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Do Multiple Solutions Matter? Prompting Multiple Solutions, Interest, Competence, and Autonomy

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Encouraging students to develop multiple solutions for given problems is an important way to improve mathematical knowledge. However, the influence of this teaching element on students' interest-related motivational orientations is an open question. In this article, we report the results of an experimental study ($N = 145$) that was carried out to investigate the influence of prompting students to construct multiple solutions for real-world problems with vague conditions on students' interest in mathematics as well as on their experiences of competence and autonomy and the number of solutions developed. A positive effect of the intervention on students' interest was found using inferential and path analyses. The number of solutions developed by students and their experiences of competence and autonomy mediated the effects of prompting multiple solutions on interest. Implications for teaching practices and future research are discussed.

Key words: Autonomy; Competence; Interest; Modelling problems; Multiple solutions; Real-world problems; Teaching methods

For several decades, mathematics educators have encouraged teachers to use real-world problems as well as to introduce, compare, and reflect on multiple solutions for mathematics problems in the classroom. Promoting the construction of multiple solutions is included in *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000), and studying the effects of multiple solutions on students' learning is an important goal for research in mathematics education. Many studies that have investigated the use of multiple solutions have reported positive results with regard to students' achievement (Große & Renkl, 2006; Rittle-Johnson & Star, 2007, 2009; Rittle-Johnson, Star, & Durkin, 2009; Star & Rittle-Johnson, 2009). However, there are not many results about the effects of prompting students to construct multiple solutions on students' interest- and motivation-related constructs. Because motivational orientations are important outcomes of mathematics education (NCTM, 2000) and there is some strong evidence about the positive connection between motivational constructs (e.g., interest) and academic achievements or career aspirations (Fisher,

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Dobbs-Oates, Doctoroff, & Arnold, 2012; Heinze, Reiss, & Rudolph, 2005; Köller, Baumert, & Schnabel, 2001; Renninger, Ewen, & Lasher, 2002; Wang, 2012), the evaluation of teaching principles such as the prompting of multiple solutions in the classroom should take motivational variables into account.

In this article, we report on a study in which we used a randomized research design to investigate the effects of prompting students to construct multiple solutions for real-world problems on students' individual interests in mathematics, experiences of competence and autonomy, and the number of solutions developed. More specifically, the real-world problems posed in this study did not specify the conditions needed to reach a solution. We consider these to be problems with *vague conditions* and hereafter refer to them to as *modelling problems*. We assume that prompting students to construct multiple solutions for modelling problems improves students' interest and that the number of solutions that students develop, as well as their experiences of competence and autonomy, mediate the effects of this teaching element on students' interest. Another important question we discuss in this article is the influence of students' interest in mathematics prior to the instructional intervention (prior interest) on students' autonomy, competence, and their individual interest measured after the instructional intervention.

MultiMa Research Project

The study reported in this article was conducted within the framework of the Multiple Solutions for Mathematics Teaching Oriented Toward Students' Self-Regulation (MultiMa) project. The main goal of this project is the evaluation of learning environments for real-world problems in which students construct multiple solutions for given problems. More specifically, the goal is to determine the effect of prompting students to construct multiple solutions for real-world problems on students' achievement, beliefs, emotions, attitudes, and solution strategies. In the first stages of the project, we analyzed the solution processes of students in a laboratory study, finalized the assessment tasks and questionnaires, and conducted a pilot study to examine whether prompting students to construct multiple solutions could be applied in the regular classroom (Schukajlow & Krug, 2013a). Next, we conducted the experimental study that we report here. The analysis of different aspects of this experimental study showed the positive effect of prompting students to use multiple solutions on their self-regulation, planning, monitoring, and preferences for solving problems with multiple solutions (Schukajlow & Krug, 2012, 2013a, 2013b, 2013c). No effects were found for students' self-efficacy, values, or uncertainty orientation (Schukajlow & Krug, 2012, 2013c). In this article, we focus on the effects of prompting students to construct multiple solutions for modelling problems on interest and interest-related measures. In the next stages of the project, we will investigate the effect of prompting students to construct multiple solutions for real-world problems in which the conditions needed to solve the problem are specified and the application of different mathematical procedures is prompted.

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Multiple Solutions and Modelling

Multiple solutions are often discussed in the context of research on problem solving (Becker & Shimada, 1997). The experience of solving problems in different ways is seen as crucial for learning new solution strategies and constructing knowledge about problem solving (Silver, Ghousseini, Gosen, Charalambous, & Font Strawhun, 2005). Developing multiple solutions improves students' creativity and fosters mathematical knowledge (Ervynck, 1991; Leikin, 2007; Pólya, 1945). Several instructional approaches that stimulate students to construct multiple solutions in the classroom have been proposed. One instructional approach is based on posing the questions *What if?* and *What if not?* for the given problems (Brown & Walter, 2005). Another approach includes discussions about different solution methods for addressing challenging problems in the classroom (Becker & Shimada, 1997) or the implementation of problems that have a range of acceptable solutions—called open (Silver, 1995) or open-ended problems—in school mathematics (Stacey, 1995).

In this article we focus on students' development of multiple solutions while solving so-called real-world problems. In real-world problems, a situation outside mathematics is given, and students have to construct a mathematical model and use mathematical procedures to solve the problem (Blum, Galbraith, Henn, & Niss, 2007). Although there are many features that characterize real-world problems (see the summary by Maaß, 2010), the core of modelling activities is a demanding transfer process between reality and mathematics. Blum, Galbraith, Henn, and Niss (2007) underlined two important reasons for teaching modelling problems in the classroom. First, solving real-world problems helps a student to understand mathematics better. The second reason to use real-world problems is that students learn how they can apply mathematics and build mathematical models in their current and future lives. Because the ability to solve real-world problems is an important goal of mathematics education, we used this type of problem to investigate the effects of prompting students to construct multiple solutions on students' interest.

Analyses of activities that people engage in while solving real-world problems (e.g., Blum & Leiß, 2007; Burkhardt, 1994; Pollak, 1979; Verschaffel, Greer, & De Corte, 2000) show that there are different ways to construct multiple solutions. We distinguish three categories of multiple solutions (for a similar approach, see Tsamir, Tirosh, Tabach, & Levenson, 2010). One type of multiple solutions results from applying different mathematical procedures or strategies while solving a problem and usually leads to the same mathematical outcome. Another type of multiple solutions includes making different assumptions about vague conditions within the problem and thereby obtaining different outcomes or results for the same problem. The combination of these two types of multiple solutions—different mathematical procedures or strategies *and* different results—is also possible and may be seen as a third type of multiple solutions. In this study, we addressed the second type of multiple solutions (for the importance of making assumptions when solving real-world problems, see Galbraith & Stillman, 2001). The development and discussion of multiple solutions that result from assumptions about vague

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conditions within the problem have an important role in the learning of modelling activities and may contribute to the adequate application of mathematics in real life (Peled & Balacheff, 2011). To demonstrate the development of multiple solutions for real-world problems, we briefly analyze solutions for the parachuting task (Figure 1).

Parachuting

When "parachuting," a plane takes jumpers to an altitude of about 4,000 meters. From there, they jump off the plane. Before a jumper opens his parachute, he free falls about 3,000 meters. At an altitude of about 1,000 meters, the parachute opens, and the sportsman glides to the landing place. While falling, the jumper is carried off target by the wind. Deviations at different stages are shown in the table below.

Wind speed	Side deviation per thousand meters during free fall	Side deviation per thousand meters while gliding
Light	60 meters	540 meters
Medium	160 meters	1,440 meters
Strong	340 meters	3,060 meters

What distance does the parachutist cover during the entire jump?

Figure 1. Modelling problem on parachuting.

Because the conditions that should be considered when solving this problem are not stated, students have to make at least the following assumptions:

- The deviation remains constant at the different stages of the jump;
- The wind has the same direction during the entire jump;
- The wind speed is either light, medium, or strong during each stage; and
- The parachute opened, for example, at 1,000 m above the earth.

Using these assumptions, the problem could be solved by modelling two right-angled triangles and applying, for example, the Pythagorean theorem; the length of the hypotenuses in the triangles could be calculated to provide information about the distances covered in the free-falling and gliding stages, respectively. After adding the two distances, the problem solver could provide an answer to the

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question posed in the problem. If students made different assumptions about the vague conditions in the problem, they could arrive at different results, even when using the same mathematical procedure. The data they did not use for their solution would then appear to be superfluous. Including superfluous data is seen as an important enrichment for constructing cognitively demanding tasks and for preventing students from using superficial problem-solving strategies (Greer, 1997).

Interest and Experiences of Autonomy and Competence

Interest is a motivational construct of specific relevance for learning. An *interest* represents a specific person–object relationship and is characterized by a person engaging and reengaging with this object over time (Hidi & Renninger, 2006; Krapp, 2005). Interest plays an important role in the learning process and is connected to strategy use (Schiefele & Schreyer, 1994), self-regulation (Pintrich, 1999), performance goals (Harackiewicz, Durik, Barron, Linnenbrink-Garcia, & Tauer, 2008), and achievement (Fisher et al., 2012; Köller et al., 2001; Schiefele, Krapp, & Schreyer, 1993). Heinze, Reiss, and Rudolph (2005) found that many students are not intrinsically motivated to learn mathematics and only a few students develop an above-average interest in mathematics at the lower secondary level. Moreover, a low interest in mathematics at the middle school level is believed to exacerbate the trend in the declining number of students who enroll in high-level mathematics courses and thus also the difficulties that universities have in obtaining suitably qualified graduates (Carmichael, Callingham, Watson, & Hay, 2009; Trewin, 2005). However, we do not know how different methods of teaching in the classroom influence students' individual interest (Heinze et al., 2005) and how individual and classroom factors work together to increase or decrease interest (Carmichael et al., 2009). In the traditional method of teaching, students are left with little room for realizing their own interests (Köller et al., 2001). Some research findings have indicated stronger improvement in students' interest in student-centered learning environments, where students work in groups, rather than in teacher-centered instructional settings (Lerkkanen et al., 2012; Schukajlow et al., 2012). Group work on a cognitively demanding problem may produce a *situation of collective interest* (Bikner-Ahsbahs, 2004; Cramer & Bikner-Ahsbahs, 2009), which may improve students' individual interest. Another important factor for interest development may be the connection of problems to real-life situations. Research in the area of statistics showed that giving real-life examples of problems by using videos or authentic short stories improved students' positive attitudes, thus it was suggested that this may also increase their interest (Allredge, Johnson, & Sanchez, 2006; D'Andrea & Waters, 2002; Leong, 2006).

Theories of interest usually distinguish between emotional and cognitive systems that regulate the development of interest and are connected to situational and personal or individual interest dimensions (Hidi & Renninger, 2006; Krapp, 2005). If a person has positive experiences while engaging in mathematics, his or her individual interest in this domain can be improved (Mitchell, 1993; Renninger & Hidi, 2002). Experiences of competence and autonomy are crucial factors that

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provide emotional feedback about students' states of functioning and thus influence interest (Krapp, 2005). These experiences are closely connected to basic psychological needs, which are defined in the self-determination theory of human motivation as "innate, organismic necessities" (Deci & Ryan, 2000, p. 229; see also Ryan & Deci, 2000). According to self-determination theory, every living organism has a system of basic psychological needs that are integrated into the complex system of behavior and motivational control (Krapp, 2005).

The empirical results from vocational education and from research in regular classrooms have indicated the positive influence of experiences of competence and autonomy in learning situations on students' interest-related motivational orientations (Hänze & Berger, 2007; Krapp, 2005).

The use of modelling problems and prompting students to develop multiple solutions for each problem can have positive effects on students' autonomy, competence, and interest. Posing tasks that allow students to develop different solutions and giving students the opportunity to choose their individual solutions can give students a sense of autonomy (Reeve, Bolt, & Cai, 1999; Reeve & Jang, 2006). Requesting that students develop multiple solutions to a problem can provide a valuable experience of competence compared to requiring them to find only one solution. If students have positive experiences of autonomy and competence, their interest in mathematics can also be increased. Furthermore, prior interest is an important factor for interest-related motivational orientations and can positively influence experiences of autonomy, competence, and interest (Marsh, Trautwein, Lüdtke, Köller, & Baumert, 2005; Minnaert, Boekaerts, & Opdenakker, 2008).

Research Questions

In the study reported here, we addressed three research questions:

1. Does prompting students to construct multiple solutions for modelling problems influence students' interest in mathematics?
2. Does prompting students to construct multiple solutions for modeling problems influence students' experiences of competence and autonomy?
3. How does prompting students to construct multiple solutions for modelling problems work together with the number of solutions developed and students' experiences of competence and autonomy to improve students' interest in mathematics?

Hypothesized Mediation Model

The advantage of mediation models compared with measuring only the direct effects of a treatment on an outcome using a black box model is the ability to test assumptions about how the treatment works. The black box model was thus named because it does not contain a mediation effect between the treatment condition (e.g., the teaching method) and the outcome (e.g., individual interest at posttest) or a mediator between interest at pretest and posttest (Figure 2).

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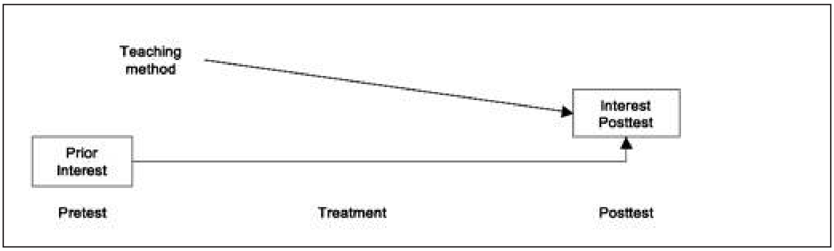

Figure 2. Graphical illustration of the black box model.

Mediation models are derived from theoretical and empirical research and link the treatment condition (e.g., the teaching method) with outcomes (e.g., individual interest at posttest) via mediators (e.g., the number of solutions developed, students' experiences of competence and autonomy). A variable is considered to be a mediator if it transmits the effect of the treatment condition or of students' dispositions (such as prior interest) to the outcome variables. To show a mediation effect of a treatment on an outcome, it is necessary to find first that the intervention influences the mediator and second that the mediator influences the outcome (Judd & Kenny, 1981; Preacher & Hayes, 2008).

Based on theories about interest, the self-determination theory of motivation, and previous research, we constructed a mediation model that represented the influence of prompting multiple solutions on students' interest (Figure 3). Prompting students to construct multiple solutions when solving modelling problems was hypothesized to lead to a higher interest in mathematics than prompting students to construct only one solution for real-world problems in which conditions for solving the problem were specified. Furthermore, we hypothesized that a set of three variables would mediate the effect of prompting multiple solutions on interest. We hypothesized that prompting multiple solutions in the classroom would increase the number of solutions that students would find, the number

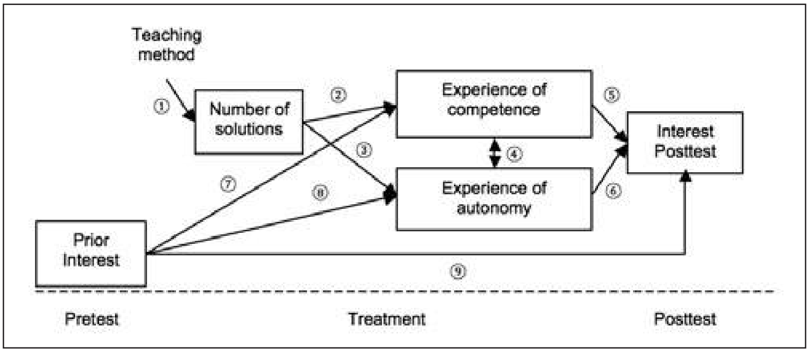

Figure 3. Hypothesized path analytic mediation model (Model 1) predicting how the prompting of multiple solutions would lead to higher interest in mathematics. The double arrow (Path 4) represents a correlation between two measures, and the directed arrows (other paths) represent a direction of regressions of the respective measures.

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of solutions would then positively influence experiences of competence and autonomy, and the positive experiences of competence and autonomy would improve interest in mathematics at posttest. We further hypothesized that prior interest would influence interest at posttest through the experiences of competence and autonomy as well as directly. We also hypothesized that there would be a correlation between experiences of competence and autonomy. Although there is a lot of evidence that supports the direct paths used in our mediation model, no study has yet tested the proposed mediation model.

In this study, variables were assessed using either trait or state scales. Trait scales assess data about the same constructs across time. State scales measure constructs at a specific point in time. We measured interest in mathematics with a trait scale (e.g., Interest: “Doing mathematics is one of my favorite activities”; Frenzel, Pekrun, Dicke, & Goetz, 2012). The number of solutions developed and experiences of competence and autonomy were measured using state scales (e.g., Autonomy: “Today I was able to set my own goals”; Schukajlow et al., 2012).

Linking Treatment Condition to the Number of Solutions Developed

Path 1 illustrates a link between the teaching method (defined by treatment condition) and the number of solutions developed by students. In the treatment condition *multiple solutions*, the development of two solutions was required from every student, whereas in the condition *one solution*, only one solution had to be developed by each student. We predicted that the teaching method in which the development of multiple solutions was required would lead to the development of more solutions than the environment in which only one solution had to be found.

Links Among Putative Mediators

The number of solutions developed in the classroom is connected to students’ experiences of competence and autonomy in the hypothesized Model (Paths 2 and 3). If students develop more solutions, their experiences of competence and autonomy should be higher than when students develop fewer solutions. Because of the lack of studies that have investigated the influence of multiple solutions on students’ need-related measures and interest-related motivational orientations, our assumption about this issue was based on theoretical considerations. The number of solutions that students develop on their own provides them with feedback about their competence and thus may influence students’ experience of competence. If students find more solutions, they may experience themselves as self-regulated and autonomous learners.

A correlation between the experiences of competence and autonomy (Path 4) would support the hypothesis that these measures are related to each other. Indeed, Krapp (2005) emphasized that the basic needs system works holistically. Although it is possible to distinguish between the components of basic needs, there is a relation between them (Minnaert et al., 2008). Students who experience competence feel autonomous and vice versa.

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Linking Putative Mediators to Interest at Posttest

The links from experiences of competence and autonomy to interest at posttest (Paths 5 and 6) have been supported by a number of empirical studies. Marsh, Trautwein, Lüdtke, Köller, and Baumert (2005) found that students' self-concept at the beginning of the school year, which is conceptually related to their experiences of competence and autonomy (Wigfield & Cambria, 2010), predicted their interest at the end of that year. As noted before, there is empirical evidence for the positive influence of students' experiences of competence and autonomy on their interest in vocational education (Krapp, 2005) and at school (Hänze & Berger, 2007). The development of students' interest in the topic of statistics is influenced by students' perceptions of their own competence in the classroom (Carmichael et al., 2009). Students who report increases in their experience of competence also often report a higher intrinsic orientation (Guay, Boggiano, & Vallerand, 2001; Harter, Whitesell, & Kowalski, 1992), which, according to Krapp (2005), can be viewed as an interest-related motivational orientation.

Linking Prior Interest to Putative Mediators and to Interest at Posttest

We further hypothesized that prior interest would influence students' experiences of competence (Path 7) and autonomy (Path 8) as well as their interest at posttest (Path 9). Because interest is a stable construct, the link between interest at pretest and at posttest is obvious and does not need special grounding (for empirical results, see Minnaert et al., 2008). A longitudinal study showed that students with well-developed academic interests at the beginning of the year reported higher self-concepts (Marsh et al., 2005) and positive experiences of competence and autonomy at the end of that year.

Alternative Models

Because there is some theoretical and empirical evidence for alternative mediation models, we also examined two alternative views (Figure 4) of how the above mediators might work together to result in interest in mathematics. We tested the fit of models with paths that were deleted from (Model 2) or added to (Model 3) the hypothesized model (denoted as Model 1).

In Model 2, the correlation between the experiences of competence and autonomy (Path 4, Figure 4) was deleted. This means that the two experiences were assumed to be independent from each other. Support for this model was based on an intervention study in physics that investigated the effect of different teaching methods on students' intrinsic motivation (Hänze & Berger, 2007), and the tested mediation model fit the empirical data well.

The other alternative model (Model 3) was based on the assumption that prompting students to construct multiple solutions influences students' experiences of competence and autonomy not only via the number of solutions developed but also directly (Paths 11 and 12, Figure 4). The additional paths are grounded in the evidence that pointing out the possibility that several solutions can be

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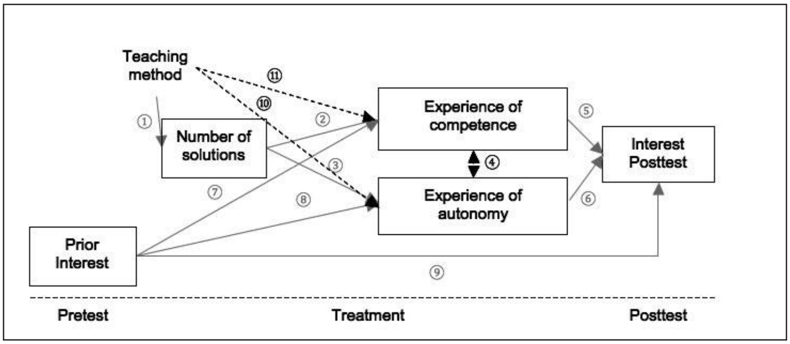

Figure 4. Alternative models to the hypothesized path analytic mediation model. Model 2: Deletion of Path 4. Model 3: Additions of Paths 10 and 11.

developed and allowing students to choose their own method of solving a problem can promote their experiences of autonomy (Reeve et al., 1999; Reeve & Jang, 2006) and competence.

Method

Participants

One hundred forty-five German ninth-grade students (43% female; mean age 15.2 years) participated in the study. They were from three comprehensive schools (German Gesamtschule) that were participating in the MultiMa project. These schools had two middle-track classes each, and the students were selected from these classes. In the middle-track classes, there are students with different levels of knowledge in mathematics, from low- to high-achieving students. Middle-track classes were chosen for the study to increase the representativeness of the sample.

Study Design

Each of the six classes was divided into two parts with the same number of students in each part in such a way that the average achievement—estimated by using students' marks from the previous term—in the two parts did not differ, and there was approximately the same ratio of males and females in each part. One part of each class was assigned randomly to the one-solution or to the multiple-solutions condition (conditions are described in the next section), and the other part of the class was assigned to the other condition (Figure 5). Thus, six groups of students were taught according to the multiple-solutions condition and six groups according to the one-solution condition.

All instruction was delivered by four mathematics teachers between 25 and 54 years of age (two females). Each instructor taught the same number of student groups in the multiple-solutions condition as in the one-solution condition, so the influence of a teacher's personality on students' learning did not differ between treatment conditions. For the teaching unit, the teachers were given all modelling

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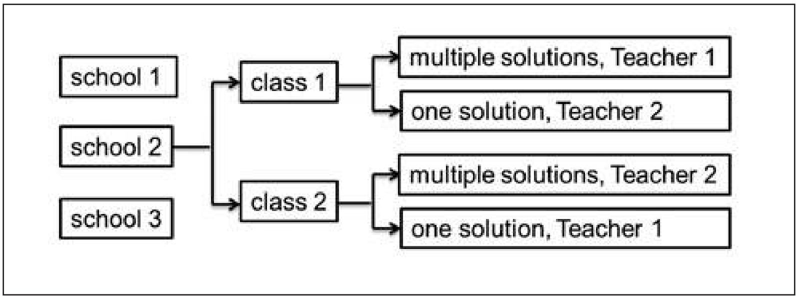

Figure 5. Assignment of students to the treatment conditions.

tasks, including possible solutions; were instructed about the major teaching principles of each treatment condition; and received instruction manuals with lesson plans for both conditions. When instructing teachers about the teaching methods, we informed them that which treatment condition is preferable for students' learning is still an open research question.

The teaching unit was carried out during regular classes and consisted of three thematically different sessions: the first and second lessons (45 and 40 minutes, respectively), the third and fourth lessons (45 and 40 minutes, respectively), and the fifth lesson (40 minutes). After the second and fourth lessons, the participants were asked about the number of solutions they developed in the previous lesson as well as about their experiences of competence and autonomy. Students were also asked about their interest in mathematics before and after the five-lesson teaching unit, which was conducted over 1 week (Figure 6).

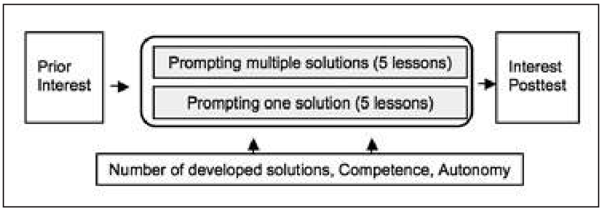

Figure 6. Overview of the study design.

Treatment Conditions

For the purposes of the study, two treatment conditions were implemented. In the multiple-solutions condition, modelling problems were posed, and students were prompted to find two solutions. In the one-solution condition, similar real-world problems were posed, but the questions were phrased to be clearly defined and to require only one solution. All of the problems in both conditions could be solved using the Pythagorean theorem. The topic of the Pythagorean theorem had already been taught using tasks that were not connected to real-world problems.

Both treatment conditions utilized a student-centered learning environment for

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teaching modelling problems that was based on the work of the Didactical Intervention Modes for Mathematics Teaching Oriented Towards Self-regulation and Directed by Tasks (DISUM) project (Schukajlow et al., 2012). This learning environment was complemented by instruction that outlined the specific aims of the teaching unit and the key features of the problems in each condition. The number of solutions that were prompted in the classroom was the critical difference between the teaching methods, and all other instructional procedures, such as methodological order, were held constant across conditions (Table 1). The description of the modelling problems in each condition and the applied instructional procedures are presented in the following sections.

Multiple-solutions condition. Solving the problems given in the condition multiple solutions required substantial modelling activities for transitioning between the real world and mathematics. In addition to the Parachuting problem (see Appendix A), five other problems were posed during the teaching unit (see Appendix B). Four of the six problems in this treatment condition included the request to create and to note two solutions as follows: "Find two possible solutions. Write down both solution methods." Because the development of one solution is typically enough to solve authentic problems in the real world, the two problems that were given in the last lesson did not contain the request for two solutions.

The five-lesson teaching unit consisted of three thematically different sessions: setting goals and the first experience of solving modelling problems with multiple solutions (first and second lessons), practicing the development of multiple solutions (third and fourth lessons), and solving modelling problems that did not request multiple solutions (fifth lesson).

Because students often have difficulties in solving real-world problems (Blum, 2011; Verschaffel et al., 2000) and are not familiar with tasks that allow more than one solution, the instructors directed students' attention toward the following features of modelling problems in the first and second lessons of the teaching unit: connection of the problems to the real world, consideration of superfluous data as well as vague problem conditions, and the possibility that different solutions could be developed. Students solved the first problem in groups of three or four according to the fixed cooperation script (Schukajlow et al., 2012): (a) working individually, reading the text and getting a first idea about how to solve the problem; (b) working cooperatively, exchanging ideas with other students in the group; and (c) working individually, writing down an individual solution. Afterwards, teachers showed how this task could be solved and answered students' questions. Students solved another problem according to the same cooperation script, presented their solutions to the whole class, discussed the estimations they had made to account for vague problem conditions, and explained how their estimations influenced their results. After that, the teachers summarized the focal features of the problems presented in the first two lessons. In the third and fourth lessons, students worked in groups to practice developing multiple solutions while

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Table 1
Instructional Procedures

Instructional procedure	Multiple solutions	One solution
Methodological order: Goals of teaching unit; group work (alone, together, alone); presentation and discussion of solution/solutions by students; summarizing key points of the lesson	+	+
Problem features: Using six real-world problems in each condition with the following features:		
• Problems that require substantial transition between the real world and mathematics and have superfluous data	+	+
• Problems with vague conditions that require two solutions		
• Problems with specified conditions that require one solution	+	+
Multiple-solutions condition		
• Solving modeling problems that require two solutions	+	
• Discussing different solutions during group work	+	
• Presentation of different solutions by students and discussing their differences and similarities	+	
• Summarizing the link between different solutions	+	
One-solution condition		
• Solving real-world problems with specified conditions that require one solution		+
• Discussing the solution during group work		+
• Presentation of the solution by students		+
• Summarizing the development of the correct solution		+

solving the third and fourth modelling problems and presented and discussed their solutions. In the fifth lesson, the same method was applied, but the problems did not include the statement to find two solutions.

One-solution condition. Solving problems in the one-solution condition also required knowledge of the transition between the real world and mathematics. We used similar problems in this teaching condition as we did in the multiple-solutions condition, but we provided all of the important data for solving the problem in the task. Information about the data that students needed to develop a solution to problems was collected in the previous study (Schukajlow & Krug, 2013a). For example, for the parachuting problem, the exact altitude data (e.g., “3,000 meters” instead of “about 3,000 meters”) were presented in the problem, and the information about wind power was specified in the question: “What distance does the parachutist cover during the whole fall if a wind of medium power blows? Write down your solution method” (see

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Appendix A). All of the problems in Appendix B were similarly modified.

The timing and instructional procedures applied in the one-solution condition were the same as in the multiple-solutions condition. The five-lesson teaching unit consisted of three sessions similar to the other treatment condition: setting goals and the first experience of solving modelling problems that required only one solution (first and second lessons), practice in solving such problems (third and fourth lessons), and continued work on problems that required only one solution (fifth lesson).

After identifying the connection of the problems to the real world and superficial data included in the problems as the core feature of the tasks, students solved the first problem according to the same cooperation script as in the multiple-solutions condition. The teacher demonstrated how the correct solution could be found and discussed the solution with students. In the second lesson, problems were solved in groups, and students presented their solutions and discussed the data they used to solve the problem. Finally, teachers summarized the features of the problems solved in the lesson. In the third, fourth, and fifth lessons, new problems were presented in the same way as in the second lesson.

Treatment Fidelity

The following procedures served to ensure a high degree of fidelity. Instructors who took part in the study had at least 2 years of teaching experience and were familiar with the teaching of modelling problems as well as with the teaching methods used in this study. In each lesson, at least one person from the research group was present in order to administer tests and assessment scales and to observe the implementation of the treatment. All of the lessons were video recorded, and students' written work was collected; analysis of both showed that the treatment conditions were implemented appropriately. No differences between treatment conditions in the amount of time on tasks were reported by observers. However, in the multiple-solutions condition, students used the time for solving problems in two different ways. In the one-solution condition, students discussed the solution longer than in the multiple-solutions condition.

Measures

Students' mathematics interest and experiences of competence and autonomy were assessed using three separate measures (Table 2). Each measure was assessed on a 5-point Likert scale ranging from 1 (*not at all true*) to 5 (*completely true*). For the measurement of interest, three items from the Project for the Analysis of Learning and Achievement in Mathematics (PALMA-Project) were used: (a) "I am interested in mathematics," (b) "I like to read books and solve brain teasers related to mathematics," and (c) "Doing mathematics is one of my favorite activities" (Frenzel et al., 2012). These items measured students' interest in mathematics and have been validated with qualitative and quantitative methods. Analyses of interviews with 70 students from Grades 5 and 9 showed that each item assessed the emotional and personal dimensions of the students' individual interest (Frenzel

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et al., 2012). In the current study, the reliabilities (Cronbach's alpha) were .80 and .74 at pre- and posttest, respectively, and were similar to the reliabilities found in other studies—Schukajlow et al. (2012) reported reliabilities of .77 and .83 in a study with 209 students.

Table 2
Items Used in the Study to Assess Interest and Experiences of Competence and Autonomy

Scale	Item
Interest	I am interested in mathematics.
(Frenzel et al., 2012)	I like to read books and solve brain teasers related to mathematics.
	Doing mathematics is one of my favorite activities.
Experience of competence	I noticed that I really understood things (Hänze & Berger, 2007).
	I felt able to master the work (Hänze & Berger, 2007).
	I feel able to deliver the required performance.
	I felt confident about my knowledge about the topic today.
Experience of autonomy	Today I was able to . . .
(Schukajlow et al., 2009)	• set my own goals.
	• organize my time.
	• decide how to participate in the lesson.
	• decide how I wanted to work.
	• check my solutions myself.
	• correct my solutions myself.

Both measures for competence and autonomy have been used in previous studies. The scale experience of competence was based on four items that refer to understanding and mastering the assigned work. Two items were taken from the study by Hänze and Berger (2007). Another two items were developed for the current study. The reliabilities of this scale measured after the second and fourth lessons were .91 and .90, respectively. Students' experiences of autonomy were assessed using a scale that refers to setting goals, time management, participating in classroom work, and choosing solution methods as well as the monitoring and correction of results. This six-item scale has been tested several times (Schukajlow & Blum, 2011; Schukajlow et al., 2009) and also showed good reliabilities in the current study (.81 and .79 after the second and fourth lessons, respectively).

We collected data on the number of solutions students developed for each of the problems they solved. At each of the two assessment points, students considered their solutions to the problems that had been taught in the previous lessons. They responded using a 4-point scale; for example, "While solving the problem 'Parachuting,' I developed (0 = no solution; 1 = one solution; 2 = two solutions,

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3 = more than two solutions)." The last two categories were aggregated into the category *two and more solutions*, so this scale ranged from 0 to 2.

Results

Statistical Analyses

First, we analyzed the influence of treatment conditions on the state measures (number of solutions, experience of competence, and experience of autonomy) and on the trait measure (interest in mathematics). Second, the hypothesized mediation model and alternative mediation models were examined. In all regression analyses, we used dummy codes for the treatment factor (0 = one solution; 1 = multiple solutions).

To answer the research questions, three path models with different numbers of parameters were applied to the data. In the current study, the ratio of subjects and parameters in the examined mediation models ranged from 6.3 to 10.4 and were above the critical value of 5 for obtaining correct results (Kline, 2005). Thus, the sample of 145 participants was sufficient for addressing the research questions.

Clustering data. To increase the external validity of the results, the students were instructed in groups of 11–14 students from the same mathematics class rather than individually; thus, the clustering of students within treatment conditions was possible. To avoid clustering effects, students attending the same class were randomly assigned to the two treatment conditions, taking into account their achievement and gender. To examine the baseline equivalence of student groups ($n = 12$), or rather possible clustering effects, for interest at pretest, we computed the intraclass correlation coefficient (ICC) using the statistical program Mplus (Muthén & Muthén, 2012). The ICC was very low (0.03). The within-group variability was significant ($p < .001$), and the between-group variability was not significant ($p > .10$). Thus, no significant clustering effects of interest within treatment conditions were found, and no statistical control of these effects using, for example, clusters as nested factors (Hopkins, 1982), was needed.

Missing values. Missing values are an important issue that is typically found in longitudinal studies. The percentage of missing values differed in the current study across the measures from 10% for prior interest to 1.3% for experiences of competence and autonomy during the treatment. The use of multiple imputation or the full information maximum likelihood estimator (Peugh & Enders, 2004) are the preferred solutions for this problem. The missing values in the current study were estimated using the maximum likelihood algorithm implemented in Mplus (Muthén & Muthén, 2012). This algorithm uses the full information of the covariance matrices to estimate the missing values.

Preliminary Analyses and Analysis of Treatment Effects

Students in the multiple-solutions condition did not differ from students in the one-solution condition in their prior interest (Table 3). The comparison of interest

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Table 3

Students' Interest on the Pretest and Posttest in the Multiple-Solutions Condition and One-Solution Condition

	Pretest		Posttest		Adjusted posttest[a]				
	MS	OS	MS	OS	MS	OS			
Variable	*M*(*SD*)	*M*(*SD*)	*M*(*SD*)	*M*(*SD*)	*M*(*SD*)	*M*(*SD*)	$t(df = 143)$	p	d[b]
Interest	2.55(.92)	2.53(.99)	2.85(.94)	2.54(.92)	2.85(.74)	2.54(.73)	2.310	$< .05$	0.42

Note. MS = multiple-solutions condition. OS = one-solution condition.

[a] Adjusted by pretest. [b] Effect size (Cohen's *d*) for adjusted posttest measures.

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at posttest, taking into account the respective prior interest, showed that the students in the multiple-solutions condition were more interested in mathematics than the students in the one-solution condition ($p < .05$). Thus, prompting them to find multiple solutions while solving modelling problems was found to have a nearly medium-sized positive effect on students' interest (Cohen's $d = 0.42$).

Students in the multiple-solutions condition felt more autonomous ($p < .05$, $d = 0.40$). The experience of competence tended to be higher in the multiple-solutions condition ($p = .21$, $d = 0.21$; see Table 4). Students' means in autonomy and competency differed between the multiple-solutions and one-solution conditions by more than one third and more than one fifth of a standard deviation, respectively.

Table 4
Number of Solutions Developed and Experiences of Competence and Autonomy During the Teaching Unit in the Multiple-Solutions Condition and One-Solution Condition

Variable	MS	OS			
	M(*SD*)	*M*(*SD*)	*t*(*df* = 143)	*p*	*d*
Number of solutions	1.55(.38)	1.16(.34)	6.496	< .05	1.08
Experience of competence	3.83(.71)	3.65(.95)	1.268	.21	0.21
Experience of autonomy	3.64(.67)	3.35(.78)	2.486	< .05	0.40

Note. MS = multiple-solutions condition. OS = one-solution condition.

We investigated how many solutions students in the multiple-solutions and one-solution conditions developed across all problems. In the multiple-solutions condition, 4% of the students did not find a solution, 38% reported finding one, and 58% reported finding two or more solutions. In the one-solution condition, 5% of students did not find a solution, 75% reported finding one solution, and 20% of the students reported finding two or more solutions. The numbers of solutions developed within the teaching units differed markedly (more than 1 *SD*) between the two groups ($p < .05$, $d = 1.08$). As we assumed in the first research question, students in the multiple-solutions condition developed more solutions than students in the one-solution condition.

Analyses of Mediation Models

Statistical procedure. For the statistical analyses of the mediation models, we used the maximum-likelihood algorithm implemented in Mplus (Muthén & Muthén, 2012). This algorithm allows the calculation of fit values for the mediation models. The comparative fit index (CFI) and the standardized root mean squared residual (SRMR) are most adequate for sample sizes with fewer than 250 persons

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(Hu & Bentler, 1999). We applied the combination of cutoff values of CFI > .95 and SRMR < .09 to test the goodness of the fit of the models. Additionally, we report the chi-square goodness of fit, which is often used to verify the fit of proposed mediation models. In order to test the relative fit of nested models, we used chi-square difference tests.

Model fit. Table 5 presents the correlation matrix of the variables measured in the current study. All the calculated correlations were in the expected direction (e.g., interest measures were significantly correlated with experiences of competency and autonomy).

Table 5
Correlations Among Measures

Variable	1	2	3	4	5
Pretest					
1. Prior interest	–				
Treatment					
2. Number of solutions	-.06	–			
3. Experience of competence	.25*	.26*	–		
4. Experience of autonomy	.19*	.37*	.64*	–	
Posttest					
5. Interest at posttest	.62*	.09	.35*	.28*	–

* $p < .05$.

The analysis of model fit values was computed for the black box model (illustrated in Figure 2). The only independent measures in the black box model are prior interest and treatment. The black box model showed good fit; 40% of the variance in interest at posttest could be explained by this model (Table 6).

As expected, the hypothesized mediation model (Model 1) and the alternative mediation models (Models 2 and 3) explained more variance in interest (42%) than the black box model. Thus, the mediation models seem better able than the black box model to explain students' interest in mathematics. However, the fit values of the mediation models differed significantly (Table 6). The hypothesized model (Model 1) fit the data well according to all fit indexes. When the correlation between the experiences of competence and autonomy was removed, the resulting alternative model (Model 2) did not fit the data well. The chi-square value, the CFI, and the SRMR were all poor for Model 2. Therefore, this model had to be rejected. The analysis of the fit statistics for the other alternative model (Model 3) revealed better results. If paths from the treatment condition to experiences of competence and autonomy were added to the hypothesized model (Model 1), the resulting model (Model 3) fit the data well. To test the relative fit of Model 3

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(Model 1 vs. Model 3), we used a chi-square difference test. The test value showed no significant differences between the models $\chi^2(2, n = 145) = 0.61$, $p = .74$. Because both models fit the data well according to the chi-square statistic, the more restrictive hypothesized model (Model 1) was thus a better choice than the alternative model (Model 3).

Table 6
Fit Values for the Black Box Model and the Mediation Models

	Black box	Mediation Model 1	Mediation Model 2	Mediation Model 3
R^2	.40	.42	·42	.42
$\chi 2$	0.021	6.01	65.177	5.492
df	1	6	7	4
p	> .05	> .05	< .05	> .05
CFI	> .95	> .95	·70	> .95
SRMR	< .09	< .09	.10	< .09
AIC	879	1371	1428	1374
BIC	903	1427	1482	1436

Note. R^2 = explained variance of interest at posttest; CFI = comparative fit index; SRMR = standardized root mean squared residual; AIC = Akaike; BIC = Bayesian.

Direct and mediated effects on students' interest. In this section, we present the results for the analysis of the direct and indirect effects estimated in the hypothesized mediation model (Model 1) and in the black box model. Parameter estimates and significance levels for the black box model and the hypothesized mediation model are presented in Table 7. In the black box model, the multiple-solutions condition directly predicted students' interest in mathematics ($\beta = .16$). As expected, students' prior interest was a strong direct predictor of interest at posttest ($\beta = .60$).

In the mediation model, the experience of competence ($\beta = .16$), but not the experience of autonomy, had a nearly significant influence on students' interest ($p = .08$). Prior interest remained a valued factor that directly predicted interest at posttest ($\beta = .56$) in the mediation model. The correlation between experiences of competence and autonomy measured within the teaching unit was high ($\beta = .58$) and thus represents an important part of the hypothesized model. Prior interest directly influenced the experiences of competence ($\beta = .26$) and autonomy ($\beta = .21$) and was a strong predictor of these factors in addition to the number of solutions developed. The number of solutions predicted the experience of competence ($\beta = .28$) and autonomy ($\beta = .38$) and was strongly predicted by the treatment condition ($\beta = .96$).

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Table 7

Estimates for the Black Box Model and Mediation Model 1

Paths and correlation parameter	Black box		Mediation Model 1	
	β	p	β	p
Direct effects				
From treatment to interest at posttest	.16	< .05	-	-
From treatment to number of solutions (1)	-	-	.96	< .05
From number of solutions to exp. of competence (2)	-	-	.28	< .05
From number of solutions to exp. of autonomy (3)	-	-	.38	< .05
Between exp. of competence and autonomy (4)	-	-	.58	< .05
From exp. of competence to interest at posttest (5)	-	-	.16	.08
From exp. of autonomy to interest at posttest (6)	-	-	.07	.44
From interest at pretest to exp. of competence (7)	-	-	.26	< .05
From interest at pretest to exp. of autonomy (8)	-	-	.21	< .05
From interest at pretest to interest at posttest (9)	.60	< .05	.56	< .05
Indirect (mediated) effects				
From treatment to interest at posttest	-	-	.07	< .05
From number of solutions to interest at posttest	-	-	.07	< .05
From interest at pretest to interest at posttest	-	-	.06	< .05
From treatment to exp. of competence	-	-	.26	< .05
From treatment to exp. of autonomy	-	-	.36	< .05

Note. Because treatment conditions represented a binary factor (one-solution condition vs. multiple-solutions condition), the StdY values were used in Mplus to calculate the standardized estimates. Thus, β coefficients may be interpreted as the predicted change in (residualized) criterion measures (in standard deviation units) when the treatment changes from 0 (one solution) to 1 (multiple solutions). According to the hypothesized mediation model, we can see that if the prior interest increases one standard deviation ($SD_{\text{pre\_i}}$), the interest in posttest changes by $\beta_i * SD_{\text{post\_i}} = .56 * SD_{\text{post\_i}}$.

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The mediation effects from three factors on students' interest at posttest were also tested in the mediation model: from treatment condition through the number of solutions and experiences of competence and autonomy, from the number of solutions students developed through experiences of competence and autonomy, and from prior interest through experiences of competence and autonomy. All tested (total indirect) mediation effects on interest at posttest were significant. Prompting students to find multiple solutions influenced students' interest through the number of solutions and students' experiences of competence and autonomy (β = .07). The number of solutions students developed indirectly influenced students' interest through experiences of competence and autonomy (β = .07). Students' prior interest indirectly influenced interest at posttest through experiences of competence and autonomy (β = .06). When combined with the direct influence of interest at pretest to interest at posttest (β = .56), the total effect of interest at pretest is β = .62.

The mediation effect from the treatment condition on experiences of competence and autonomy within the teaching unit was also examined. The indirect effect of the treatment condition on students' experiences of competence and autonomy through the number of solutions was found to be significant, with β = .26 and β = .36 respectively.

Discussion

The effects of prompting students to find multiple solutions for real-world problems on students' interest-related motivational orientations address an important issue in mathematics education. Although there are some theoretical models about the development of interest and empirical results about the connection between different interest-related motivational orientations, there has been a lack of information about how these measures work together and influence students' interest. In this study, we analyzed the importance of prompting students to find multiple solutions for modelling problems with regard to students' interest and identified important factors that mediated the effects of doing so.

We hypothesized that prompting multiple solutions for modelling problems would have positive effects on the number of solutions students developed, their experiences of autonomy and competence, and their reported interest in mathematics. Indeed, we found positive effects of prompting students to find multiple solutions on the number of solutions developed and experiences of autonomy and competence during the teaching unit as well as on students' interest at posttest. These results are in line with other studies on this issue that have revealed the importance of giving students choices about individual solution routes with respect to students' autonomy (Reeve et al., 1999; Reeve & Jang, 2006) and interest (Guberman & Leikin, 2013; Guthrie et al., 2006; Reynolds & Symons, 2001). Furthermore, we were able to extend research on interest-related motivational orientations (e.g., Krapp, 2005; Renninger & Hidi, 2002) by finding that students' experiences of competence tended to be higher with the multiple-solutions teaching method. Our findings confirm the view that instructional conditions can

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have an effect on students' motivational orientations (Hidi & Renninger, 2006) and students' individual interest (Heinze et al., 2005). Because students in both the multiple-solutions and one-solution conditions all worked in groups and experienced the same methodological order but on different tasks, we argue that the types of problems addressed in the classroom play an important role in fostering students' interest and experiences of autonomy and competency (see Cramer & Bikner-Ahsbahs, 2009). However, the increase in interest cannot be expected from only the connection of the task to reality. Instead, it is from the connection of the task to reality in combination with the cognitive demand placed on the tasks due to the vague conditions.

The relevance of students' prior interest to experiences of autonomy and competence in addressing modelling problems as well as for interest at posttest was another point we investigated. Previous studies have shown the relationship between students' interest and motivational experiences or competence-related measures (Köller et al., 2001). However, the direction of this connection was not completely clear. The results of our study show that students' prior interest positively influences their experiences of competence and autonomy. Interest at posttest tended to be higher when students felt autonomous. Moreover, students' interest increased significantly if they felt competent during the teaching unit.

Finally, the factors that mediated the effect of prompting students to find multiple solutions and of prior interest on students' interest at posttest were identified. The results indicate how the mediators work together to transmit the effect of prompting multiple solutions for modelling problems and prior interest on students' final interest. These findings contribute to understanding of an open research question (see Carmichael et al., 2009; Heinze et al., 2005) about how classroom and individual factors work together to improve students' interest in mathematics.

The Hypothesized Mediation Model

As expected, the predicted mediation model explained more variance in interest than the black box model and showed a good fit to the data. A theory-derived addition or deletion of paths led to the deterioration of fit values. Thus, the hypothesized model adequately described the influence of prompting students to find multiple solutions for modelling problems and of prior interest on students' final interest.

Treatment and development of multiple solutions and students' experiences of autonomy and competence. Researchers have identified the quality of problems posed and the solution methods applied to those problems as critical components for student learning (e.g., Hiebert et al., 2003). In our study, the number of solutions that students developed on their own was a crucial factor that influenced students' learning outcomes. We expected that the number of solutions that students developed would provide students with feedback about their competence in the learning situation and about their ability to act autonomously and to

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positively improve both measures. This assumption was confirmed in our study: The number of solutions that students developed positively influenced students' experiences of autonomy and competence. Prompting students to find multiple solutions to modelling problems gave students favorable opportunities to enhance the number of solutions they developed; interest and experiences of competency and autonomy were positively influenced through the number of solutions students developed. As expected, prompting multiple solutions had a strong direct effect on the number of solutions they developed and also an indirect effect on students' experiences of autonomy and competence. Thus, encouraging every student to develop several solutions to modelling problems increases the number of solutions that students construct and leads to considerably more experiences of autonomy and competence within learning situations.

Experiences of autonomy and competence and students' interest. In theories about interest, the importance of experiences of competence and autonomy has been assumed (Krapp, 2005). Our findings partially support this assumption. An experience of competence had a nearly significant effect on interest in mathematics ($p = .08$). The power of this influence is comparable to the findings of other studies (Guay et al., 2001; Hänze & Berger, 2007). Thus, mathematics instructors should take into account the opportunity to increase students' experience of competence in order to improve motivation-related measures such as interest. The influence of the experience of autonomy on interest was small and not significant, in contradiction to the theoretical assumption of theories about interest. Indeed, although research has shown that interest-related motivational orientations depend on experiences of autonomy and competence, the parts of these experiences that are important are still not completely clear. In some studies (e.g., Krapp & Wild, 1998), only one of the two experiences was crucial for interest-related measures, whereas in other studies, they both positively influenced interest-related motivational orientations (Krapp, 2005; Lewalter, 2002). We assume that context variables and students' characteristics should be taken into account to clarify the answer to this question. Whereas in some studies, content activities were directly manipulated (e.g., the treatment condition in our study) and the same methodological order was applied, in other studies, the influence of the regulation of a person's own activities (e.g., student- vs. teacher-centered teaching methods) on motivation was investigated using the same content (Hänze & Berger, 2007; Lerkkanen et al., 2012). Another possible explanation for the nonsignificant effect of autonomy on interest could be the different levels of student performance in our sample. We know from other studies that many students do not want the opportunity to make decisions about their learning process (Eccles et al., 1993), so the relation between the experience of autonomy and interest-related motivational orientations can differ for students with different achievement levels. This assumption has been confirmed in experimental studies in which autonomy support was found to be positively connected to students' expectancies or academic self-concept for high but not for low achievers (Patrick, Skinner, &

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Connell, 1993; Wang, 2012). Because our sample included middle-track and not only high-achieving students, a weak and statistically nonsignificant effect of autonomy on interest is in line with the results of these previous studies.

Treatment and development of multiple solutions for modelling problems and students' interest. Another important result of our study concerns the interplay of teaching methods, state-related measures within instructional settings, and individual interest. Effects of the multiple-solutions treatment on students' interest were mediated through the number of solutions that students developed and their experiences of autonomy and competence. These mediating factors have to be taken into account in order to achieve the effects of prompting multiple solutions for modelling problems on interest-related motivational orientations. Instructional settings that require students to construct multiple solutions for modelling problems provide students with the opportunity to discuss individual solutions during group work, to present and discuss differences and similarities between solutions, and to summarize the links between solutions. This in turn can improve students' interest, a claim supported by researchers who have reported on the effectiveness of these instructional settings for developing students' interest in mathematics (Bikner-Ahsbahs, 2004; Cramer & Bikner-Ahsbahs, 2009; Schukajlow et al., 2012).

One implication of our findings about measures that mediate the effects of instructional settings on students' interest-related measures pertains to the role of having students actively construct solutions for cognitively demanding tasks. Our findings show the crucial role of this factor for experiences of competence and autonomy and thus also for interest. Lower positive effects on experiences of competence, autonomy, or interest could be expected for students who are not successful at developing multiple solutions for modelling problems. Thus, we argue that teachers should support students in developing multiple solutions during problem solving. However, further investigations are needed to clarify the influence of solving cognitively demanding tasks on students' interest-related measures. On the one hand, students are more interested in solving cognitively demanding tasks (e.g., Guberman & Leikin, 2013), presumably because solving these tasks supports feelings of autonomy and competence. On the other hand, if students perceive tasks as too difficult, their interest could decrease (e.g., Horvath, Herleman, & McKie, 2006).

Strengths and Limitations

The effect of treatment condition and prior interest on experiences of autonomy and competence as well as on interest was explored using inferential and path analyses. Because the teaching method was actively manipulated in our study, causal interpretations of effects of the multiple-solutions treatment on the outcome measures are allowed. However, one important limitation of the present study concerns causal interpretations of paths in the hypothesized mediation model. To examine the hypothesized mediation model, we collected data before, during, and

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after the teaching unit. Therefore, the data used in our analyses were ordered along a timeline. Such an ordering of data allows some conclusions to be drawn about the direction of influence of measured factors, but it cannot replace randomized experimental studies (Spencer, Zanna, & Fong, 2005). Thus, the results of the path analysis should be interpreted carefully.

The hypothesized mediation model was derived from theories about interest. The results of previous empirical studies have supported the assumed links and paths between the treatment, the numbers of solutions developed, students' interest in mathematics, and their experiences of competence and autonomy. One limitation derives, however, from the possible incomplete specification of mediators. Other important mediators could be student performance or level of students' mathematical discourse during group discussion (Rittle-Johnson & Star, 2009), which could influence the development of solutions and students' experiences of autonomy and competence during instruction. Linking interest-related motivational orientations and student performance or level of students' mathematical discourse in a hypothesized mediation model should be an important issue for future research. In addition, students' self-efficacy, which was shown to mediate the relation between performance and interest in statistics (Carmichael, Callingham, Hay, & Watson, 2010), should be taken into account in future research. Additionally, we encourage researchers to investigate the effects of prompting multiple solutions for modelling problems on other important measures such as self-concept, value, uncertainty orientation, metacognitive knowledge, and performance (Schukajlow & Krug, 2012, 2013b, 2013c).

Also, our analyses were conducted using a relatively small sample and only a few items for each scale. More students are needed for better estimation of path values and for increasing the stability of the analyses of mediation models, and more items are needed to increase the validity of the scales. Investigations of the stability of changes in students' interest using follow-up measures as well as analyses of the effect of longer interventions on students' interest are other important issues that should be assessed in future research.

Conclusion

Promoting the development of multiple solutions in mathematics classrooms is an important teaching element that has not often been investigated using experimental designs. The aim of the present study was to explore the effects of this teaching element on students' interest and experiences of competence and autonomy. Our results complement the findings of previous studies with regard to the positive effect of prompting students to find multiple solutions for modelling problems on students' learning. The inferential and path analyses, which were based on theories about interest, showed the positive influence of prompting students to find multiple solutions on individual interest in mathematics. Thus, the prompting of multiple solutions for modelling problems has to be paid more attention in teachers' everyday teaching practice to provide positive experiences of autonomy and competence and to increase students' interest in mathematics.

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APPENDIX A

Sample problems presented in the treatment conditions multiple solutions and one solution.

Parachuting (Multiple-Solutions Condition)

When "parachuting," a plane takes jumpers to an altitude of about 4,000 meters. From there, they jump off the plane. Before a jumper opens his parachute, he free falls about 3,000 meters. At an altitude of about 1,000 meters, the parachute opens, and the sportsman glides to the landing place. While falling, the jumper is carried off target by the wind. Deviations at different stages are shown in the table below.

Wind speed	Side deviation per thousand meters during free fall	Side deviation per thousand meters while gliding
Light	60 meters	540 meters
Medium	160 meters	1,440 meters
Strong	340 meters	3,060 meters

What distance does the parachutist cover during the entire jump? Find two possible solutions. Write down both solution methods.

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APPENDIX A (continued)

Parachuting (One-Solution Condition)

When "parachuting," a plane takes jumpers to an altitude of 4,000 meters. From there, they jump off the plane. Before a jumper opens his parachute, he free falls 3,000 meters. At an altitude of 1,000 meters, the parachute opens, and the sportsman glides to the landing place. While falling, the jumper is carried off target by the wind. Deviations at different stages are shown in the table below.

Wind speed	Side deviation per thousand meters during free fall	Side deviation per thousand meters while gliding
Light	60 meters	540 meters
Medium	160 meters	1,440 meters
Strong	340 meters	3,060 meters

What distance does the parachutist cover during the whole fall if a wind of medium power blows? Write down your solution method.

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APPENDIX B

Problems presented in the treatment conditions multiple solutions: Fire Brigade, Broadcasting Tower, Half-Timbered House, Parachuting (cf. section “Multiple solutions and modelling”), Salt Mountain, Ironman Austria

Fire Brigade

In 2004, the Munich fire brigade got a new fire engine with a turn-ladder. Using the cage at the end of the ladder, the fire brigade can rescue people from great heights. According to the official rules, while rescuing people, the truck has to keep a distance of at least 12 meters from the burning house.

Technical data of fire engine

Engine model:	Daimler Chrysler AG Econic 18/28 LL - Diesel
Construction year:	2004
Power:	205 kw (279 HP)
Cubic capacity:	6374 cm³
Dimensions of fire engine:	Length 10 m width 2.5 m height 3.19 m
Dimensions of ladder:	Length 30 m
Weight of unloaded truck:	15540 kg
Total weight:	18000 kg

From what maximal height can the Munich fire brigade rescue people with this fire engine? Find two possible solutions. Write down both solution methods.

Source: Reprinted with permission of Springer. The fire brigade task was taken from Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in the teaching and learning of mathematical modelling: Proceedings of the 14th Biennial International Conference on the Teaching of Mathematical Modeling and Applications* (pp. 15–30). New York, NY: Springer.

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APPENDIX B (continued)

Broadcasting Tower

There is a broadcasting tower at Ras Al Zawr in Saudi Arabia. It was built for radio and television broadcasting in the abutting area. The tower has a power of 743 kHz and is located close to the Persian Gulf. It is 208 meters high and is anchored by 1,098 meters of steel rope in a desert.

At what distance from the tower are the steel ropes positioned? Find two possible solutions. Write down both solution methods.

Half-Timbered House

In Germany, there are more than one million half-timbered houses. In such constructions, wooden beams often form rectangular areas. In a lumber mill, only one piece of timber is made from each log to minimize the amount of wood chips.

In the image below, you see a log. What is the maximum length and width of a rectangular cross section of timber that can be cut from a log with a diameter of 20 centimeters? Find two possible solutions. Write down both solution methods.

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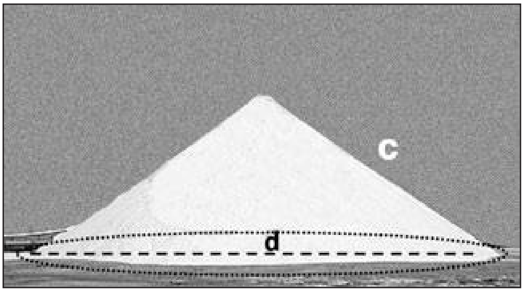
APPENDIX B (continued)

Salt Mountain

In medieval times, salt was obtained from seawater by evaporation. Currently, it is extracted from salt mines. Salt is transported on a 1.2-meter-wide conveyer belt and piled up into huge mountains.

See the salt mountain diagram below. It is *c* meters long and has a diameter of approximately 20 meters, depending on the water content in the salt.

Calculate the height of the mountain. Find one possible solution. Write down your solution method.



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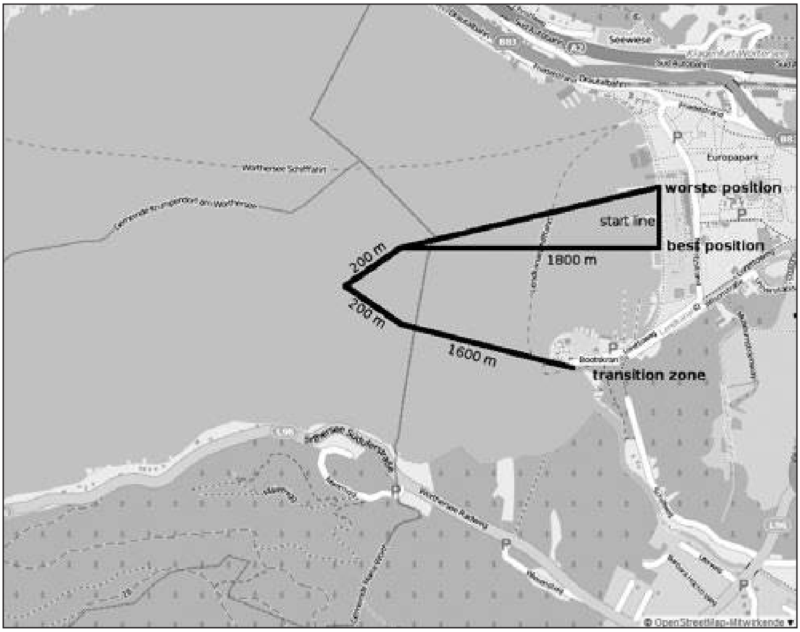
APPENDIX B (continued)

Ironman Austria

In the middle of July, the Ironman Austria (triathlon) takes place in Kärnten at Lake Wörth. Athletes have to complete the following distance events:

- 3.8 km swimming in Lake Wörthersee,
- 180.0 km cycling, and
- 42.2 km running (marathon).

More than 1,200 participants begin with the swimming competition. Because 1,200 people is a large number, the athletes start along a 410-meter-long starting line at the sea side (see the plan below). A participant then has to swim around three turning buoys. Someone who begins swimming from the right side of the starting line has to swim a longer distance than a participant who starts from the best position on the left side.



Each participant can begin anywhere in the starting line. How much more than an athlete at the best starting position does another participant have to swim? Find one possible solution. Write down your solution method.

Source: © OpenStreetMap-Mitwirkende

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