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Analyzing effective communication in mathematics group work: The role of visual mediators and technical terms

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Abstract Analyzing and designing productive group work and effective communication constitute ongoing research interests in mathematics education. In this article we contribute to this research by using and developing a newly introduced analytical approach for examining effective communication within group work in mathematics education. By using data from 12 to 13-year old students playing a dice game as well as from a group of university students working with a proof by induction, the article shows how the link between visual mediators and technical terms is crucial in students' attempts to communicate effectively. The critical evaluation of visual mediators and technical terms, and of links between them, is useful for researchers interested in analyzing effective communication and designing environments providing opportunities for students to learn mathematics.

Keywords Effective communication · Classroom discourse · Contextualization · Technical terms · Visual mediators

1 Introduction

The role of student group discussions in mathematics education has attracted much attention in mathematics education research (e.g., Cobb, 1999; Martin, Towers, & Pirie, 2005; Weber,

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Maher, Powell, & Lee, 2008). Research has produced both several diverse ways of conceptualizing interactions (e.g., Radford, 2011; Schwarz, Dreyfus, & Hershkowitz, 2009) and diverse results in terms of the benefits of group discussions. That is, while some researchers conceptualize interactions as, for instance, empowering acts in which citizenship, agency, power and political matters are constituted (cf. Radford, 2011; Valero & Stentoft, 2010), others examine how learning opportunities of content are established in interactions (e.g., Kieran, 2001; Nilsson & Ryve, 2010; Weber et al., 2008). Furthermore, several studies show positive effects of mathematical group work (e.g., Amigues, 1988; Coleman, 1998; Prusak, Hershkowitz, & Schwarz, 2012) while others find that students have substantial difficulties establishing productive group discussions (e.g., Barron, 2003; McCrone, 2005; Ryve, 2006; Sfard, 2001). In this article we focus on how students manage to establish effective group discussions about mathematical content. Kieran (2001) suggests that aspects such as face-saving (cf. Salomon & Globerson, 1989), social rules (Grice, 1968) and high status among participants (Linn & Burbules, 1993) are consequential for the students' establishment of effective communication in mathematics group work. We note that it is of interest to study and derive explanations for how aspects brought into the group work, such as high status and social rules, affect the interaction. However, in this study we chose to focus primarily on interactional aspects such as the use of visual mediators and technical terms, and how they function in the establishment of effective communication.

The diverging results of the benefits of group communication, as shown above, and the different ways of conceptualizing group discussions in the mathematics classroom, indicate that the mathematics education research community further needs to develop explicit and sound methodologies for studying effective communication (Ryve, 2011; Sfard, 2008). In this article we cumulatively build on the analytical approach of Nilsson and Ryve (2010) which, in turn, builds on Ryve (2006) and Sfard (2001). As argued by Nilsson and Ryve (2010), the analytical approach enables the researcher to conceptualize and analyze effective mathematical communication[1] by focusing on how students' individual projects constitute an important component of their collaboration. However, while Nilsson and Ryve (2010) provide analytical constructs such as focal events/projects and contextualization as tools for organizing the analysis of whether or not the students communicate effectively, they do not suggest explanations for *why* the students manage to establish common interactional foci and effective communication. In this article we aim to contribute to the understanding of how the students, during interaction, act when they manage to establish common interactional foci, and we focus especially on the role of visual mediators and technical terms in the co-construction of effective communication. The research question guiding the study is: What is the role of visual mediators and technical terms in establishing effective mathematical communication, and how could the findings be used to amend the analytical approach of Nilsson and Ryve (2010)?

2 Theoretical framework

The analytical approach used to conceptualize and analyze the group communications consists of both theoretical constructs used from the very start when first approaching the

[1] The concept of effective communication is used in accordance with Sfard and Kieran (2001), and is discussed and defined below.

data, and constructs emerging as usable during the process of analyzing the data. As indicated above, we use the analytical framework developed by Nilsson and Ryve (2010) to approach the data and analyze *whether* the communications are effective. Other constructs emerged throughout the process, and we especially needed a research language for talking about *how* and *why* interactional aspects influence the establishment of effective communication. Key constructs within this research language are visual mediators (Sfard, 2008) and technical terms (Mason, 1998; Wertsch & Kazak, 2011).

2.1 Constructs used for analyzing whether the communication is effective

Kieran and Dreyfus (1998) argue that students need to enter each other's *universe of thought* during group work. Sfard and Kieran (2001) specify this as defining communication as *effective* if "the different utterances of the interlocutors evoke responses that are in tune with the speakers' meta-discursive expectations" (p. 49). Meta-discursive expectation should be understood as an indication that a certain reaction does not have to contain specific content but rather content of an expected kind. Nilsson and Ryve (2010) show how the framework of *contextualization* helps in modeling universes of thought and, based on such modeling, in conceptualizing the meaning of meta-discursive expectations in order to account for effective mathematics communication in group work. In practice this means that, in a mathematics group activity, we ascribe to the students individual focal projects (FP). A student's FP refers to the problem or the project the student engages in and interprets as his or her obligation to solve. However, a project can be dealt with in many different ways, and how an individual chooses to deal with his or her FP depends on how he or she contextualizes the project; that is, in which personal and mental context the individual is operating in order to develop an understanding of the project (Nilsson & Ryve, 2010). For instance, Ryve (2006) shows how engineering students did not communicate effectively as they developed different focal projects and contextualizations of focal projects in an activity involving the construction of a concept map. In the study, one student (Student A) engages in the FP of constructing a concept map in linear algebra and locates this FP within a conceptual context of discussing concepts and their relationships. In contrast, Student B, Student A's collaborator, locates the FP of constructing a concept map in the context of making as many connections as quickly as possible. The discrepancy between these two overall patterns of FP and contextualizations of FP makes it hard for the students to respond within the frame of each other's meta-discursive expectations and, based on this, to relate to and challenge or contribute to each other's turns and suggestions.

In short, Nilsson and Ryve (2010) dig deeply into the students' contextualizations of learning tasks and provide us with analytical tools for studying how contextualizations produce focal projects and how such focal projects, in turn, are contextualized into new, more local contexts of interpretations. These chains of contextualizations and focal projects have been proven useful for studying how students relate to each other's meta-discursive expectations and, as such, for conceptualizing and examining whether students communicate effectively (Nilsson, 2009; Nilsson & Ryve, 2010).

2.2 Constructs for analyzing how and why the communication is effective

Kieran (2001) engages in the development of analytical tools for examining whether and why students manage to establish effective communication. Kieran's study shows that interactional aspects such as a high number of interpersonal object-level turns as well as tightly connected personal and interpersonal projects in the beginning of the interaction are typical of effective communication. But how do students manage to establish such tightly connected interpersonal projects? Sfard (2008) elaborates on the importance of images of objects for students to be able to develop tightly connected interpersonal projects. Sfard (2008) denotes realizations of objects as *visual mediators* and notes that colloquial discourses are often mediated by images of concrete objects that preexist the discourse, while in the literate discourse of mathematics visual mediators are symbolic artifacts such as algebraic expressions, tables, graphs, etc. (see also, e.g., Arcavi, 2003; David & Tomaz, 2012) that constitute the discourse. In this study we continue the work of Sfard (2008) by further analyzing and critically reflecting upon how visual mediators affect students' communication, that is, the role visual mediators may play in the development of common focal projects and contextualizations of those in mathematics small-group work.

Sfard (2008) stresses that the same visual mediator can be scanned in different ways by different individuals. This claim is supported by earlier studies, for instance in which Bergqvist and Säljö (1994) found, when analyzing students engaging in optics laboratories, that critical features in institutionalized learning situations are often of a discursive nature and are therefore also beyond immediate perception. From such a perspective, the use of *technical terms* in directing students' attention to critical features is stressed as crucial (e.g., Ryve, Nilsson, & Mason, 2012; Wertsch, 2007; Wertsch & Kazak, 2011). A technical term refers to a term that typically belongs to a specific discourse, such as mathematics education discourse or mathematics discourse, and has specific meanings within this discourse. Mason (1998) suggests that "each technical term marks a particular way of seeing" (p. 252) and that the use of technical terms therefore has the potential to change the structure of attention and indicates certain ways of perceiving, talking about, and making sense of a phenomenon. As such, the use of technical terms is of importance to us as researchers in understanding effective mathematics communication (cf. Mason, 1998; Ryve, Larsson, & Nilsson, 2011). In particular, we study how students contextualize visual mediators by using technical terms and how this affects the establishment of effective communication.

3 Methodology

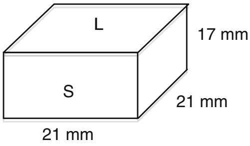
3.1 Data

In using and amending the analytical approach of Nilsson and Ryve (2010), we use data from two different settings. These two settings were chosen because, from an a priori perspective, they constitute two very different cases (cf. Bryman, 2004). Each case is introduced below, and we can see how the two groups differ concerning aspects such as prior knowledge of the mathematics, the number of participants, time spent on the problem and the age of participants. The idea of choosing different cases relates to our interest in understanding how the use of visual mediators and technical terms affects students' communication in different kinds of settings. We could by no means claim that the data enable us to carry out an empirically grounded saturation; instead, the data serve as a source for critically reflecting on how these aspects function in relation to effective communication.

3.2 Dice group

The entire game activity consisted of four sets of dice and lasted about 70 min (Nilsson, 2009). Eight Swedish sixth-grade students (12–13 years old) were divided into four groups.

Fig. 1 Index *S* refers to the four small faces of the dice and index *L* to the two large faces



The dice were numbered as follows:

$D_1=\{2_L, 2_L, 2_S, 4_S, 4_S, 4_S\}$ and

$D_2=\{3_L, 3_L, 3_S, 5_S, 5_S, 5_S\}$.

The present analysis will be restricted to one group of students, Elin (E) and Jenny (J), and their joint efforts to develop winning strategies in the first round of the game. Elin and Jenny are friends outside school and are used to collaborating at school when possible. However, the students were not familiar with playing games in groups when being taught mathematics.

Two teams compete in the game. On the basis of the sum of two specifically designed dice (Fig. 1), each team is instructed to distribute 25 markers on a board, numbered 1–12. If one or both teams have at least one marker in the area marked with the sum of the dice, they remove one marker from this area irrespective of which team rolled the dice. The team that is first to remove all its markers from the board wins the game.

Prior to this activity the students have not been introduced to probability within the school setting, and nothing about chance, probability, randomness, etc. was mentioned to them in the presentation of the activity. Instead, the information only concerned the rules of the game and how to play it. The small-group discussions of each team took place in their respective corners of a classroom, and were video- and audiotaped. In these groups, the students had access to paper and pencils, a picture of the board and the two dice to play with. When they had finished their discussions the teams approached the board in the middle of the classroom, and started to play against one of the other teams.

One of the authors was present as an observer during the session. The main task of the observer was to present the game to the students and keep a record using the audio and video equipment. If the students asked about the rules of the game, he gave clarifications.

3.3 University students—proof by induction

A group of four students, two women and two men, in their first year of a mathematics program at tertiary level worked with a challenging task that included the concepts of function, derivative and proof by induction (Pettersson, 2008). The problem was to decide how many zeros exist when you know that the derivative of order n is nonzero. The problem was not specified in full; that is, the students needed to decide whether the function should be regarded as differentiable for all real numbers. In this case the group talked about the function as it was differentiable n times in all **R**. The task was:

Let f be a function defined on all of **R**.

a) How many zeros at most can the function have if $f'(x) \neq 0$ for all x?
b) If instead $f''(x) \neq 0$, what can you say about the number of zeros of the function?
c) If we have $f^{(n)}(x) \neq 0$, what can be said about the number of zeros of the function? Use induction to prove your statement.

The students had almost finished the program’s first year, which was comprised of an introductory course including proof by induction, courses in linear algebra and calculus (both single- and multi-variable) and a course in computer programming. Three of the students (Students A, C and D) knew each other well and were used to working together. At the time of the study Student B was taking the same course as the others, but it was the first time B had collaborated with A, C and D. The reported problem-solving session was set up outside ordinary lectures, and the students volunteered to take part in it.

The group work was videotaped, and notes from the students were collected and also used in the interpretation of the group work. The group was not given any maximum time for their work, and as it turned out they spent about 2 h working on it. The students created a proof by induction, but did not believe it fit the usual pattern of such proof by induction as they remembered it from textbooks and teaching.

4 Results

4.1 The dice group

We start by reviewing the data presented in Nilsson and Ryve (2010), and study the two instances when the pupils managed to establish compatible focal projects and contexts. In particular, we are interested in studying how and why they managed to establish effective communication.

> [7]. E: Then there is a higher chance it will end up showing these…[pointing to the large faces of one die and looking around to see if anyone has heard her]
>
> [8]. J: Yes…
>
> [9]. E: …that is…
>
> [10]. J: … 'cause there's…
>
> [11]. E: …larger.
>
> [12]. J: …many that…
>
> [13]. E: But there's a higher, there's a higher chance that it [the die] will show these [takes one die from Jenny and points to one of the large faces] than these [shows Jenny one of the small faces and turns her face to Jenny, smiling].
>
> [14]. J: Yes, there is [looking at the die in her hand].
>
> [15]. E: Well.
>
> [16]. J: But it's on this too [taking the die back from Elin].
>
> [17]. E: Yes.
>
> [18]. J: 'Cause it can just be… The highest chance is that it will be five.
>
> [19]. E: Yes.

As shown in Nilsson and Ryve (2010), during turns [7]–[12] the communication is not effective. While Elin locates the FP of the chance of sums in the context of individual dice and the physical and geometrical features of the dice, Jenny talks about 'many' and seems to locate the FP of 'the chance of sums' in the context of favorable outcomes. However, in [13], something changes: Nilsson and Ryve (2010) suggest that the FP and its contextualization start to harmonize between the girls. Why do the pupils manage to establish a common FP and context here? First, in [13], Elin clearly marks a shift using ‘But’ and then uses the visual mediator (Sfard, 2008) of the dice to establish a common focal project regarding the chance of a sum and, subsequently, a common context for this FP in terms of the physical shape of the dice. However, it is not only the visual mediator of the dice that helps them establish a common focus but also the use of technical terms (cf. Mason, 1998; Wertsch & Kazak, 2011) such as higher chance, larger and five. The combination of using the visual mediator of the dice and technical terms helps Elin make explicit her location of the FP of the chance of a sum in the context of the physical size of the sides of the dice. Hence, in this setting, the sequence of producing a shift to get attention and using a visual mediator and technical terms to specify the focal project and how it is contextualized is significant for the establishment of effective communication. We shall return to the finding of how a visual mediator serves as a common focal object (cf. Sfard, 2008; Sfard & Kieran, 2001) and how the use of language and especially technical terms seem necessary for establishing common FP and contexts (cf. Wertsch & Kazak, 2011).

A few moments later the second instance occurs, which is interesting from the point of view of how visual mediators and technical terms support the establishment of effective communication.

[32]. J: OK, but how many should we put on five? About seven?

[33]. E: Wait, look here! There’s one, two, three. One, two three… [pointing to the faces of one of the dice]

[34]. J: There are three twos.

[35]. E: And then one, two, three… [counting the faces of the other die]. Yes…

[36]. J: But…

[37]. E: …there’s not that big a chance of getting nine, ‘cause there are only two…

[38]. J: There are…

[39]. E: But, there are…[looking at the dice]

[40]. J: But, the highest chance is still five.

[41]. E: [Elin continues investigating the dice, counting quietly to herself]

[42]. J: But if we put seven on five.

In [32] Jenny is engaged in the FP of distributing the markers, which she now locates in the conceptual context of ‘the quantitative distribution’ and has consequently left the context of aspects that can affect the probability distribution of the sums (size of sides, frequency of numbers). This context leads to the new, more local FP of quantitatively deciding the number of markers on 5. Elin, on the other hand, is still focused on discussing the chance of sums and locates it in a combinatorial context of the number of ways the sums can be arrived at. The clearly marked shift in [33], 'Wait, look here', serves as a starting point for establishing this context as a common context for interpreting the chance of the sums, and the dice serve as a visual mediator to accomplish this. Elin [33] uses her finger to point to the faces of the dice and uses technical terms such as one, two and three to indicate her contextualization (specific numbering) of the FP of the chance of the sums. Turn [34] indicates that Jenny is able to enter Elin's universe of thought, but in [36], [38] and [40] Jenny tries to return to her FP of quantitatively deciding how many to put on 5. Hence, Elin refers to a visual mediator and uses technical terms in [33] to make explicit her FP and context, but Jenny chooses not to deepen this interactional focus. Interestingly, here it seems that Jenny and Elin have the potential to establish effective communication but that other forces and considerations come into play. Therefore, one may argue that to more fully understand this interactional sequence it is productive to also conceptualize the interaction in terms of, for instance, positioning, agency and identity construction (Radford, 2011). That is, Elin's and Jenny's interaction is not only about effectively engaging in winning the game but is also an interpersonal act in which identity and agency are at stake. From such a perspective, one may also understand the use of a shift as a move to dominate the interaction (e.g., West & Zimmerman, 1983).

To recap, as shown in Nilsson and Ryve (2010), throughout the interaction Elin and Jenny are often engaged in compatible FP but locate them in different contexts and do not fully enter each other's universe of thought (Kieran & Dreyfus, 1998). However, the two instances when their FPs and contexts harmonize, albeit only for a short period of time, follow the same pattern: a shift of the interactional focus using visual mediators and technical terms.

4.2 University students—proof by induction

The first excerpt is taken from the first 2 min of the students' work. In this part of the problem-solving, the students interpret the task and solve the first part of it: how many zeros at most a function can have when you know that $f'(x) \neq 0$.

[1]. C: We'll write one proof each and then compare them to each other.

[2]. B: OK.

[3]. D: OK.

[4]. C: a.

[5]. D: Should one prove…

[6]. B: It's only the last one that should…

[7]. D: Yeah, the first one should not be proven…

[8]. B: It should only include the discussion.

[9]. C: Mm.

[10]. B: How many zeros, at most …

[11]. B: OK.

[12]. D: OK.

[13]. B: OK, if the derivative is nonzero…

[14]. D: If the derivative…

[15]. B: Then…

[16]. C: Feels like…

[17]. D: Then it means that, yeah…

[18]. D: We've probably had something similar, but …

[19]. D: If the derivative isn't allowed to be zero, then the derivative won't change.

[20]. B: No [hence, agree].

[21]. D: No.

[22]. B: And then it's an increasing

[23]. D: Yeah, increasing or decreasing.

[24]. C: Mm.

[25]. A: Mm.

[26]. D: On the whole of R. And then we couldn't have that many…

[27]. B: At most…

[28]. C: …one approximately…

[29]. D: Yes.

After a first comment (probably a joke) made by C about the way of working, the students start contextualizing the task. The FP developed in the beginning concerns how to tackle the problem. C indicates in [4] that he allocates this FP to an educational context of working chronologically through problems, which is typical of Swedish classrooms, and this context gives rise to the new, more local FP of starting with Subtask a. This context differs from B's and D's context. They, instead, put 'tackling the problem' in a context of forming an overall picture of the whole problem ([5]–[8]). In [10], B changes the context into focusing on the first subtask and D adjusts to it (cf. [14]), making this explicit by repeating parts of B's turn in [13]. B's turns of changing the context ([10] and [13]) are well marked by the technical terms *derivative* and *nonzero* and are mediated by the visual mediator of the written text.

Hence, the introduction of visual mediators and technical terms are of importance when making explicit FPs and contexts. Note also that D in [19] yet again repeats after B in [13]. This repetition functions as an explicit way of marking and telling the others 'I'm working with the same FP in the same context as you are'. In line with the dice group and Sfard (2008), we can see how mediators are important in the communication and that the symbolic expressions ($f'(x) \neq 0$) given in the text are referred to several times and, therefore, function as a means of establishing effective communication. In the rest of the transcript we can see how the students use technical terms (increasing, decreasing, **R**) and how C becomes engaged in the interaction (e.g., [16] and [28]) when B and D start focusing on Subtask a, which harmonizes with C's FP.

The second excerpt is also from the beginning of the discussion, in the third and fourth minutes. The students are working with the second part of the problem, Subtask b, looking for how many zeros, at most, a function can have when knowing that the second derivative is nonzero.

[44]. C: When f double prime x is nonzero.

[45]. B: Mm…

[46]. D: Then, the same thing holds that…

[47]. B: Is it the case that, that's what I was also thinking about, 'cause can…

[48]. D: That f, that is, 'cause then it's, if f prime is nonzero, then it means that, that the function is increasing or decreasing.

[49]. B: Mm.

[50]. D: Then it means that f double prime is nonzero, that the derivative is increasing or decreasing.

[51]. C: Mm.

[52]. B: Mm…

[53]. D: Which means.

[54]. C: Go on.

[55]. D: Which means that the derivative is allowed to change sign once at most, or change sign, yeah change sign once.

[56]. A: But, it doesn't have to change sign.

[57]. D: No, thus, once at most, that's what we're talking about here.

[58]. A, B, C: Mm. [A nodding.]

[59]. D: And, …

[60]. B: If it changes once at most then, how…

[61]. D: … and then the derivative looks like this [draws and points at it (see Fig. 2)] and then it's minus here and plus here.

[62]. A: Mm.

[63]. B: Mm, and then it could have?

[64]. D: Have you drawn that many? [looks at C's drawing (see Fig. 3)]

[65]. C: I've drawn a lot.

[66]. D: You've moved ahead here [laughter].

[67]. C: Yeah.

[68]. B: Hm.

[69]. D: [draws (see Fig. 4)] Or, that's the way isn't it? Or…

[70]. A: Yes.

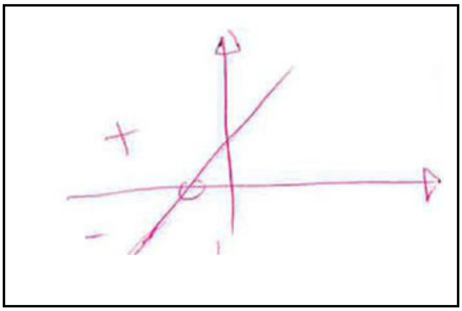
[71]. B: Then it becomes…

[72]. D: …at most…

[73]. A, B, C, D: …two.

In [44], C engages in the FP of looking for zeros of the function by focusing on the algebraic expression in the written text. Hence, the visual mediator of the algebraic expression becomes the focus of attention and, together with oral technical terms, serves as means of establishing compatible FPs and contexts between the participants. The data show that D also allocates the solution of Subtask b to an algebraic context. Further, D develops her algebraic context for understanding the solution of the task by engaging in the more local FP of comparing the algebraic expressions involved in Subtask a and Subtask b. She explicitly introduces this FP in [48] by repeating the reasoning used in relation to Subtask a. D is given

Fig. 2 D's drawing in [61]



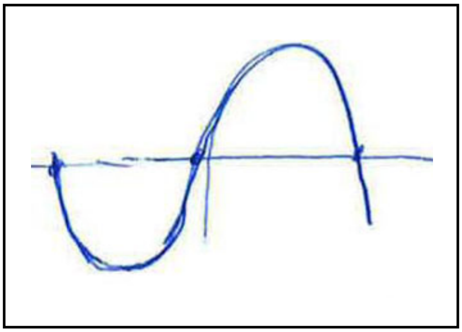


Fig. 3 C's drawing in [64]

time to publicly develop this FP through both minimal responses ([49], [51] and [52]) and explicit encouragement ([54]) from the other group members. In both [56] and [58], the other students initiate turns that indicate that their FP and contexts harmonize with D. In [60], B expresses some uncertainties and, at this stage of the communication, further mediating tools seem necessary to support a common context and effective communication. D draws a figure in [61], probably mostly for her own thought process, but this figure functions as a visual mediator creating possibilities for the other students to follow her contextualization. Note also that D both uses technical terms like 'minus' and 'plus' and points at the visual mediator of figure to make explicit her contextualization. In [63], B asks about what conclusions could be drawn from the figure, showing that the FP of looking for zeros of the function is still her focus and that she places this FP in a compatible context as D's (what conclusions can be drawn from the knowledge about the first derivative).

There is a sharp shift concerning focal project and context in [64]. The graph drawn by C and observed by D prompts this shift. But it is also a graph, drawn by D and referred to in [69], that makes it possible for the students to shift back to D's original common FP of zeros of the function. This new graph seems to become the new object of the discussion, not only because it is a visual mediator but also because D clearly marks its introduction by using 'Or' and asking a question. Hence, the shift of 'Or' and an explicit question together with the produced visual mediator of the graph are used to establish effective communication. It is also interesting to note that the Cartesian graph itself belongs to the mathematical discourse and contains mathematical objects and representations of relationships between mathematical objects (cf. Radford, 2008; Roth & Lee, 2004). Therefore, the graph is useful for including several crucial objects and relationships within a single visual mediator and, in relation to the effectiveness of the communication, it is also crucial for the individuals in specifying and making explicit which objects or relationships are in focus and how they are contextualized.

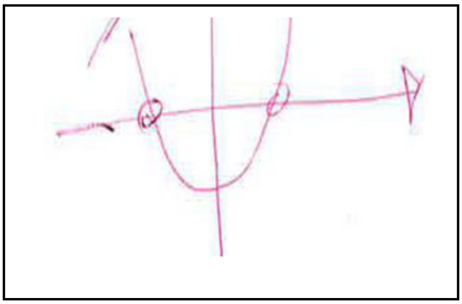


Fig. 4 D's drawing in [69]

In the next excerpt, the students start to solve the third and last part of the task. They begin by discussing how to do proof by induction. They also pose their hypothesis about how many zeros the function can have at most when they know that the nth derivative is nonzero.

[75]. B: But now we got induction here, it's going to get exciting.

[76]. D: Well, it's … I don't know, I don't really get it, yeah what is it you should base induction on, what is it that…

[77]. C: Well, we got…

[78]. B: Isn't it this, this…

[79]. C: … a and b as inductions …

[80]. D: … hypotheses.

[81]. C: Yeah.

[82]. B: Then it's n [pointing to the written text] over there, yes…

[83]. C: … so then you get all the way to f prime…

[84]. C: Yeah, start on n and then, yeah…

[85]. B: … n, yeah if you put, p minus one and then…

[86]. C: Mm.

[87]. B: … and you check if it's correct then, but I don't know how you write this down. [B and C laugh]

[88]. D: Yeah, but, or what is it, what should we do induction over, sort of, thus what we've been doing now isn't some kind of reasoning, sort of that…

[89]. C: I'd like to assume that a function gets n zeros, perhaps…

[90]. D: Mm.

[91]. C: … that … or … what do you assume?

[92]. D: But, what happens if you get…

[93]. C: Yeah, it's like that a statement, if f n x is nonzero … yeah but it becomes the statement in some way anyway.

[94]. A: Yeah.

[95]. D: Mm.

[96]. C: [writes] Yeah, so what … Yeah, exactly, about zeros, n zeros … that's what we think isn't it?

[97]. A, B, D: Yes.

The focal project in this part is to start formulating the proof by induction (cf. [75]). D is engaged in this FP in [76] and locates it in the conceptual context of what to base the induction on. This context of FP is not continued by C, who instead is about to locate it within another conceptual context of formulating base cases by referring to Subtasks a and b. In [80], D seems to relate to this new conceptual context by filling in C's turn in [79] but locating it in yet another conceptual context by introducing an induction hypothesis, which from a mathematical point of view is wrong, but is treated by C as acceptable. Between [82] and [87], B and C try to establish a common context but do not succeed. Why? In relation to the analysis of the other transcripts, a sensible guess is the lack of a precise use of technical terms as well as the difficulty in producing any visual mediators. 'Yeah, but…' in [88] indicates a clear shift in focus, and D returns to her initial conceptual context of what to base the induction on. C relates to this shift of context and tries to answer [88], but we can see that C's exact context is hard to figure out for an outsider and reasonably also for the other participants in the interaction. C mixes technical terms like 'assume' and 'the statement', and explicitly marks that he is not sure of his ways of expressing aspects of how to formulate an inductive hypothesis 'in some way anyway'. We can also see several instances of short utterances, as they simply test the technical terms from the area of proof by induction: assume, inductive step, hypothesis, induction over and so on. Instead of trying to elaborate further on the different contextualizations of approaching Subtask c, C starts to formulate a statement about possible numbers of zeros of the function ([93] and [96]), using written text. For the establishment of a common FP and context, such a move is important since the other participants can point and refer to the written text, and [97] indicates that they are all engaged in the conceptual context of formulating a hypothesis. Interestingly, the analysis of the group shows that the participants strive to use visual mediators, and when visual mediators are not provided they produce the visual mediator of a graph. When this is not possible they produce other visual mediators, such as written text.

5 Discussion

While we note that there are important strands of research studying how aspects of interaction are constitutive of students' and teachers' formation of identity and agency (cf. Radford, 2011), the research presented in this article should be seen as a contribution to ongoing efforts to develop analytical approaches for studying mathematics classroom communication (e.g., Nilsson & Ryve, 2010; Ryve, 2006; Sfard & Kieran, 2001) and to the findings of what make mathematical communications effective (e.g., Kieran, 2001; Sfard, 2008). That is, the main contribution of our study is the elaboration upon how the constructs of focal project, contextualization, visual mediators and technical terms are useful in analyzing effective communication; as such, the study could be seen as an amendment of the framework of Nilsson and Ryve (2010). Further, in relation to prior studies (Kieran, 2001; Sfard, 2008) that present aspects consequential for effective communication such as high status, social rules and time limits, this study complements the picture by elaborating upon how face-to-face interactional features like the use of visual mediators and technical terms, and how they function together, affect the process of establishing effective communication. Here we would also like to deepen the discussion of these concepts by further critically reflecting on their role in group work.

As Sfard (2008) suggests, our data analysis shows that visual mediators are important for establishing effective communication. In the analysis of the dice group above, we saw how instances when Jenny and Elin managed to establish common FPs and locate them in compatible contexts were mediated by the dice. In the study of the group of university students we could also see how visual mediators, especially graphs, function as a way of making FP and context explicit in the interaction. In this sense, visual mediators serve not only as means for establishing effective communication but also as structuring devices for the interactional activity (e.g., David & Tomaz, 2012). However, group work is not just about establishing effective communication but also engaging in productive argumentation/communication, in which students engage in multiple forms of mathematical representations and deductive reasoning (Prusak et al., 2012). From this perspective and in relation to our analysis, we see a possible conflict between, on the one hand, establishing and maintaining effective communication by means of specific visual mediators such as the dice and graphs and, on the other hand, the groups' motivation to engage in other forms of mathematical representations. That is, if specific visual mediators strongly mediate the communication and facilitate its effectiveness, as in our data, there seems to be a risk that students will hesitate to use other forms of representations that might jeopardize the effectiveness of the communication. This conjecture must be substantiated through further empirical studies, and we suggest that conceptualizing the phenomenon as a relation between effective communication (Sfard & Kieran, 2001) and productive argumentation (Prusak et al., 2012) is a possible fruitful avenue for such studies.

The study shows that technical terms, in combination with visual mediators, are important for establishing effective communication. That is, visual mediators are important in establishing a common focal object but, since critical features of institutionalized learning are often of a discursive nature (Bergqvist & Säljö, 1994; Wertsch & Kazak, 2011), the students' use of words is crucial for establishing and making explicit the focal projects and contexts. It is worth noting that the pupils in the dice game had not been formally introduced to any technical terms belonging to the mathematical discourse of probability (e.g., sample space, probability) whereas the university students were already familiar with several technical terms related to the area of calculus and proof by induction. The results of our study and the consideration of the relationship between effective communication and technical terms led us to a non-trivial consideration of when and how to introduce technical terms in a teaching sequence. That is, if teaching is about initiating students into a disciplinary dialogue, as Forman suggests in Wegerif, Boero, Andriessen, and Forman (2009), and technical terms constitute an important part of the disciplinary dialogue, we must consider when and how to introduce technical terms in group work. Our analysis of effective communication and Forman's ideas of disciplinary dialogue suggest that without access to technical terms it is unlikely that students will engage in effective communication of disciplinary content. Further, as Howe (2009) notes in relation to science teaching, "no matter what its significance, group work among children will never be sufficient to deliver the science curriculum" (p. 93) and group work must therefore be understood as a component of a teaching sequence. There thus seems to be a complicated relationship between effective communications in group work, the appropriate time to introduce students to technical terms, and the role of group work in a teaching sequence. For instance, although the dice group had a hard time establishing effective communication, we could imagine that this communication could be seen as potentially productive in relation to a subsequent whole-class discussion in which the teacher returns to the game and conceptualize it in terms of key technical terms of probability (sample space, combinatorics, and frequency).

Synthesizing the critical elaborations on visual mediators and technical terms leads us to suggest that while visual mediators are crucial for effective communication (Sfard, 2008) they should be integrated into multi-channeled argumentation to lead to productive communication (Prusak et al., 2012). This is not enough, however; discursive practices could hardly be regarded as productive unless they integrate technical terms (Wertsch & Kazak, 2011). The emergence of technical terms within group discussions is, in turn, heavily dependent on the teacher (Howe, 2009). Consequently, definitions and analyses of effective and productive communication should reasonably be understood in relation to learning goals and the role of group communication in a teaching sequence. We therefore suggest that future studies analyze series of activities—group work as well as sessions led by the teacher aiming at acculturating students to technical terms—to better understand the teachers' role in facilitating productive group discussions by providing, making explicit and connecting visual mediators and technical terms.

The focus of the analysis in this article is how aspects of and within interaction affect students' possibilities to communicate effectively. Several studies (Kieran, 2001; McCrone, 2005; Weber et al., 2008) discuss important aspects for creating environments for productive group work, and several analytical approaches have been introduced for conceptualizing and analyzing the interaction of students working with mathematical tasks. This study shows how we could complement and amend the analytical approach of Nilsson and Ryve (2010) to study the effectiveness of mathematical communication. As discussed above, the study suggests that two aspects are important to consider when conceptualizing and analyzing how students manage to create effective mathematical communication: first, the use of visual mediators from within and outside mathematics; and second, the use of technical terms connected to the specific mathematical area under consideration and how they are used in relation to visual mediators. However, the discussion of this article also situates the findings of this study in relation to other perspectives on interactions, and elaborates on how effective communication might be seen in relation to productive communication and group work in a teaching sequence. This elaboration, together with prior studies (e.g., Howe, 2009; Prusak et al., 2012; Wegerif et al., 2009), suggests that a considerable amount of empirical research and further development of analytical approaches are needed to better understand the relationship between effective communication, productive communication, and opportunities for learning.

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