ZDM Mathematics Education (2013) 45:901–909
DOI 10.1007/s11858-013-0531-5

COMMENTARY PAPER

A US perspective on the implementation of inquiry-based learning in mathematics

Alan H. Schoenfeld · Jeremy Kilpatrick

Accepted: 9 August 2013 / Published online: 17 August 2013

Abstract This paper discusses issues related to the potential adoption of inquiry-based learning (IBL) projects in mathematics in the United States. To explain the challenges faced in making a place for IBL in the mathematics curriculum, we describe the historical demands of working with a diverse, highly distributed educational system (that is, a system that does not have a central educational decision-making agency with the authority to mandate nationwide changes), the impact of high-stakes tests to either open or limit the potential for curricular changes, and the changing context in the United States owing to the emergence of the Common Core State Standards in Mathematics (CCSS-M) and nationwide high-stakes assessments designed to be consistent with the CCSS-M. We identify a number of dimensions along which there would be challenges for the implementation of IBL in US school mathematics, including: perceived societal needs; schooling traditions; the specific framing of CCSS-M goals pertaining to problem solving, communicating and reasoning, and modeling and data analysis; and the readiness of the US teaching force to implement IBL. We then consider the issue of scaling up interventions such as IBL, and the politics involved therein.

A. H. Schoenfeld (✉)
Education, EMST, University of California, Tolman Hall MC #1670, Berkeley, CA 94720-1670, USA
e-mail: alans@berkeley.edu

J. Kilpatrick
University of Georgia, 105 Aderhold, Athens, GA 30602-7124, USA
e-mail: jkilpat@uga.edu

Keywords Inquiry-based learning · Problem solving · Standards · Social contexts of curricular reform

1 Introduction

The present volume describing the implementation of inquiry-based learning using the PRIMAS (Promoting Inquiry in Mathematics and Science Education across Europe) Project contextualizes and describes a notable cross-national collaboration in science and mathematics education. For two mathematics educators in the United States, the natural question to ask is the following: “Could it happen here?” And in this context: “What is the ‘it’—what is meant by inquiry-based learning *in mathematics*?”

The paper by Artigue and Blomhøj (2013) in the present volume addresses the “What is it?” question historically and analytically by comparing the concept of inquiry-based learning (IBL) with some established theoretical frameworks in mathematics education. The paper shows clearly that IBL, when addressing mathematics, needs to be seen against an abundant background of theoretical work that has been done in the field ever since John Dewey (1916, 1938) argued that we learn by doing (inquiry) and by thinking about what we do (reflection). Artigue and Blomhøj (2013) make clear that inquiry-based learning as it is shaped by the PRIMAS Project is set within long traditions of research and practice, much of it coming from the teaching of science, that vary across the contexts supplied by different countries’ educational systems.

In their paper, Bruder and Prescott (2013) point out some additional variations in IBL: the variations in learner independence from *structured inquiry* through *guided inquiry* to *open inquiry,* depending on how the teacher formulates the task and supports the learner. These variations, along with variations in *task difficulty,* also noted by Bruder and Prescott, complicate further the question of what IBL means. Engeln et al. (2013) cite these differences in their paper and add additional dimensions of variation: "IBL can be differentiated according to the type and the complexity of the problems, the degree of student-centered learning and also to the order of problem and information presentation." The question then becomes: What does IBL mean in this context for this teacher, this learner, and this mathematics?

In this paper, we first ask what would happen if an attempt were made to implement IBL in the United States on the same scale at which PRIMAS was implemented in Europe (roughly 100 teachers per country across 12 countries). We then ask what issues are raised by any attempt to implement such change on a large scale across multiple jurisdictions. What would developers of any project intended for large-scale implementation have to worry about? Addressing this issue requires a substantial discussion of what education policies in the "United" States have been, and what they may be on the verge of becoming. For many years, the 50 states have had education policies far more diverse than those of the European Union—but that may be changing. Understanding what has been, and what might take place over the next few years, is essential to understanding the potential for IBL (or, for that matter, any significant approach to mathematics and science instruction) to take hold in the United States.

2 Implementing inquiry-based learning in the United States

In their paper, Maaß and Doorman (2013) emphasize how dissemination and implementation of inquiry-based learning using the PRIMAS model had to be designed so that the uptake of IBL could occur in a variety of different contexts. Although one often speaks of a single US educational system, it is actually a collection of systems, thereby also presenting a variety of different contexts. In fact, one can reasonably argue that instead of being treated as a single entity in international comparative studies such as the Trends in International Mathematics and Science Studies (TIMSS), the Program for International Student Assessment (PISA), and the Teacher Education and Development Study in Mathematics (TEDS-M), the United States would have been better served if individual states had been the units of study. To give just one example, consider the following scores on the 2011 TIMSS eighth-grade mathematics assessment. Overall, the international average for the test was 500; the national score for the United States was 509 (slightly above the international average); and the highest average national score, 613, was earned by Korea.

As it happens, nine states in the United States also participated in the 2011 TIMSS eighth-grade mathematics assessment. Statewide averages ranged from lows of 466 and 493 in Alabama and California, respectively, to highs of 545 and 561 in Minnesota and Massachusetts, respectively. The bottom two states averaged significantly lower than both the TIMSS and US averages, whereas the top two states averaged significantly higher than both the TIMSS and US averages (see Mullis et al. 2011).

These data indicate why it is essential to understand the current (and dynamically changing!) context in the United States before considering whether something like PRIMAS might be possible in the context of the US political system.

2.1 Contexts, past to present

Ever since its founding as a republic in 1776, the United States has had a strong tradition of "states' rights." These were enshrined in Article 10 of the 1789 "Bill of Rights," which amended the US Constitution. Article 10 stated that "the powers not delegated to the United States by the Constitution, nor prohibited by it to the States, are reserved to the States respectively, or to the people." That is, the federal government has only those powers specifically granted by the Constitution; for example, the power to declare war, to collect taxes, and to regulate interstate business activities. Setting educational policy was not one such power. Hence, the individual states had the right to set their own educational policies.

What this arrangement has meant is that each of the 50 states has had its own requirements for credentialing teachers (certifying their readiness to teach), setting curriculum policy, setting "standards" for student performance, and assessing student performance. Although, as explained below, some homogenizing forces are at work, any attempt to undertake a project like PRIMAS (or for that matter, any curricular research-and-development project of national scope) in the United States would need to deal with individual states and their departments of education, and would need to identify and recruit partner universities in those states. In the absence of an authoritative document such as the report by Rocard et al. (2007) urging problem-based or inquiry-based learning as *the* method of choice for school mathematics instruction, such recruitment might be quite difficult. Here, in part, is why.

2.2 The recent past: an environment of high-stakes testing

Although the federal government cannot *compel* states to adopt particular educational policies, it can *induce* them to adopt particular policies by offering fiscal incentives—and the incentives are often large enough that the states will adopt the policies required by the federal government.

On January 8, 2002, President George W. Bush signed into law the No Child Left Behind Act of 2001 (NCLB). The law continued federal funding to low-income students (known as Title 1 funding), a substantial source of funding for states. But NCLB also included, for the first time in the nation's history, the requirement that all public schools receiving federal funding administer a state-wide standardized test annually to all students. Each school's scores had to meet a minimum threshold each year—and the threshold was increased each year. (This minimum annual target was called "Annual Yearly Progress," or AYP.) If a school failed to meet AYP, there were penalties; and the penalties became increasingly severe each year the school failed to attain the (continuously increasing) AYP. If a school fell below threshold for 2 years in a row, students in the school were given options to transfer to other schools; missing AYP for 4 years in a row was a warrant for "corrective action," which might mean some combination of replacing the staff, changing the curriculum, or making the school day longer. Missing AYP for 6 years in a row resulted in mandatory re-structuring—a complete loss of autonomy and job guarantees for all staff associated with the school. Options for restructuring included closing the school, changing its administrative structure and staff entirely, hiring a private company to run the school, or assigning a state-chosen administrator (with almost unlimited administrative authority) to run the school.

In short, the penalties were draconian. The tests became known as "high-stakes" tests because so much was at stake in terms of student performance. In various districts, whether a student would be promoted to the next grade, whether the teacher would earn a raise in salary or even retain his or her job, and whether a school would remain open—all hinged on students' test scores. As a result, "teaching to the test" became an increasing aspect of instruction over time. To put things simply: When so much is at stake, a large part of instruction becomes preparation for the test or "test prep." On a state-by-state basis, test blueprints and sample test items were made public.[1] Across the nation, teachers and students focused on the content of their state tests.

That focus had a number of consequences. What one had, in essence, was a form of institutionalized chaos: There was wide variation in the 50 sets of standards and assessments across the United States. Moreover, each set of standards and assessments served practically as a very tight set of constraints. If any curriculum project was seen as being at variance with the goal of test performance, the likelihood of its adoption was close to zero. Thus, if an IBL-inspired curriculum did not, at face value, look as if it would prepare students for a particular state's test items, the chance of that project being welcomed in that state's schools would be very small.[2] As we discuss in the following section, the national context is changing—but the relationship of IBL-inspired projects to high-stakes tests is still a major issue.

2.3 The present and near future in the United States: revolutionary or evolutionary change?

The educational landscape in the United States changed once again when, in July 2009, President Barack Obama announced a $4.35 billion educational initiative called Race to the Top (RTT). Like No Child Left Behind, RTT offered significant incentives for the creation of (new and improved) standards and testing—but with a twist. In an attempt to undo the institutionalized incoherence of No Child Left Behind (50 states, each with its own standards and assessment), RTT offered funding to *consortia* of at least five states that could agree upon standards and assessments. Given the very short timetable for meeting RTT's requirements, the National Governors Association and the Council of Chief State School Officers joined together in an effort to support the development of an optional set of "Common Core State Standards" (CCSS) in Mathematics (CCSS-M) and English Language Arts that states could choose to adopt if they wished (for details of the CCSS-M, see National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). The optional nature of the adoption respected the concept of states' rights: Each state could join a consortium that built its own standards if it chose to do so. But as the enormity of the challenge of building consensus standards on a short time frame became apparent, more and more states adopted the CCSS. As it stands now, forty-five of the fifty states have adopted the Common Core State Standards. Thus, for the first time in the history of the United States, there is a de facto national set of standards.

Alongside this dramatic change is an equally significant change in testing—one, again, tailored to unique aspects of the political system in the United States. Although there is in essence one national set of standards, those were

[1] See, for example, California's "testing and accountability" Web pages at http://www.cde.ca.gov/ta/, or Georgia's discussion of "testing and performance," at http://www.doe.k12.ga.us/External-Affairs-and-Policy/AskDOE/Pages/Testing-and-Performance.aspx.

[2] In California, for example, the high-stakes assessments were skills-oriented multiple-choice tests that did not ask for reasoning or meaningful problem solving. In the months before the test, students in classrooms across the state would be seen practicing for it. It is true that research indicates that students who study from a curriculum that gives attention to skills, concepts, and problem solving will do quite well on a skills-oriented test. It takes a brave teacher, however, to maintain a broad focus in instruction when the fates of his or her students—and possibly those of the teacher and the school—depend on how the students will do on a narrow skills-oriented test.

voluntarily adopted. It would have been an infringement on states' rights to offer only one assessment. Hence, two "parallel" assessments are being developed, and each state can opt to use one or the other. It remains to be seen how consistent the two assessments, developed by consortia known as PARCC (Partnership for the Assessment of College and Career Readiness; see http://www.parcconline.org) and SBAC (Smarter Balanced Assessment Consortium; see http://www.smarterbalanced.org), will be. Pilot versions of the assessments will be available for use at the end of the 2013–2014 academic year, and the first high-stakes implementation of the tests will be in the 2014–2015 academic year.

The existence of the CCSS-M, and the aligned assessments, promises yet more tightening of the system—in particular directions. To the degree that those directions are consistent with those of any IBL-inspired program such as PRIMAS, the pathway to development and adoption might be eased; to the degree that the directions are perceived as being in conflict, there would be challenges.

The CCSS-M and associated assessments portend two main changes to current practice. The first is that the CCSS-M list content and learning goals by grade level. This arrangement will have a homogenizing impact across the United States in that the sequence of topic coverage in the curriculum will have much less latitude than under the more distributed system enforced by No Child Left Behind. A priori, there is no reason to believe that this constraint would have a significant impact on the potential adoption of PRIMAS-like materials; such materials would simply have to be developed and placed carefully within the curriculum sequence. The second change, however, has significant implications. The CCSS-M emphasize not only mathematics *content*, but mathematical *practices.* Those practices are listed in Table 1.

The first four of these practices will be emphasized in the scoring schemes of the two assessment consortia, PARCC and SBAC. In particular, for the first time in large-scale testing, SBAC plans to return not just one score in a mathematics test, but four (Smarter Balanced Assessment Consortium 2012). On the Smarter Balanced assessments, students (and schools, and districts, in the aggregate) will receive separate scores on each of the following:

- Concepts and procedures
- Problem solving
- Communicating reasoning
- Modeling and data analysis

To the degree that these categories are transparent[3] and are reinforced by the tests, they will have a very significant impact on curricula and classroom practice in the United States.

Thus, the likelihood of a PRIMAS-like, or inquiry-based, mathematics project taking root over the next few years in the United States would depend, in significant measure, on how well the project could "sell" its version of inquiry as being consistent with (1) perceived societal needs, (2) schooling traditions, (3) the specific framing of CCSS-M goals pertaining to problem solving, communicating and reasoning, and modeling and data analysis, and (4) the readiness of the US teaching force to implement IBL.

Table 1 Mathematical practices highlighted in the CCSS-M

Make sense of problems and persevere in solving them
Reason abstractly and quantitatively
Construct viable arguments and critique the reasoning of others
Model with mathematics
Use appropriate tools strategically
Attend to precision
Look for and make use of structure
Look for and express regularity in repeated reasoning

Source. National Governors Association Center for Best Practices, Council of Chief State School Officers (2010, pp. 6–8)

3 On fit, real and potential

Here we explore the potential for IBL, or a PRIMAS-like curriculum, to be viewed positively within the context described in Sect. 2.

3.1 Fit with perceived societal needs

The central motivation for the PRIMAS project with respect to mathematics was a desire to increase the number of pupils who continue their study of mathematics and ultimately seek to be employed in a mathematics-related field. The need for a strong work force in Science, Technology, Engineering, and Mathematics (STEM) is well recognized in the United States. Indeed, in competitions for Race to The Top funding, extra credit (15 points out of a possible total of 500) was given to states that prioritized STEM funding. In the United States, however, efforts to increase enrollments in mathematics courses have typically depended on raising mathematics requirements for school or university graduation. Efforts to increase employment in mathematics-related fields have relied on market forces to attract employees through means such as attractive wage packages, employment benefits, and working conditions.

[3] By virtue of their being reporting categories for SBAC, the four dimensions will obviously be focal for states using SBAC assessments. Whether they will be salient in the case of PARCC remains to be seen.

Efforts to raise the numbers of pupils studying and continuing in mathematics have not typically been made by changing the nature of instruction in school mathematics, and it is an open question how successful similar efforts might be in the US educational environment.

3.2 Fit with schooling traditions

In their review, Artigue and Blomhøj (2013) make it clear that there is significant philosophical overlap between the notions of "inquiry" and "problem solving." Within the curricular and epistemological traditions in the United States, however, those two terms are worlds apart. It would not be much of an exaggeration, if any at all, to say that *inquiry* is considered the province of science education, whereas *problem solving* is considered the province of mathematics education. Within the United States, the separation is almost complete. In mathematics, the recent round of "standards-based" curricular reform began with the publication of the 1989 *Curriculum and Evaluation Standards* by the National Council of Teachers of Mathematics. The very first standard in the volume is called "mathematics as problem solving." And a search reveals that the word *inquiry* does not appear once in the volume! This is not an isolated occurrence. As noted above, the first mathematical practice described in the CCSS-M is that students will "make sense of problems and persevere in solving them." A search reveals that the word *inquiry* is nowhere to be found within the CCSS-M.

In US school mathematics, *problem solving* is understood to mean engaging in a task whose solution method is not known in advance. It is not simply a curricular goal but also a means for reaching that goal. The standards-based reform promoted by the NCTM (2000), therefore, takes problem solving as one of its so-called process standards and expects that throughout their study of mathematics, US students will have many opportunities to formulate, work on, solve, and reflect on complex mathematical problems. It is not clear how much actual problem solving US students are engaging in while studying mathematics, but curricular materials and teacher development programs have been promoting attention to problem solving in mathematics instruction for over three decades. There has been no such attention to the pursuit of inquiry in school mathematics.

In 1996, the US National Research Council produced a set of *National Science Education Standards*, which played somewhat the same role for the science community that the NCTM Standards did for the mathematics community. The phrase *problem solving* does not appear in the index to the *National Science Education Standards.* When you look up *inquiry* in the index, you are told "See *scientific inquiry*," and when you do, the index devotes a half page to listings throughout the volume. When the teaching standards are introduced, on pp. 29 ff., the largest print in that section says, "Inquiry into authentic questions generated from student experiences is the central strategy for teaching science" (p. 31).

In sum, there is a tremendous gulf between the language and traditions of problem solving in mathematics and inquiry in science. Although it is quite possible that that IBL would find a welcome reception in the science education community in the United States, our sense is that some major work would be necessary to bridge the linguistic and epistemological differences between problem solving and IBL.

And that is not to mention some cultural differences. Of course, there is great cultural diversity across the European nations that collaborated in the PRIMAS project. But, for example, there are very few cultural contexts in the United States where it would be feasible to implement the kind of parental involvement called for in the paper by Mousoulides (2013). And to the degree that radically different cultural contexts across Europe resulted in different implementations of IBL across the nations that participated in PRIMAS, one has to wonder just how large the umbrella encompassing *inquiry based learning* actually is. In the United States, after the publication of the 1989 NCTM *Standards*, many states instituted their own standards. There was so much variation in the implementations, however, that it was not clear what *standards-based instruction* actually meant. Given that IBL would have a similar lack of precision if implemented in any US context, the question would naturally arise as to whether the "same" curricular approach was being taken as had been implemented in the PRIMAS project. As is so often the case with studies of innovative instruction, one would need to be concerned with *fidelity of implementation* before one could draw firm conclusions.

3.3 Fit with CCSS-M

As Sects. 2.3 and 3.3 make clear, there would have to be a significant amount of "boundary crossing"—with regard to both language and substance—before an IBL-based project could hope to receive a warm welcome in the current context of mathematics instruction in the United States. In terms of substance, much about mathematics curricula and assessment is unsettled as we write. What is clear, however, is that in the vast majority of jurisdictions across the United States, anyone proposing a unit for inclusion in the curriculum will need to make a very clear case that students who study that unit will be developing the mathematical understandings (both at the level of content and practices) that are called for by the Common Core State Standards. To date, only a small selection of sample assessment items have been released by PARCC and SBAC. Once the full assessments have become public, instruction (and practice for the assessments) will become even narrower.[4] Hence, proponents of IBL will have to make the case that the understandings developed by any proposed curriculum unit are both a good fit with CCSS-M *and* will help prepare students for the assessments aligned with it.

We believe that such a case can be made for many inquiry-based units in mathematics. Indeed, an overarching perspective toward mathematics is that mathematics should be encountered as a sense-making discipline; that the formal mathematics one learns can be seen as the codification of patterns of inquiry and sense making. One definition of long standing is that mathematics is the "science of patterns" (see, e.g., Devlin 1994; Steen 1988). Some famous examples within the mathematics education literature fit within this tradition; for example, Deborah Ball's class of third graders who noticed, conjectured, and then proved that the sum of two odd numbers is always even (Stylianides 2007). Thus, a substantive case can be made that *inquiry* can and should be seen as an essential component of mathematics instruction. But the case will have to be both substantive (i.e., teachers, students, and the public will have to see that the mathematics learned in an inquiry-based unit is aligned with the CCSS-M and the assessments) *and* linguistic: Either the unit must be presented as a content-related problem-solving unit, or the label *inquiry* will have to be validated in some way. Names and traditions matter. Indeed, the wrong name can place one at a disadvantage: US teachers of mathematics might be resistant to efforts to promote guided inquiry in mathematics because guided discovery (which sounds similar) was a largely unsuccessful pedagogical movement some decades ago.

3.4 Teacher professional development

Teaching for problem solving, or teaching using inquiry methods, or for that matter *any* pedagogical method that calls for leading students into and then guiding them through the construction and validation of conjectures based on patterns of observations, is hard. It calls for a set of skills that few teachers in the United States—whose primary mode of instruction is what Lappan and Phillips (2009) call "show and practice" instruction—possess. A major challenge, then, is *capacity building*—providing pedagogical support for teachers so that they can develop the repertoire of skills and understandings required to teach for inquiry, or for problem solving. Of course, that is not a challenge simply for inquiry-based mathematics education (IBME); it is a challenge for any teachers hoping to teach in the spirit of the CCSS-M as well. Classrooms in which students engage in legitimate inquiry, or which are designed to foster the development of the mathematical practices listed in Table 1, require a very different set of pedagogical skills than classes that operate in show-and-practice mode.

4 Implementing inquiry-based learning on a large scale

The reports in the present volume of experiences implementing IBL in Europe, together with our speculation on how such implementation might operate in the United States, raise some larger issues that anyone attempting IBL implementation on a large scale would need to face.

The papers by Dorier and Garcia (2013) and by Wake and Burkhardt (2013) point to some of those larger issues: rejection of change by teachers and some parents owing to a succession of reforms in recent years; inertial pressure for a more traditional pedagogy; narrowly conceived assessments oriented toward easily quantifiable learning outcomes; pedagogical training programs with limited attention to mathematics and to IBL; and restricted opportunities for, and lack of coherence in, continued professional development. We see these issues as important regardless of the project undertaken to implement IBL.

4.1 Tensions

A major issue raised by any implementation of IBL is that when school mathematics moves away from the so-called deductive approach to teaching and toward an organization according to the development of proficiency in inquiry, the usual curriculum model is disturbed: One is no longer treating a progression of mathematical topics from simple to more complex. Teachers—and curriculum developers—begin to worry that implementing IBL will not allow students to see mathematics as a unitary structure of related ideas. Artigue and Blomhøj (2013) make that point as well as identifying some other tensions:

> These are tensions [inherent to IBME] between the development of inquiry habits of mind and the progression of mathematical knowledge paying the necessary attention to curricular progressions, tension between internal and external sources of mathematical activities, tension between scientific and real life interests.

[4] Of course, high-stakes assessment is a double-edged sword. That it directs and narrows the focus of instruction is a given. During the time of No Child Left Behind, however, there was a wide range of assessments—some mathematically rich, some narrowly skills-oriented. If the assessments tied to CCSS-M are mathematically rich, they could move mathematics instruction in productive directions.

4.2 Goals

Any implementation of IBL must deal with the question of goals for student learning in mathematics. The goals used by PRIMAS are apparently those of the European Reference Framework (European Commission 2007), which defines *mathematical competence* as follows:

> Mathematical competence is the ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations. Building on a sound mastery of numeracy, the emphasis is on process and activity, as well as knowledge. Mathematical competence involves, to different degrees, the ability and willingness to use mathematical modes of thought (logical and spatial thinking) and presentation (formulas, models, constructs, graphs, charts). (p. 6).

It goes on to indicate the essential knowledge, skills and attitudes relevant to that competence:

> Necessary **knowledge** in mathematics includes a sound knowledge of numbers, measures and structures, basic operations and basic mathematical presentations, an understanding of mathematical terms and concepts, and an awareness of the questions to which mathematics can offer answers.
> An individual should have the **skills** to apply basic mathematical principles and processes in everyday contexts at home and work, and to follow and assess chains of arguments. An individual should be able to reason mathematically, understand mathematical proof and communicate in mathematical language, and to use appropriate aids.
> A positive **attitude** in mathematics is based on the respect of truth and willingness to look for reasons and to assess their validity. (p. 6).

Although *competence in science* is defined in the framework so as to include recognizing "the essential features of scientific inquiry" (p. 6), inquiry is not mentioned in the discussion of mathematical competence.

The challenge, on an international scale, is to rationalize this kind of framework against the specifics of nationally mandated standards or goals. We have illustrated the challenge of IBL in the United States, but the same is the case within the European framework as well. As this paper was being written, for example, the British government issued a framework document for the National Curriculum in England (Department for Education 2013b) and for the General Certificate of Secondary Education (GCSE; Department for Education 2013a) in mathematics. The focus in those documents is much more skills-oriented, and less inquiry-oriented, than advocates of IBL would like—and they are undoubtedly going to make it that much more of a challenge to implement PRIMAS-like materials on a large scale in the United Kingdom.

4.3 Politics, including issues of alignment

It is one thing to find a niche within a political system, to find a way for a 100-teacher pilot of a program such as PRIMAS (suitably modified to work within the local context) to take hold and flower. It is quite something else to scale up. Our description of recent politics surrounding the Common Core State Standards, and the changes in the United Kingdom referred to in the previous paragraph, suggest the challenges of operating on a large scale. A change of government, or ministry, or highly placed official who wants to leave his or her mark on educational policy; a new set of standards; or a new set of assessments can change the context in a short amount of time, with the unfortunate result that attempts at curriculum change can be upended. This issue is larger than one that researchers or curriculum developers can handle on their own, but it is one that we must be sensitive to, and one that we as a profession should be concerned about "getting on the agenda" of our national and international professional organizations.

4.4 Communication

Once attempts to implement change become large scale, they enter the public as well as the political arena—and image matters. The so-called math wars in the United States and elsewhere were fought on the basis of image rather than substance (Schoenfeld 2004), and misinterpretations (that standards-based instruction is a reincarnation of the "new math," a slander that caused significant difficulties for advocates of standards-based instruction; or that "inquiry math" would be a reincarnation of "discovery math") could easily cause public rejection of a meritorious approach. A major challenge is to provide parents, teachers, administrators, and politicians an image of what instruction might look like so that there is a shared sense of understanding and so that potentially valuable approaches are not undermined for the wrong reasons.

4.5 Professional development

We discussed this situation vis-à-vis the United States in Sect. 3.4. Of course, contexts vary substantially from nation to nation. But there is some consistency: Few countries have in place systems that can help, at a national scale, to prepare teachers for forms of instruction that vary in significant ways from current practice. How to address this issue is an unsolved problem—a problem certainly not solvable by the development of curricular materials alone.

(Ann Brown, 1992, noted that all curriculum projects undergo mutations when they are put into practice. The challenge, she said, was to avoid lethal mutations.)

4.6 Research questions

How does one take a design research perspective seriously (as Maaß & Doorman, this volume, suggest), so that one contributes to knowledge as well as to improved materials? Maaß and Doorman list seven aspects of design research:

1. **Interventionist**: it aims at designing an intervention in the real world;
2. **Utility-oriented**: the merit of a design is measured, in part, by its practicality for users in real contexts;
3. **Iterative**: it incorporates a cyclic approach of design, evaluation and revision;
4. **Theory-based**: it is based upon theoretical propositions;
5. **Context-oriented**: it considers the context as central for the intervention;
6. **Process-oriented**: a black box model of input–output measurement is avoided; the focus is on understanding and improving interventions;
7. **Theory-oriented**: field testing of the design contributes to theory building.

Each of those aspects needs to be considered carefully. To these, we would add some refinements drawn from Cobb et al. (2003). Design research is most effective when it is not grounded in global theory (e.g., "constructivism") but rather local theory—hypotheses about how and why engaging with the materials will result in students' development of specific understandings. As such, design research puts forth a set of materials *and* a theoretical rationale for how and why they should work. Attempts at intervention should thus provide two things: revised materials *and* a revised theoretical understanding of how students come to grips with that particular topic.

5 Conclusion

Superficially, problem solving in mathematics and inquiry in science are much the same phenomenon. Problem solving in science is simply the finding of answers to scientific questions by generating hypotheses, gathering evidence to test them, and drawing explanations from the evidence. Inquiry in mathematics is no more than finding connections between mathematical concepts and procedures by exploring how that mathematics might be used inside and outside school. Yet at a deeper level, as Artigue and Blomhøj (2013) note, there is an important difference between inquiry-based mathematics education (IBME) and inquiry-based science education (IBSE): "The terrain for inquiry in IBME is broader than that of IBSE". Sources of questions for IBME come not only from applications of mathematics in almost all domains of human activity but also from mathematical objects themselves. Moreover, in our view, although mathematics is not a purely deductive science, its deductive aspect looms much larger than any deductive aspect of science. Whereas mathematical problem solving entails conjecture and plausible reasoning just as scientific inquiry does, once a solution has been obtained, it is necessarily presented as a deduction from what was given in the problem to what was to be found or proved.

In school practice, nonetheless, scientific inquiry and mathematical problem solving look much alike. In both, the learner is an active agent exploring aspects of the subject by using them to address situations as a scientist or mathematician might. The teacher guides the learner's experience so that it will presumably fit within the school curriculum. In both cases, the purpose is to attract more learners to the subject and its study. Although in this commentary, we have identified challenges faced by anyone seeking to promote inquiry-based learning in mathematics, we could not be in greater agreement with that purpose.

This volume of ZDM offers readers valuable information about inquiry-based learning and its realization in the PRIMAS project, a successful attempt to implement aspects of inquiry-based learning in mathematics and science across a dozen European nations. The challenge, given the promise of a new intervention, is to ask, "What is necessary in order to implement the ideas on a much larger scale?" We hope to have illuminated some of the challenges, using the state of mathematics instruction in the United States as an example. The more that we can understand the challenges of helping interventions "travel" and grow, the more likely we will be able to succeed in doing so.

References

Artigue, M., & Blomhøj, M. (2013). Conceptualising inquiry based education in mathematics. *ZDM—The International Journal on Mathematics Education*, *45*(6) (this issue). doi:10.1007/s11858-013-0506-6.

Brown, A. L. (1992). Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *Journal of the Learning Sciences, 2*, 141–178.

Bruder, R., & Prescott, A. (2013). Research evidence on the benefits of IBL. *ZDM—The International Journal on Mathematics Education*, *45*(6) (this issue).

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9–13.

Department for Education, United Kingdom (2013a, June). *Mathematics GCSE subject content and assessment objectives*. http://www.education.gov.uk/schools/teachingandlearning/qualifications/gcses (Accessed 7 Aug 2013).
Department for Education, United Kingdom (2013b, July). *The national curriculum in England: Framework document*. Downloaded July 4, 2013, from http://www.education.gov.uk/nationalcurriculum (Accessed 7 Aug 2013).
Devlin, K. (1994). *Mathematics, the science of patterns: The search for order in life, mind and the universe*. New York, NY: Scientific American Library.
Dewey, J. (1916). *Democracy and education*. New York, NY: Macmillan.
Dewey, J. (1938). *Logic: The theory of inquiry*. New York, NY: Holt.
Dorier, J.-L., & Garcia, J. (2013). Challenges and opportunities for the implementation of inquiry-based learning in day-to-day teaching. *ZDM—The International Journal on Mathematics Education*, *45*(6) (this issue). doi:10.1007/s11858-013-0512-8.
Engeln, K., Euler, M., & Maaß, K. (2013). Inquiry based learning in mathematics and science: A comparative baseline study of teachers' beliefs and practices across 12 European countries. *ZDM—The International Journal on Mathematics Education*, *45*(6) (this issue). doi:10.1007/s11858-013-0507-5.
European Commission. (2007). *Key competences for lifelong learning: European reference framework*. Luxembourg: Office for Official Publications of the European Communities.
Lappan, G., & Phillips, E. (2009). A designer speaks. *Educational Designer*, *1*(3). http://www.educationaldesigner.org/ed/volume1/issue3/article11 (Accessed 7 Aug 2013).
Maaß, K., & Doorman, M. (2013). A model for a widespread implementation of inquiry-based learning. *ZDM—The International Journal on Mathematics Education*, *45*(6) (this issue). doi:10.1007/s11858-013-0505-7.
Mousoulides, N. (2013). Facilitating parental engagement in school mathematics and science through inquiry-based learning: an examination of teachers' and parents' beliefs. *ZDM—The International Journal on Mathematics Education*, *45*(6) (this issue). doi:10.1007/s11858-013-0524-4.
Mullis, I. V., Martin, M. O., Foy, P., & Arora, A. (2011). *TIMSS 2011 international results in mathematics*. Boston, MA: TIMSS and PIRLS International Study Center. See also http://timssandpirls.bc.edu/timss2011/international-results-mathematics.html (Accessed 7 Aug 2013).
National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). Common Core State Standards: Mathematics. Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers.
National Research Council. (1996). *National science education standards*. Washington, DC: National Academies Press.
Rocard, M., Csermely, P., Jorde, D., Lenzen, D., Walberg-Henriksson, H., & Hemmo, V. (2007). *Science education now: A renewed pedagogy for the future of Europe (EU 22845)*. Brussels: European Commission, Directorate-General for Research.
Schoenfeld, A. H. (2004). The math wars. *Educational Policy, 18*(1), 253–286.
Smarter Balanced Assessment Consortium (2012) Content specifications for the summative assessment of the Common Core State Standards for Mathematics. http://www.smarterbalanced.org/wordpress/wp-content/uploads/2011/12/Math-Content-Specifications.pdf (Accessed 7 Aug 2013).
Steen, L. (1988). The science of patterns. *Science, 240*, 611–616.
Stylianides, A. (2007). Proof and proving in school mathematics. *Journal for Research in Mathematics Education, 38*, 289–321.
Wake, G. D., & Burkhardt, H. (2013). Understanding the European policy landscape and its impact on change in mathematics and science pedagogies. *ZDM—The International Journal on Mathematics Education*, *45*(6) (this issue). doi:10.1007/s11858-013-0513-7.