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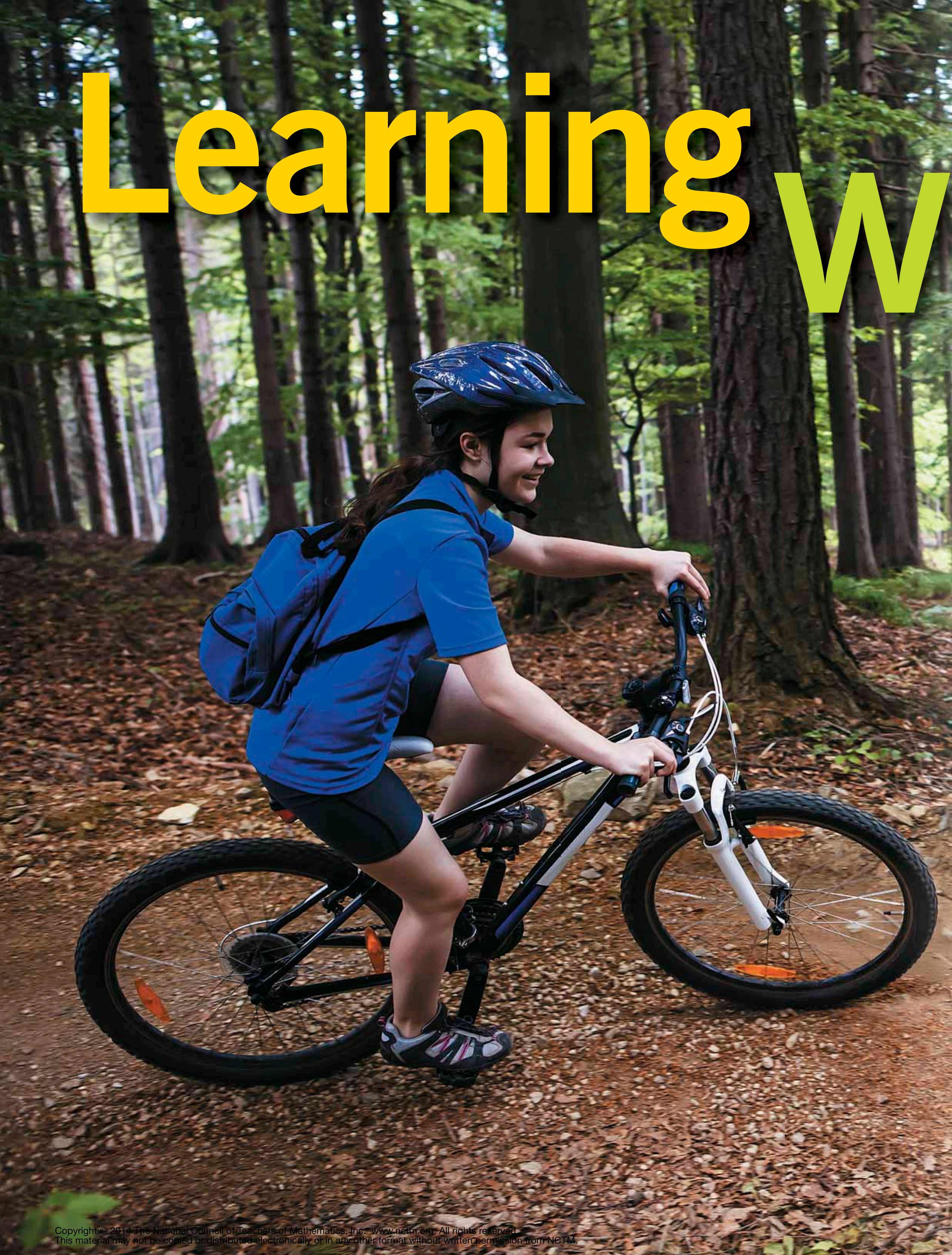

hat Works:

Promoting Small-Group Discussions

Elicit productive discourse from students as they work through a bicycle rate problem.

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Many teachers have designed lessons for students who will be working in groups to discuss and solve a problem. After investing time in constructing an interesting problem, creating strategically designed groups, and introducing the problem carefully, teachers may be left wondering how to help students collaborate to make sense of mathematical ideas. Group work in which students labor together on open-ended problems has become a regular feature of many mathematics classrooms (Jackson et al. 2012; Lappan et al. 1998; Zahner 2012). This format allows students to communicate their mathematical thinking as well as listen carefully and respond to the ideas of their peers (CCSSI 2010; NCTM 2000).

That said, teachers can find it difficult to facilitate productive interactions in several groups of students simultaneously. Teachers may wonder, "Are my students equipped with the necessary strategies to interact productively and respectfully with their peers?"

We present excerpts from student conversations during a middle school mathematics lesson in which students worked in small groups on a problem about rates of change and linear relationships. The teacher had used group work previously, and the students had already experienced working in groups on an open-ended problem. However, working together on one or two problems that would require an entire class period to solve was an unusual experience for them. Although many students struggled with this format of activity, the groups displayed aspects of productive collaborative behavior that we identified when thinking about how to better support students' group work.

Some students in the class used strategies for maintaining productive mathematical discussions within their groups. These strategies included—

1. asking questions about the problem;
2. sharing the mathematical authority within the group; and
3. challenging one another's mathematical ideas.

Not all groups used all these strategies, but the tools gave students a way

to maintain productive conversations about the problem. By teaching these strategies to students, they will have more resources for working effectively in groups.

PRESENTING THE PROBLEM

The problem explored a fictional character, named Isabel, and her arrival time at a campsite called Yellow Park. The Isabel task was part of a larger problem-based lesson in which students analyzed piecewise linear relationships in the context of two friends traveling to a common destination from different starting locations. Isabel rode her bike to the campsite and left a series of postings on a social networking site (see **fig. 1**). The students had to determine whether Isabel would make it to the campsite before 8:00 p.m.

Because they were expected to determine exactly what time Isabel would arrive and to show how they established that time, students had to decide within their groups how to approach the problem. The teacher, Mr. Jenkins (all names are pseudonyms), introduced the problem-based lesson in four different seventh-grade math classes, giving students one class period to work on the problem in pairs or small groups. The following excerpts are examples of students' productive interactions.

ASKING A QUESTION ABOUT THE PROBLEM

One strategy that students used was to *ask a question about the problem*. Some students formulated their own questions about the content of the problem or problem-solving process and asked those questions to members of their groups. This did not happen immediately, and many students initially attempted to work out a solution independently. It may seem obvious that, when working together, students would ask questions of one another. However, this is a departure from more traditional classroom interactions in which

Fig. 1 Students used Isabel's posts to decide if she would reach the campsite before 8:00 p.m.

Isabel made some postings on Facebook©:

Isabel Riley Beautiful day for a bike ride. 12 mph, keeping the cadence!
August 19 at 2:44pm Like

Isabel Riley No rain on my parade! A flat tire, but I'm prepared with my kit!
August 19 at 3:44pm Like

Isabel Riley Back on track! I have to go faster now! 16 mph.
August 19 at 4:14pm Like

Isabel Riley At the gas station. I'm halfway there! Going to rest and have a snack.
August 19 at 4:59pm Like

Isabel Riley 10 mph. The snacks slowed me down, but nothing will stop me!
August 19 at 5:30pm Like

the teacher controls the discussion and asks all the questions (Cazden 2001; Mehan 1979). When students work together in groups, asking questions is the responsibility of the students. It can be challenging for students to establish this practice.

In the excerpt that follows, Blake, Allen, and Kristen were trying to decide how far Isabel had traveled up to the halfway point of her trip. They had already concluded that Isabel traveled 12 miles between her first and second posting, and they were trying to decide how far she went between her third and fourth posting. Blake initiated a conversation about Isabel's distance with a question to Allen.

Blake: But how much to go from there to there?
Allen: Well, you have to find out how long her distance is.
Blake: Yeah.
Allen: That's 45 minutes, so that's like a fourth, or 3/4 of an hour.
Blake: Twelve, right?
Allen: Yeah.
Blake: Twelve miles? Well it couldn't because—it was 3/4 right?
Allen: Yeah.
Blake: So 3/4 of that is 12, right?
Allen: Yeah.
Blake: Twelve, 12 that's 24. So she went 24 miles, and she's halfway there, so 48 is the whole way.
Allen: Yeah.
Kristen: Forty-eight is the whole thing.

Blake's question made explicit a part of the task for which the group did not yet have a solution. He also helped position his group to find the answer. Allen responded by clarifying that they needed to determine Isabel's distance, which helped the pair better define the task they had to accomplish. Blake and Allen decided that Isabel had traveled 12 miles during that portion of her trip, or 24 miles total in the first half. By asking a question, Blake initiated a joint problem-solving effort with Allen that allowed them to clarify what they needed to do and to

successfully achieve part of the solution to the problem. Although Blake initiated the conversation by asking a question, the dialogue was not ideal because Kristen sat silently while Blake and Allen established how far Isabel had traveled.

The exchange above illustrates a common challenge that we observed among students working together. Namely, in posing a question, students were likely to initiate a back-and-forth dialogue with one other member of the group. Although the interaction above seemed productive for Blake and Allen, Kristen appeared to have little meaningful interaction in the conversation. Asking their peers questions created opportunities for students to discuss the problem and monitor one another's understanding. However, students did not always pursue strategies to include all voices of the group. For this reason, we looked for ways in which students were able to distribute the problem-solving process more evenly among members of a group.

SHARING THE MATHEMATICAL AUTHORITY IN THE GROUP

We found that some students took turns asking and answering questions, and different students presented different parts of the solution. We identify these interactions as *sharing the mathematical authority within the group*. In the excerpt below, Kayla, Stephen, and Karen were trying to decide how far Isabel would travel if she went 16 miles per hour for 45 minutes.

Kayla: Well, because she only goes 45 minutes, so that wouldn't be a full hour.
Stephen: So that means we only have 45 minutes, not a full hour.
Kayla: Exactly. So she won't go exactly 16 miles.
Karen: She'll go about . . .
Stephen: She'll go like 14 and a half, or something like that.
Karen: How would you figure that out? 'Cuz she was on that for 45 minutes.
Stephen: I don't know. I just kind of guessed.

At the beginning of the conversation above, Kayla took on a position of authority in asserting that Isabel would not travel a full 16 miles. Stephen then pursued an authoritative position in predicting that Isabel would travel 14.5 miles. Although his prediction was incorrect, Stephen initiated an idea about how far Isabel might have traveled. By questioning Stephen, Karen acknowledged that his idea was the starting point, but she did not accept it as the final answer. The group continued their discussion.

Kayla: So if you divide 16 in half that's 8. So, for half an hour, and then you add . . .
Stephen: But it was 45 minutes.
Kayla: Exactly. So you have to add that onto this.
Karen: So it'd be like 14. I mean 12. It'd be 12, in 45.
Stephen: Yeah.
Kayla: It would? [Checks computations on calculator.] Yeah, so it'd be 12 miles.

After Stephen acknowledged that he did not have adequate reasoning for his prediction of 14.5 miles, Kayla offered a new strategy for finding the distance. Once Kayla identified that Isabel would travel 8 miles during the first 30 minutes, Karen offered that she would, therefore, travel 12 miles during the entire 45 minutes. Karen provided the final answer to that segment of the problem, which Stephen and Kayla accepted.

The three students shared the authoritative role. No one person in the group had a complete solution to the problem, and no one person asked all the questions. Since no single student was the sole authority, all three students were able to contribute parts of the solution and build on one another's ideas. The collaboration helped them work toward a solution and experience ownership.

Sharing the mathematical authority was one of the most challenging aspects of group work for the students in our study. Students were often more likely to accept the authority of one or two students who found a solution to the problem. The conversation above among Karen, Stephen, and Kayla illustrates students' potential to jointly establish the solution to a problem. By taking turns asking questions, more students in the group have an opportunity to contribute part of the answer.

CHALLENGING ONE ANOTHER'S SOLUTIONS AND STRATEGIES

We found that some students productively *challenged one another's solutions and strategies*. Rather than immediately accepting the truth of the ideas proposed by their peers, students pushed one another to explain or justify their solutions. Students challenged group members to develop a more sophisticated solution to the problem or to make the underlying mathematical argument more explicit for the group. In the following excerpt, Melanie and Bryant were discussing how long it had taken Isabel to get to the halfway point of her trip and when she would arrive. Melanie and Bryant challenged each other in the course of developing a complete solution to the problem. Bryant insisted that they "have to figure it out." The group worked quietly for about 30 seconds, doing computations on the page.

Bryant: It took her, from here to here when she was going 12 miles an hour, it took her exactly 1 hour.
Melanie: You don't know how far she is there.
Bryant: Yeah you do, because 12 miles per hour. She traveled exactly 1 hour.

Table 1 Examples of teaching moves support students' collaboration.

Before group work: Set expectations	When you are working in your groups, I expect everyone to ask at least one question to the rest of the group about the problem.
	Once you have answered a question, I expect you to give other group members opportunities to answer questions. No one person should have all the answers.
	Remember that when we challenge one another's ideas, we should be challenging the mathematical content only and doing it respectfully.
Before, during, and after group work: Model the strategies	Jocelyn had a good question about how to find how far Isabel traveled in 45 minutes. I want her to ask that question to the class and see if anyone can answer it.
	Abe has convinced us that it will take Isabel less than 2.5 hours to travel the last 24 miles. Can anyone pick up where Abe left off and build on his answer?
	Manuel, you said you think that Isabel traveled 14.5 miles in 45 minutes, but what is your justification for that number?
During group work: Make strategies explicit	I just heard Ming ask a good question about how Jason knew the last 24 miles would take less than 2.5 hours. I want to make sure that the question gets answered before you move on.
	I appreciate that after Maurice figured out that 20 miles would take Isabel 2 hours, Rachel pointed out that the last 4 miles would take less than 30 minutes. Now work on deciding exactly how long those last 4 miles will take.
	Taylor, you do not look convinced by Mitchell's solution. This would be a good opportunity to challenge him if you disagree.

Melanie: So, so she traveled 12 miles there.
Bryant: Yeah.

Melanie estimated that Isabel would get to the campsite around 7:00. By saying, "I think we actually have to figure it out," Bryant challenged Melanie to develop a more complete solution to the problem. Several seconds later, Bryant pointed out that Isabel went 12 miles per hour for 1 hour. Melanie challenged Bryant with "you don't know how far she is there." In response, Bryant explained the importance of traveling 12 miles per hour for 1 hour. By challenging Bryant, Melanie pushed him to make his reasoning clear. In so doing, Melanie understood where the 12 miles came from, and they continued solving the problem.

By challenging one another's solutions, the students pushed each other to develop a mathematically sound solution and make their reasoning explicit. Similar to sharing the mathematical authority, we found that the act of *challenging* one another was a less common strategy among the groups in this study. However, when students used this strategy, it effectively engaged them in explaining their ideas and listening to those of the group.

PROMOTING COLLABORATION

The examples above illustrated strategies that were effective for students in maintaining productive, collaborative work on the problem. This is not to say that all students engaged in these practices or that students were experts in knowing how to discuss mathematics with their peers. Rather, we found that, among the groups working on the problem, the strategies of asking questions, sharing the mathematical authority, and challenging one another's mathematical ideas supported those students who used these strategies in maintaining their joint work on the problem. Recognizing the strategies that worked for students can help teachers pursue those approaches to maintain productive discussions. We suggest ways that teachers can promote discourse strategies among more students in their groups (see **table 1**). These moves can add to the repertoire of actions that mathematics teachers can perform to promote productive classroom discussions (Chapin, O'Connor, and Anderson 2003; Herbst 2011; Smith and Stein 2011).

Set Expectations for Students' Interactions

Setting expectations for interactions before students form groups can provide a road map for their conversations. Such expectations are not meant to impose rigid rules on group work but to structure students' interactions and help them know what is expected when they work together. Some students may not have had many experiences talking about mathematics with their peers during group work. In this setting, students may not know how

to interact or what types of conversations they should be having in their groups. The teacher can help students learn how to talk about mathematics with their peers by providing guidelines for their conversations.

Model the Strategies That Students Use

Before, during, and after group work, the teacher can model the types of discussion strategies that students may use when they work in groups and make explicit the strategies that some students may already be using. Modeling shows students the value of asking thoughtful questions about a math problem and helps in establishing clear expectations.

Make Students' Strategies Explicit during Group Work

Finally, while students work in groups, the teacher can point out instances when students use discourse strategies productively or have an opportunity to use such a strategy. Students may not always recognize opportunities to pose questions to their group members. They may not realize when they are in a position to share the mathematical authority with their group or may not feel empowered to challenge another group member's idea. The teacher can encourage students to maintain their own mathematical discussions and provide them with specific strategies to engage in productive discussions on their own in the future.

LEADING PRODUCTIVE DISCUSSIONS

Mathematics educators have identified teaching actions for leading productive mathematical discussions (Chapin, O'Connor, and Anderson 2003; Herbel-Eisenmann and Cirillo 2009; Horn 2012; Smith and Stein 2011). In this article, we have identified strategies that students used to manage their own discussions during group work. By teaching students to collaborate productively in the mathematics classroom, we can foster the skills and strategies for supporting students' mathematical learning.

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Any thoughts on this article? Send an e-mail to ***mtms@nctm.org***.—*Ed*.

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