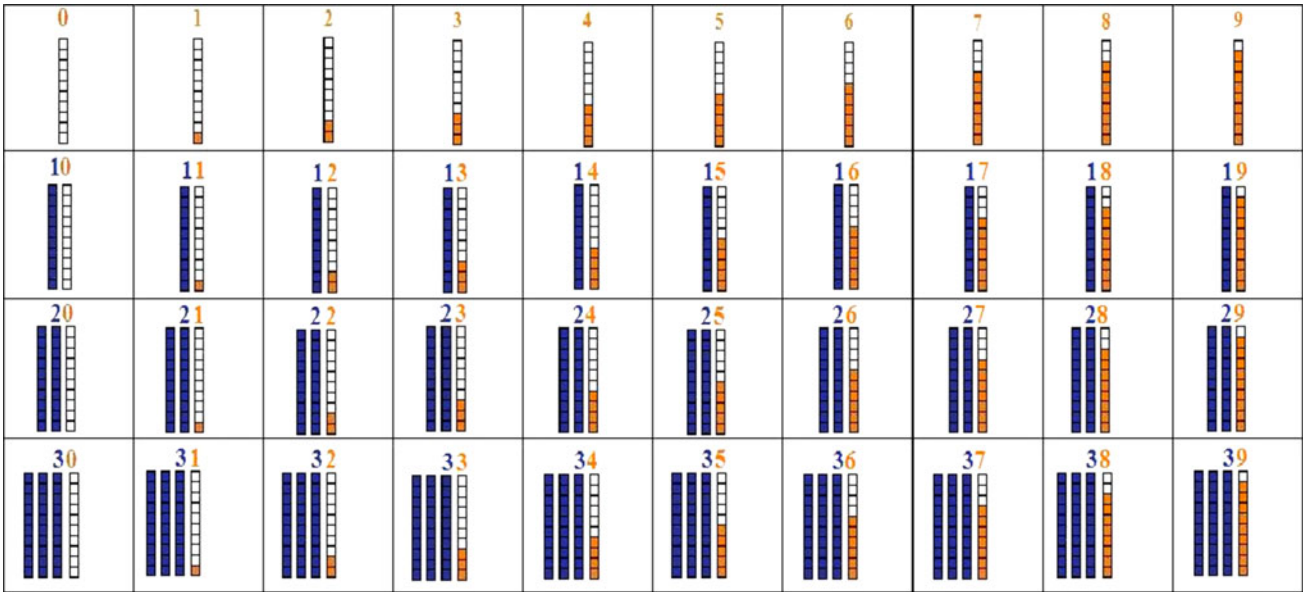


Fig. 2 MTP-math number chart with ten-frame representations

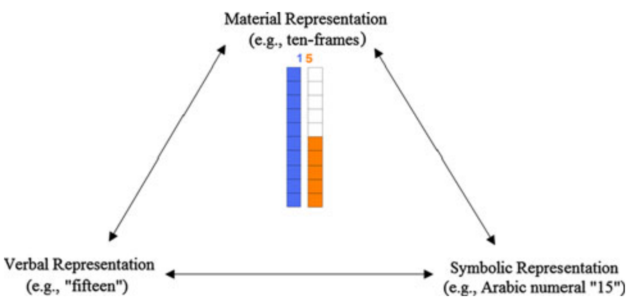


Fig. 3 The three basic numerical representations of the number 15

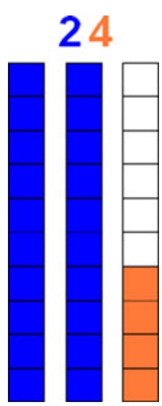


Fig. 4 Part-to-whole relationship of the number 24; two groups of ten and four leftovers (ones)

Selection of Effective Teachers

To select the classrooms examined in this study, a purposeful selection was employed to identify the two highest quality teachers from the eight participating in the curricular pilot testing. The rationale for selecting only the highest quality teachers is that two-digit place value is a challenging mathematics concept to teach in Pre-K. We wanted our video analysis to constitute the highest possible quality of teacher-child interactions. Teaching quality was assessed using both the Classroom Assessment Scoring System (*CLASS*; Pianta et al. 2008), as well as a subset of items chosen from a math-specific observational measure, Classroom Observation of Early Mathematics—Environment and Teaching (*COEMET*; Sarama and Clements 2007) in a separate study, and those same data were used to select two highest quality teachers for this study.

Procedures

All two-digit place value activities described in this analysis were videotaped in their entirety, enabling us to analyze the relationships between teacher practice and student involvement across the entire duration of the activity. An analysis was conducted on the implementation of four out of the five *MTP Math* place value activities across the two effective teachers (note that neither teacher recorded videotape for the *How Many Hands* activity), and therefore it was impossible to analyze the teacher-child interactions for this specific lesson). In total, nearly 120 min of *MTP Math* two-digit place video data were analyzed (76 min extracted from Teacher One and 41 min from Teacher Two).

The specific video analysis procedure used in this study was adapted from that advanced by Powell et al. (2003). This model was developed after extensive research related to theories of developing mental representations and ideas about students' mathematical understanding and was specifically designed to examine interactions involving the development of students' mathematical thinking using videotaped data. The process involves (1) attentively viewing video, (2) identifying and transcribing critical events, and (3) extracting key themes.

To ensure that we effectively captured the full depth of the interactions and discussions surrounding two-digit place value, we identified teacher-child interactions occurring at any point in the lesson as critical events; including those directly involving the *MTP Math Number Chart*, use of concrete or semi-concrete ten-frame representations, or conversations within an activity related to the positional parts of two-digit numerals. Finally, instances where student utterances indicated incorrect understandings of two-digit place value were noted, as Powell's model suggests the importance of examining students' conflicting schemes, naïve generalizations, or incorrect or erroneous applications of logic that are directly relevant to the research agenda. In total, 43 critical events (74.5 min) were identified from the video data. After specific sections of tape were identified as critical events, these interactions were transcribed in their entirety, enabling themes to emerge above and beyond those suggested by "specific, a priori guiding research questions" (Powell et al. 2003, p. 422).

Codes

The codes that emerged from our review process were split into two categories to reflect the original research questions. The first coding category dealt exclusively with all interactions involving *student understanding* of mathematical concepts related to place value. The second coding category includes codes related to *teacher scaffolding* (i.e., instructional strategies). See Table 1 for a full description of the emergent place value codes.

Results

Our qualitative video data and corresponding transcriptions, notes, and specific analysis of codes collected throughout the study revealed specific factors related to students' conceptual understanding of place value and teachers' ability to provide appropriate scaffolding. Several transcriptions from critical events are included to document the language used by both teachers and students, as well as provide an authentic context for interpreting teacher-child interactions.

Students' Two-Digit Place Value Understanding

Our results suggest that students were able to quickly and accurately respond to questions involving leftovers (e.g., *"How many leftovers are there in 13?")* with little or no teacher scaffolding. Students demonstrated this understanding as early as the first *MTP Math* activity when it was introduced with the two-digit numbers 10 and 11. We believe students were able to quickly grasp the idea of leftovers, because from a conceptual standpoint, identifying leftovers as single units is consistent with the exercise of building ten-frame representations for one-digit numbers. In other words, students had essentially been reviewing the idea of leftovers since early September, and therefore were already learning to coordinate the prerequisite number sense skills (e.g., one-to-one correspondence and cardinality) required to identify leftovers in ten-frame representations of two-digit numbers.

Challenges with Unitizing

Despite students' ability to answer questions related to leftovers early on in the sequence of *MTP Math* two-digit place value activities, related understanding of the tens place developed at a slower pace for students. Based on multiple transcriptions from critical events, particularly those extracted from the first two *MTP Math* place value activities in the instructional sequence, it was clear that students required extensive teacher support to answer questions related to the tens place or groups of ten (e.g., *How many groups of ten are there in 13*?) We hypothesize that the primary rationale for this early struggle stems from students inability to unitize, in other words, the ability to identify, make, and count groups of ten. Fosnot and Dolk (2001) outline the importance of, as well as challenges faced with, unitizing in early childhood mathematics:

> Unitizing underlies the understanding of place value; ten objects becomes one ten. Unitizing requires that children use number to count not only objects but also groups – and to count them both simultaneously. The whole is thus seen as a group of a number of objects. The parts together become the new whole, and the parts (the objects in the group) and the whole (the group) can be considered simultaneously. For learners, this is a shift in perspective. Children have just learned to count ten objects, one by one. Unitizing these *ten* things as *one* thing – one group – requires almost negating their original idea of number (p. 11).

The following excerpt (see Transcription 1) from a critical event taken from the first *MTP Math* two-digit place value activity, *Number Chart 10 & 11*, reflects students' initial understandings of the positional parts of two-digit numbers as well as the need for teacher scaffolding to help students correctly answer the question related to unitizing. In this interaction, Student One appears to be confusing the concepts of cardinality (e.g., identifying the total number of objects in a set) and unitizing (e.g., counting groups of ten). This was a common misconception displayed by students throughout multiple interactions involving unitizing.

Students' Minimal Language Use

Across all critical events, the majority of teacher-child interactions were characterized by minimal amounts of student language (i.e., students rarely responded with more than one or two word statements). The following excerpt reflects a common theme of students' often short and incomplete answers to teachers' questions about place value. In this teacher-child interaction, the teacher is encouraging a student to match the color-coded Arabic numeral for the number 18 to the corresponding ten-frame visual representation of the quantity of 18 on the *MTP Math Number Chart*.

In this dialogue, Marcus is able to quickly and correctly match the numeral 18 to the corresponding ten-frame representation; however, when asked to describe his thinking he is unable to elaborate with more than "cuz". The minimal amount of student language, typical of other responses across our video transcriptions, can perhaps be explained by several factors. First, because students are just learning about two-digit place value concepts they possess only a nascent conceptual understanding, an understanding that could be categorized as instrumental rather than relational (Skemp 1978). In other words, although this student understands conceptually the relationship between the numeral 18 and the corresponding ten-frame representation; he is still developing a relational understanding (i.e., knowing the rules and why they work) required to fully describe the match. Related to this factor, limited knowledge may also be a function of the well-documented, slower rate at which language skills develop in students at risk. Research suggests that lower income children lag behind their peers in basic language skills with knowledge gaps evident as early as age two (Hoff 2003); therefore, students may simply not yet possess the vocabulary to describe their thinking. Finally, the minimal amounts of student talk could perhaps be attributed to the type of questioning strategies typically employed by the teacher. In the following section, we more thoroughly describe teachers' scaffolding practices and specifically consider how their questioning strategies affect students' language.

Teachers' Scaffolding

Leading Questions Strategy

Video analysis and critical event transcriptions indicated a pattern of questioning employed by teachers across the interactions, one that included predominantly closed-ended *leading questions*. This type of questioning sequence, as summarized by Franke et al. (2009), is one where the teacher directly guides students to particular answers or explanations and is consistent with the questioning technique of "funneling" described by Wood (1998). With this technique, the teacher assumes much of the mathematical legwork while explicitly scaffolding students' thinking and guiding them through the correct explanation of challenging or new concepts.

Despite the fact that most discussions were dominated by teacher-talk, we assert that for a new and challenging concept such as two-digit place value, the technique of asking leading questions can serve as an effective instructional scaffold. This assertion is consistent with previous research suggesting that the amount of "math talk" provided by teachers is significantly related to the growth of Pre-K children's mathematical knowledge (Klibanoff et al. 2006). Moreover, literature in the area of children's language acquisition and vocabulary development advocates for a more teacher-centered instructional approach, especially in the earliest years of schooling (Biemiller 2001). Therefore, in a mathematical context, providing students with opportunities to engage in challenging math talk supported by leading questions may be fundamentally important to students' mathematical and vocabulary development despite the fact that it results in minimal student talk.

In the following two subsections we describe two specific strategies employed by teachers throughout the observations. Transcription 2 displays the teacher's patterns of leading questions, in other words, the teacher is the one assuming the majority of the mathematical legwork while scaffolding students' thinking.

Emphasis on Cardinality with Direct Modeling

A common strategy displayed by teachers was to use physical objects (typically snap cubes) to perform "cardinality checks". In other words, teachers explicitly connected the meaning of one- and two-digit Arabic numerals to the quantities that they represent. In the majority of instances involving a cardinality check with physical objects the teachers explicitly pointed to each individual object and asked students to join in a group count. This strategy is consistent with research suggesting that specifically pointing to the object in the count helps children to maintain a more accurate one-to-one correspondence and coordinate number words with counted objects at a greater rate of success (Alibali and DiRusso 1999). In virtually all cardinality checks involving a set of objects the teacher concluded by asking students a question such as: *How many are altogether?*

For related cardinality checks with two-digit quantities, both teachers leveraged physical snap cube representations to assist students in their ability to "count on" from the number ten. Counting on, in contrast to "counting all", involves beginning a count from a number other than one, and has been suggested to help promote preschool students' numerical flexibility (Baroody 2009) as well as underlie early elementary students' initial processes for simple addition and subtraction problems (Carpenter et al. 1981). In the event below (Transcription 3), the teacher actively models a counting on strategy to demonstrate a more efficient counting strategy.

Dynamic Construction of Two-Digit Arabic Numerals

During several instances teachers used a dry erase board to dynamically construct two-digit Arabic representations. Transcription 3 illustrates a teachers' modeling of how to dynamically write/build the number 11. Using this strategy, the teachers were able to effectively focus students' attention on the positional parts of the numbers (e.g., the first number represents how many groups of ten, and the second number representing the amount of leftovers) and how they related to the part-to-whole relationship of two-digit numbers. This strategy of dynamically writing the numbers on the dry erase board not only assisted students in their ability to answer questions about the constituent parts of two-digit numerals, but also resulted in certain students being able to predict how to build larger numbers.

Discussion

Unlike previous research efforts in later elementary grades, this study provides a window into students' earliest exposure to two-digit place value and describes how effective teachers scaffold students' emerging understanding. Our findings indicate that students experience few problems grasping the concept of leftovers, however, initially struggle with the concept of unitizing. We found that students, if properly scaffolded by teachers, were able to actively engage in complicated mathematics exercises such as counting on from ten, matching Arabic numeral representations to corresponding ten-frames, and identify the positional parts of two-digit numbers. Despite students' ability to engage in these activities, our results point to minimal language use and a lack of ability to explain their thinking.

From a teacher perspective, our findings suggest that the two effective teachers were able to best facilitate the two-digit place value activities by leveraging multiple numerical representations using leading questions to direct students' thinking. More specifically, teachers used numerical representations to model cardinality checks and demonstrate mathematical strategies such as counting on from ten. Related to these instructional strategies, it was clear that the teachers were responsible for assuming much of the mathematical legwork and dictating the teacher-child interactions. This finding has particular implications for Pre-K mathematics instruction involving challenging mathematics content, as it suggests that in order for teachers to employ this strategy they must possess both the pedagogical content in mathematics knowledge, as well as the ability to interpret the level of students' conceptual understanding. To help better guide practicing Pre-K teachers to better implement place value specific concepts with their students, a set of teaching tips (see Table 1).

Limitations

There are three primary limitations related in the study reported here. First, the video analysis involved only two publicly funded Pre-K classrooms comprised entirely of students identified as at risk of academic failure. Therefore it is not clear if the results of the teacher-interactions documented in this study can be generalized to different types of preschool environments or those that serve relatively advantaged student populations. Second, our analysis involved only effective, credentialed teachers with several years of experience in early childhood education. Research on place-value instruction suggests that both full-time and pre-service elementary teachers (McClain 2003) face challenges in encouraging the development of place value, and therefore we must also consider the practices of less skilled and/or less experienced teachers. Third, our study analyzed video data drawn only from whole group activity settings. Therefore it is possible that the quality of the teacher-child interactions may differ in smaller group settings, particularly in students' use of language and teachers' questioning techniques, because there are more opportunities for one-on-one interaction.

Recommendations for Future Research

Future research efforts must also assess the extent to which Pre-K students can acquire two-digit place value knowledge by directly measuring student achievement. Our results imply that it may be particularly useful to more carefully explore the developmental sequence and skills required to understand the concept of unitizing. For example, examining the relationship between children's early exposure to two-digit place value concepts to predict later success in more complicated place value concepts such as multi-digit addition/subtraction or regrouping. An interesting extension of this study would be to expand the ten-frame representations included in the *MTP Math Number Chart* to three-digit numbers with three different colors, to help students in kindergarten or first grade extend the concepts of place value and unitizing to groups of 100 or more. This would provide more longitudinal evidence about the evolution of students' place value understanding over time. Further, longitudinal data could perhaps indicate the potential of concrete and semi-concrete external representations and the ability to generalize the effective instructional strategies outlined in this paper to teachers across different elementary grade levels. By carefully unpacking what Pre-K children are capable of learning about place value and related prerequisite skills, this will add to the already existent knowledge base of how place value development is multifaceted and acquired over time (Fuson 1990; Ross 1986).

In addition, it is crucial that future research efforts focus on ways to not only support Pre-K students' development, but Pre-K teachers' ability to introduce place value in a meaningful, developmentally appropriate manner. Simply providing classrooms with access to a research-based mathematics curriculum and related instructional manipulatives will not automatically result in high quality teacher-child interactions in mathematics (Ball 1992). Subsequently, future research efforts that provide targeted professional development would support Pre-K teachers in their ability to guide students' thinking through mathematical discussions and manipulative-based activities will be necessary to help more teachers improve their confidence in teaching concepts that are not traditionally introduced at the Pre-K grade level. This work could also directly fill a gap in the Common Core State Standards for Mathematics, a standards set that does not exist below the Kindergarten grade level. Giving careful consideration to the earliest developmental stages will help to identify best practices which will prepare teachers to facilitate conceptually-based mathematics understanding in Pre-K classrooms.

Appendix

See Table 1.

Table 1 MTP math activities description and place value teaching tips

Activity	Activity objective(s)	Overview	Teaching tips for activity
Jan week 4 *Number chart 10 and 11*	Make and identify groups of ten Recognize numerals 10 and 11	Students create groups of ten by connecting individual snap cubes. After creating the physical representations, students review the abstract representations for the quantities of 10 and 11 on the *MTP Math Number Chart.* This lesson provides students with the first introduction to two-digit numbers and encourages them to create groups of ten with individual snap cubes. Students have the opportunity to practice the fundamentally important "exchange principle", e.g., ten ones is the same as one group of ten (Copley 2000) and explore the concept of "leftovers"	Remind students that 10 is a special number in math. Provide students with opportunities to make and count groups of 10 because this is a foundational place value skill
Feb week 4 *Anno's counting book*	Recognize numerals 0–12 Count with objects up to 12 (place value: groups of 10 and 2)	Following the reading of *Anno's Counting Book* (by Mitsumasa Anno), students use individual snap cubes to build up to the number 12. Along the way the discussion focuses on groups of ten and connections are made to the *MTP Math Number Chart* for both one- and two-digit numbers. This activity provides students with a physical representation of how two-digit numbers are created, and emphasizes the place value concepts of groups of ten and leftovers ones	Anno's counting book offers an excellent opportunity for children to make connections between the three different number formats (for example, the spoken word "three", the Arabic numeral "3" and the snap cube representation of 3 boxes filled in a 10-frame (See Fig. 3 above for more information)

Table 1 continued

Activity	Activity objective(s)	Overview	Teaching tips for activity
March week 1 *Building numbers*	Recognize numerals 0–15 Count with a running start 2–15 Count on from 2 to 15 Identify the number that comes before and after up to 14	Students work with physical snap cube representations as well as review the *MTP Math Number Chart* and ten-frame representations for the numbers 13, 14, and 15. Emphasis is placed on observing patterns in the rows and columns of the chart. Students work with the teacher to add new numbers to the chart, making predictions about what number will come next and how many ten-frames/leftovers each number will have	Provide students opportunities to compare two-digit numbers using visual representations in the Number Chart. Emphasize that each two digit number in this activity has one set of ten (blue block) and a differing amount of ones (orange blocks)
Apr week 3 *How many hands?*	Recognize patterns in the *number chart*	This activity closely relates to the patterns discussed in a previous activity (Mar Week 1). Students count the hands of students in their small group, and find that amount on the *MTP Math Number Chart*. Extension questions for this activity focus on changing the number of hands in the set by ten (more or fewer) and what patterns can be observed on the *Number Chart* in relation to ten	This activity is designed to use the Number Chart to scaffold students' understanding of 2-digit numbers by making connections to the first number (groups of 10) and second number (leftover 1's). This builds the foundation for later understanding of place value
May week 1 *Number chart matching game*	Recognize numbers 0–39 Recognize patterns in the *number chart*	This lesson represents the culminating place value and number recognition activity for the year. Students play a number matching game, matching numerals 0–39 to the corresponding ten-frame representations on the *Number Chart*. The discussion section within this activity provides a two-digit place value summary. Students practice counting groups of ten, identifying leftovers, and describing the positional meaning of two-digit numerals	In this activity it is important to emphasize the meaning of the parts of two-digit numbers; for example: in the number 23 the "2" represents two groups of 10 and the "3" represents three leftovers

References

Access Center. (2004). Concrete-representational-abstract instructional approach. *The access center* (pp. 1–6). Retrieved September 24, 2012 from http://www.k8accesscenter.org/training_resources/documents/CRAApplicationFinal_000.pdf.

Ainsworth, S. (1999). The functions of multiple representations. *Computers & Education, 33*, 131–152.

Alibali, M. W., & Dirusso, A. A. (1999). The function of gesture in learning to count: More than keeping track. *Cognitive Development, 14*, 37–56.

Ball, D. L. (1992). Magical hopes: Manipulatives and the reform of math education. *American Educator, 16*(2), 14–18, 46–47.

Baroody, A. J. (1990). How and when should place value concepts and skills be taught? *Journal for Research in Mathematics Education, 21*(4), 281–286.

Baroody, A. J. (2009). Fostering early numeracy in preschool and kindergarten. *Encyclopedia of language and literacy development* (pp. 1–9). London, ON: Canadian Language and Literacy Research Network. Retrieved June 24, 2012 from http://literacyencyclopedia.ca/pdfs/topic.php?topId=271.

Biemiller, A. (2001). Teaching vocabulary: Early, direct, and sequential. *The American Educator, 25*(1), 24–28.

Carpenter, T., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 19–32). Mahwah, NJ: Lawrence Erlbaum Associates.

Carpenter, T. P., Hiebert, J., & Moser, J. M. (1981). Problem structure and first-grade children's initial solution processes for simple addition and subtraction problems. *Journal for Research in Mathematics Education, 12*(1), 27–39.

Clement, L. L. (2004). A model for understanding, using, and connecting representations. *Teaching Children Mathematics, 11*(2), 97–102.

Cobb, P., & Wheatley, G. (1988). Children's initial understandings of ten. *Focus on Learning Problems in Mathematics, 10*(3), 1–28.

Copley, J. V. (2000). *The young child and mathematics*. Washington, DC: NAEYC.

Dienes, Z. P. (1960). *Building up mathematics*. London, England: Hutchinson Education.

diSessa, A., Hammer, D., Sherin, B., & Kolpakowski, T. (1991). Inventing graphing: Children's meta-representational expertise. *Journal of Mathematical Behavior, 10*(2), 117–160.

Fischer, F. E. (1990). A part–part-whole curriculum for teaching number in kindergarten. *Journal for Research in Mathematics Education, 21*(3), 207–215.

Flores, M. M. (2010). Using the concrete-representational-abstract sequence to teach subtraction with regrouping to students at risk for failure. *Remedial and Special Education, 31*(3), 195–207.

Fosnot, C. T., & Dolk, M. (2001). *Young mathematicians at work: Constructing number sense, addition, and subtraction*. Portsmouth, NH: Heinemann.

Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary classroom. *Journal of Teacher Education, 60*(4), 380–392.

Fuson, K. (1990). Conceptual structures for multiunit numbers: Implications for learning and teaching multidigit addition, subtraction, and place-value. *Cognition and Instruction, 7*(4), 343–403.

Fuson, K., & Briars, D. (1990). Using a base-ten blocks learning/teaching approach for first- and second-grade place value and multidigit addition and subtraction. *Journal for Research in Mathematics Education, 21*(3), 180–206.

Ginsburg, H. P., Greenes, C., & Balfanz, R. (2003). *Big math for little kids: Classroom set*. Parsippany, NJ: Dale Seymour Publications.

Hiebert, J., & Wearne, D. (1992). Links between teaching and learning place value with understanding in first grade. *Journal for Research in Mathematics Education, 23*(2), 98–122.

Ho, C. S., & Cheng, F. S. (1997). Training in place value concepts improves children's addition skills. *Contemporary Educational Psychology, 22*(4), 495–506.

Hoff, E. (2003). The specificity of environmental influence: Socioeconomic status affects early vocabulary development via maternal speech. *Child Development, 74*, 1368–1378.

Hunting, R. P. (2003). Part-whole number knowledge in preschool children. *Journal of Mathematical Behavior, 22*, 217–235.

Kamii, C. (1986). Place value: An explanation of its difficulty and educational implications for the primary grades. *Journal of Research in Childhood Education, 1*, 75–85.

Klibanoff, R. S., Levine, S. C., Huttenlocher, J., Vasilyeva, M., & Hedges, L. V. (2006). Preschool children's mathematical knowledge: The effect of teacher "Math Talk". *Developmental Psychology, 42*(1), 59–69.

McClain, K. (2003). Supporting preservice teachers' understanding of place value and multidigit arithmetic. *Mathematical Thinking and Learning, 5*(4), 281–306.

Murata, A. (2004). Paths to learning ten-structured understandings of teen sums: Addition solution methods of Japanese grade 1 students. *Cognition and Instruction, 22*(2), 185–218.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA. Retrieved April 7, 2012 from http://standards.nctm.org/document/chapter4/numb.htm.

National Research Council. (2009). Mathematics learning in early childhood: Paths toward excellence and equity. C. T. Cross, T. A. Woods, & H. Schweingruber (Eds.), *Committee on Early Childhood Mathematics, Center for Education. Division of Behavioral and Social Sciences in Education*. Washington, DC: The National Academies Press.

Peterson, S. K., Mercer, C. D., & O'Shea, L. (1988). Teaching learning disabled students place value using the concrete to abstract sequence. *Learning Disabilities Research, 4*, 52–56.

Pianta, R. C., La Paro, K. M., & Hamre, B. K. (2008). *The classroom assessment scoring system (CLASS)*. Baltimore, MD: Paul H. Brookes Publishing Co.

Powell, A. B., Francisco, J. M., & Maher, C. A. (2003). An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data. *Journal of Mathematical Behavior, 22*, 405–435.

Ross, S. H. (1986). *The development of children's place-value numeration concepts in grades two through five*. San Diego, CA: Paper presented at the Annual Meeting of the American Educational Research Association.

Sarama, J., & Clements, D. H. (2007). *Manual for classroom observation (COEMET)-version 3*. Unpublished version.

Skemp, R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher, 26*, 9–15.

Van de Walle, J, (2003). Developing early number concepts and number sense. In *Elementary and middle school mathematics: Teaching developmentally* (pp. 115–134). Boston, MA: Allyn & Bacon.

Wood, T. (1998). Funneling or focusing? Alternative patterns of communication in mathematics class. In H. Steinbring, M. G. Bartolini-Bussi, & A. Sierpinski (Eds.), *Language and communication in the mathematics classroom* (pp. 167–178). Reston, VA: National Council of Teachers of Mathematics.