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EXAMINING THE EFFECTS OF WRITING ON CONCEPTUAL AND PROCEDURAL KNOWLEDGE IN CALCULUS *

ABSTRACT. It has been claimed that writing to learn mathematics (WTLM) may benefit students' conceptual understanding as well as their procedural ability. To investigate this claim, we collected data from students in two sections of an introductory calculus course. In one of the sections, students used WTLM activities and discussed the activities after completing the writing; in the other section, students used similar activities that did not involve writing but engaged them in thinking about the mathematical ideas and in discussing the activities. The errors from the in-class and final exams of both groups of students were categorized and analyzed for information about the students' conceptual and procedural understanding. We found no significant differences between the WTLM group and the non-writing group, which suggests that the real benefit from writing activities may not be in the actual activity of writing, but rather in the fact that such activities require students to struggle to understand mathematical ideas well enough to communicate their understanding to others.

KEY WORDS: calculus, conceptual understanding, procedural understanding, writing, writing to learn mathematics

1. INTRODUCTION

It is not unusual to find students who use mathematical procedures with little or no understanding of the concepts behind these procedures (Hiebert and Lefevre, 1986; Schoenfeld, 1985). In fact, some students are not even aware that there are concepts underlying the procedures they use (Oaks, 1987/1988, 1990). Such students do not realize that there is meaning in mathematics. They believe that doing mathematics means performing pointless operations on meaningless symbols, and that everyone, including the teacher, learns mathematics by memorizing (Oaks, 1990).

Oaks (1987/1988, 1990) suggested that students' difficulties in doing mathematics can be related to their conceptions of mathematics (i.e., the ways in which the students view mathematics). Other research suggests that beliefs about mathematics can also influence the success of students in calculus and more advanced mathematics courses (Harel and Sowder,

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1998; Schoenfeld, 1985). A rote conception of mathematics can interfere with students' procedural ability. It can also prevent students from gaining an understanding of mathematical concepts. Both procedural ability and conceptual understanding are necessary for success in mathematics (Hiebert and Carpenter, 1992).

Some researchers have suggested (e.g., Brandau, 1990; Doherty, 1996; Miller, 1992; Pugalee, 1997; Rose, 1990) that one way in which students may be encouraged to see mathematics as meaningful is through the use of writing to learn mathematics (WTLM). Writing to learn mathematics can refer to any type of writing used to help students learn mathematics. The underlying assumption of WTLM is that "writing is not simply a way of expressing or displaying what one has learned. Writing is itself a fundamental mode of learning" (Stehney, 1990, p. 27). For the purpose of this study, we conceptualized WTLM as writing that involves articulating and explaining mathematical ideas for the purpose of deepening one's understanding.

In her ground-breaking study on expressive writing, Rose (1989) suggested that the perceived benefits of writing in mathematics could be divided into three general categories: (a) benefits for the student as writer, (b) benefits to the teacher as reader, and (c) benefits to the student-teacher interaction. One of the perceived benefits to students, according to the students in Rose's study, was that the writing helped them to understand the material (Rose, 1989). Others (Gopen and Smith, 1990; Nahrgang and Petersen, 1986; Pugalee, 1997) have also proposed that WTLM may improve students' conceptual understanding.

Much has been written praising WTLM, but a lack of comparative research in this area has meant that the claims of WTLM's benefits to student learning are made largely without concrete evidence (Morgan, 1998). A few comparative studies have been done, however, on some perceived benefits of WTLM. Two of these (Guckin, 1992; Youngberg, 1989/1990) studied the effect of WTLM on students' procedural ability. Both of these studies focused on college algebra, not calculus. Youngberg (1989/1990) found that the students in the WTLM class earned somewhat higher scores on their examinations, which were comprised of questions that tested algebra skills and procedural ability. Guckin (1992) found that the students who used writing achieved significantly higher scores on two of the three examinations, each of which was comprised of exercises requiring routine algebraic manipulations. One other comparative study has been done on a different perceived benefit of WTLM; Allen (1991/1992) investigated the effect of expressive writing on metacognitive skills. Her work suggested that this type of WTLM does promote students' ability to assess the correctness of their own work.

While it has been suggested that WTLM may offer a benefit to students' conceptual understanding, no studies have been done in which the conceptual understanding of students who used WTLM was compared with that of students who did not use WTLM. We address this need for comparative research in the current study.

There is also a need for a systematic method of examining the effects of a factor on conceptual and procedural knowledge. Several researchers (Heid, 1988; Judson, 1988, 1990; Palmiter, 1986, 1991) have examined the effects of a particular factor (e.g., using a computer algebra system) on conceptual and procedural knowledge, and, in order to carry out these investigations, each researcher chose one or several ways to measure conceptual and procedural knowledge. In all three of these studies, students' overall scores on the examinations or their scores on various problems on the examinations were used to measure conceptual and procedural knowledge; one study also used student interviews and analyzed error patterns for additional information about students' conceptual knowledge (Heid, 1988).

It is important to note that, in the Judson and Palmiter studies, the data used from students' examinations consisted of numerical scores, either from individual problems or from the examination as a whole. One problem with using scores like these to measure conceptual and procedural knowledge is that it is possible, perhaps even likely, that two students might earn the same score on an examination (or on a certain problem) but be very different in terms of their conceptual and procedural understanding of the material on the examination.

To avoid the problems associated with using numerical examination scores as measures of conceptual and procedural knowledge, we used error analysis, a form of measurement involving a content analysis of the students' work. Students can answer questions and solve problems correctly without necessarily understanding the material. Therefore, rather than looking at work that has been done correctly but perhaps without understanding, we tried to gain information about students' conceptual and procedural knowledge by examining the errors students make.

Recognizing that their errors can provide useful information about the students' knowledge, we examined students' errors as a means of investigating our research questions:

- Does writing to learn mathematics improve students' conceptual understanding?

- Does writing to learn mathematics affect students' ability to perform routine skills and procedures?

While we recognize that WTLM has a number of perceived benefits, in this study we focused only on the effects of WTLM on conceptual and procedural understanding.

We used Hiebert and Lefevre's (1986) framework of conceptual and procedural knowledge. They characterized conceptual knowledge as that which is part of a network comprised of individual pieces of information and the relationships between these pieces of information. Hiebert and Lefevre also defined procedural knowledge as including both a familiarity with the symbol representation system of mathematics and knowledge of rules and procedures for solving exercises in mathematics. They noted that, while procedural knowledge may or may not be learned meaningfully, conceptual knowledge must be learned with meaning.

Procedural knowledge learned without meaning is similar to instrumental understanding, a type of understanding named by Mellin-Olsen and described by Skemp (1976) as "rules without reason." Skemp used Mellin-Olsen's term relational understanding for what he described as "knowing both what to do and why"; relational understanding occurs when procedural knowledge is linked with conceptual knowledge.

Some researchers have examined student errors as part of their investigation of students' conceptual and procedural understanding in calculus (Heid, 1988; Orton, 1983a, 1983b), but neither Heid nor Orton developed a classification system for calculus errors. In this study, we built upon their work of examining students' conceptual and procedural understanding through investigating student errors in calculus; we did this by developing an error classification system grounded in our data.

2. Research design

Analyzing student errors in a qualitative manner they called 'constructive analysis', Movshovitz-Hadar, Zaslavsky, and Inbar (1987) developed an empirical classification system for errors in secondary level mathematics, in which calculus was not included. We used their work as a guiding framework in developing a classification system for errors in calculus, using their categories as starting points for our classification system while working under Hiebert and Lefevre's (1986) characterizations of conceptual and procedural knowledge.

Using constructive analysis, we classified and analyzed the errors made by students on course examinations in order to investigate the effects of writing activities on the conceptual understanding and procedural ability of students in an introductory college calculus class. We used qualitative methods to develop categories for the errors and then employed quantitative methods to analyze the data for information about student understanding.

2.1. *Sample*

The subjects of this study were students in two back-to-back sections of an introductory calculus course at a research university. One section was randomly chosen as the WTLM group; the other section served as the comparison group. Both groups were composed predominantly of first-year students of traditional college age. The WTLM group consisted of 3 females and 12 males; the comparison group had 4 females and 14 males. In both groups, about two thirds of the students had taken a calculus course at the secondary level before this one.

On the first day of class, the students in both groups were given the Mathematical Association of America (MAA)'s Calculus Readiness Test, a standardized test given to all students in this course to determine whether or not they are prepared for this course. A t-test indicated that the difference between the mean scores of the two groups (WTLM $M = 14.67$; Comparison $M = 13.39$) on this test was not statistically significant ($t = 0.79$, $df = 31$, $p = 0.44$). Thus, in terms of their preparedness for the course, the two groups were considered to be comparable when the course began.

2.2. *Instruction*

The instruction for the two groups was similar in that both groups were taught with an emphasis on concepts and both groups had the same instructor. The instructor was the first author, who had eight years of experience teaching calculus and other mathematics courses at the college level, and had used writing to learn mathematics in several courses, including calculus, before the study began. Because the first author was both researcher and instructor in this study, the possibility of researcher bias was minimized as much as possible by having the course activities for the two classes be as similar as possible. The difference in activities, as discussed in detail below, was that for one class the tasks involved writing and for the other class the tasks did not involve writing.

Students in the WTLM group participated in a variety of writing activities. Occasionally, the students were given writing tasks to be done as in-class activities. Because class time was limited, the students were also sometimes given writing activities to be done outside of class. The WTLM students were asked to write about topics related to course concepts and procedures. In these writing activities, students were asked to explain course ideas in their own words, to discuss the relationship between course concepts, and to think, on paper, about concepts and procedures of the course.

TABLE I.
Sample writing (W) and non-writing (N) tasks

W: (Students were encouraged to work in groups on this, and they were told that they should be prepared to discuss their answers next time in class.)

(1) Write in words, in terms of *distance* (not absolute value) what $\mid x - 8 \mid < \partial$ means.
(2) Explain in words what it means to *solve* an inequality.
(3) Explain your steps and reasoning as you solve: $\mid 3x - 2 \mid < 4$.
(4) Write $2 < x < 7$ in the form $\mid x - x_0 \mid < \partial$, and explain how you did it.

N: (Students were encouraged to work in groups on this, and they were told they did not need to write anything down but to be prepared to discuss their answers next time in class.)

(1) Discuss with your group: What does $\mid x - 8 \mid < \partial$ mean, in terms of *distance* (not absolute value)?
(2) Discuss with your group and define what it means to *solve* an inequality.
(3) *Show your steps* as you solve: $\mid 3x - 2 \mid < 4$.
(4) Write $2 < x < 7$ in the form $\mid x - x_0 \mid < \partial$, and show your work.

W: (1) You are asked to design a small box-shaped container that will hold 12 cubic inches of a fruit drink. The fruit drink company wants the front of the box to be square, so the box will look like this: [Sketch of box, with square front, is given.] Given these constraints, find the dimensions of the box that will require the least material, and briefly **explain** your steps so that another student could understand you.
(2) Based on your work in the previous problem and on the examples done in our textbook and in class, write a note to another student, answering these questions. Explain carefully, so the student can understand what you mean.

- What is the goal of an optimization problem? (What are you trying to find?)
- How does one solve an optimization problem?
- Write a careful explanation describing this procedure in your own words.

N: (1) You are asked to design a small box-shaped container that will hold 12 cubic inches of a fruit drink. The fruit drink company wants the front of the box to be square, so the box will look like this: [Sketch of box, with square front, is given.] Given these constraints, find the dimensions of the box that will require the least material.
(2) *NOT to be handed in, but be ready to discuss in class next time:*

- What is the goal of an optimization problem? (What are you trying to find?)
- How does one solve an optimization problem?

The comparison group did not participate in any writing activities. However, whenever the WTLM group was given a writing activity, the comparison group was given an assignment or activity that did not involve writing, but which focused on the same concepts and procedures as the WTLM group's writing activity. For a sample of the writing and non-writing activities given to students, see Table I.

Both groups were given feedback on their work, in the form of written comments, and both groups discussed the activity during the next class meeting. While there were some differences between the way the tasks were worded, in order to make it seem natural to either write or not write, the tasks were constructed to be similar in what was required mathematically of the students. The difference between the groups was that the activities of the WTLM group involved writing and those of the comparison group generally did not.

The WTLM group was given a writing activity, either in class or out of class, on 17 of the 38 non-examination class days during the semester. As discussed above, whenever the WTLM group had a writing activity, the comparison group was given a corresponding activity that did not require writing. For both groups, these were collected and graded and, for each student, the average of these grades comprised the student's assignment and quiz grade, which was 14 percent of the total course grade. The completion rate for both classes on these assignments was near 100%.

3. Data collection and analysis

During the semester, the students were given three in-class examinations and one final examination, and these examinations were the same for both groups of students. The first author was responsible for creating the three in-class examinations, which included both routine exercises and non-routine problems. The routine exercises included exercises that focused on basic course procedures and skills, similar to those typically found in traditional calculus textbooks and examinations. The non-routine problems on the examinations included problems for which the students had not been taught a method of solution as well as questions in which students had to explain course concepts and relationships between such concepts in their own words.

These in-class examinations were designed to test what had been taught in the classes. It was not difficult to design the examinations to reflect what both groups had focused on because, in both groups, the concepts of the course were emphasized and the activities and other experiences of the WTLM and comparison groups were quite similar. Since both classes emphasized conceptual and procedural understanding of the mathematical content, the examinations were designed to assess these.

Another mathematics instructor, the course chairperson, was responsible for creating the final examination used in this study. All of the course instructors submitted possible problems for this examination; the course chairperson decided which problems would be included in the examination.

Based on our guiding frameworks and on the data collected in this study, we developed a classification system for errors in calculus. We then used this system in analyzing the data, which consisted of the in-class and final examinations of all of the students in the WTLM and comparison groups; students' work on the writing and non-writing activities was not part of the data analyzed in this study. Note that we recognize the potential problem in examining the effects of WTLM on conceptual and procedural understanding by looking only at students' errors; we do not view understanding as only the absence of errors. However, since the classes were taught with an emphasis on conceptual ideas as well as procedures and the examinations were designed to assess both conceptual and procedural understanding, in examining students' errors we were able to gain some insight into how much they understood mathematically and to discern if the writing activities affected the conceptual understanding and/or procedural ability of the students in the WTLM group.

4. RESULTS AND DISCUSSION

We began the statistical data analysis by using analysis of variance to compare the number of calculus-level errors made by each of the two groups on each examination; students' algebra and other pre-calculus-level errors were not a focus of this study. Although the WTLM group's mean number of calculus-level errors was higher on each examination than that of the comparison group, the difference between the means of the two groups was not statistically significant for any of the four examinations.

Next, we investigated differences related to the two varieties of calculus-level errors: procedural and conceptual. Procedural errors were comprised of syntax errors and errors in carrying out procedures, while conceptual errors included such things as the selection of inappropriate procedures, misinterpretation of mathematical terms, and errors in logic. For each examination, we used a multivariate analysis of variance (MANOVA) to compare the two groups in terms of their conceptual and procedural errors. The results are presented in Table II.

TABLE II

MANOVA comparisons of groups on conceptual and procedural errors per person on each examination

	WTLM Group ($n = 15$)		Comparison Group ($n = 18$)		MANOVA	
	M	*SD*	*M*	*SD*	$F(2,30)$	*p*
Exam 1						
Conceptual	6.67	3.77	5.22	3.26	0.75	0.48
Procedural	1.00	1.31	1.17	1.25		
Exam 2						
Conceptual	15.40	5.64	11.28	4.47	3.87	0.03*
Procedural	1.93	2.25	2.61	2.30		
Exam 3						
Conceptual	12.87	6.35	10.00	4.24	1.30	0.29
Procedural	4.67	2.38	3.89	2.61		
Final Exam						
Conceptual	21.33	7.63	17.22	5.79	1.58	0.22
Procedural	9.4	3.94	9.06	3.87		

* significant difference between mean vectors.

For three of the four examinations, the results of the multivariate analysis of variance for conceptual and procedural errors were not statistically significant. Only on Exam 2 did the multivariate *F*-test indicate a statistically significant difference between the mean vectors of the two groups ($F = 3.87$, $df = 2/30$, $p = .03$). Following this, we constructed simultaneous confidence intervals to determine which of the two dependent variables, conceptual errors or procedural errors, might have contributed to this statistically significant difference. The 95 percent simultaneous confidence intervals for the difference in the WTLM and comparison means for the two dependent variables are as follows:

Conceptual errors: $-0.54 \leq \mu_{WC} - \mu_{NC} \leq 8.78$

Procedural errors: $-2.79 \leq \mu_{WP} - \mu_{NP} \leq 1.43$

Hence, while the mean vectors of the two groups were significantly different, the confidence intervals indicated that the groups did not differ significantly on either of the individual components of the vectors. Further classification of the errors into types of conceptual and procedural errors, respectively, revealed no statistically significant differences between the groups.

An interesting result of this study was that no statistically significant differences between the WTLM group and the comparison group were found, other than the one significant MANOVA on Exam 2. In particular, we found no significant differences between the two groups in terms of their overall calculus-level errors, their conceptual errors, their procedural errors, or the particular types of their conceptual and procedural errors.

While this result is interesting, it should not be unexpected. Since the writing and non-writing activities given to the two groups of students, together with the in-class discussions of these activities, were very close in nature, the experiences of the WTLM and comparison groups were quite similar. Both groups were taught calculus with an emphasis on the concepts of the course, and the instructor was the same for both groups. In both sections, students did activities that involved concepts, procedures, and language (written and sometimes oral for the WTLM group, oral for the comparison group). The only real difference between the two groups was that the WTLM group did activities that involved writing; the comparison group generally did not write as part of their activities.

The purpose of the study was to investigate the effects of WTLM on conceptual and procedural understanding in calculus. Since we found no significant differences between the two groups in terms of their procedural errors on any of the examinations, this suggests that the WTLM activities did not have a different effect than the non-writing activities on students' procedural ability. The same can be said of the students' conceptual errors and conceptual understanding.

Not only are these responses to the research questions interesting, they also have an important implication for mathematics classrooms. If students who engage in non-writing activities that focus on concepts and involve discussion can achieve the same level of conceptual and procedural understanding as students who use WTLM activities and discussion, then mathematics instructors have a viable alternative to using writing activities. While writing has been praised as a useful tool for students' learning, instructors who use writing in their classrooms also know that writing activities can mean longer hours spent reading students' written work and grading or responding in writing to it. Students, too, might enjoy these non-writing activities as an alternative to writing, especially those who bristle at the mention of doing writing in a mathematics class. Thus, these results suggest that the real benefits of using writing to learn mathematics may be due, not to the actual activity of writing, but rather to the fact that it requires students to spend time thinking about mathematical ideas and then communicating these ideas to others.

However, we propose it is not sufficient to have students spend time thinking about mathematical concepts, but rather students must be engaged intellectually with mathematical tasks that require them to articulate (either in writing or aloud) a description or justification for the mathematical ideas involved. This study provides a direct challenge to Stehney's (1990) contention that "writing is not simply a way of expressing or displaying what one has learned. Writing is itself a fundamental mode of learning" (p. 27) by suggesting that it is the act of articulating, in writing or aloud, that is the essential part of this mode of learning, not the act of writing.

There is a need for further research work to investigate the two assumptions that (a) writing is a fundamental mode of learning, and (b) it is the act of articulating that is fundamental, not the act of writing. One such study might involve WTLM and non-writing groups, as in this study, but have researchers analyze the students' written work and class discussions on the writing or non-writing activities – their explanatory written and oral 'texts' – to investigate the conceptual and procedural understandings that students developed as a result of the activities. When analyzing such texts, however, Morgan (1998, p. 30) cautions that readers must pay careful attention to their "assumptions about which forms of language are 'appropriate' and which are taken as signs of more or less 'advanced' thinking," lest the reader/researcher interpret a student's lack of facility with language as a lack of understanding of the subject.

It may also be the case that the effect of WTLM activities or non-writing activities with discussion depends upon the learning styles of individual students. It may be useful to have a study in which students choose which group they will be in (WTLM or non-writing with discussion), and also use a control group in which students are randomly assigned to one of these two groups. By conducting research investigating assumptions (a) and (b), we can learn more about effective modes of learning and, with sufficiently large sample sizes, gain results that will be more generalizable.

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