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“Playing the game” of story problems: Coordinating situation-based reasoning with algebraic representation

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ABSTRACT

This study critically examines a key justification used by educational stakeholders for placing mathematics in context –the idea that contextualization provides students with access to mathematical ideas. We present interviews of 24 ninth grade students from a low-performing urban school solving algebra story problems, some of which were personalized to their experiences. Using a situated cognition framework, we discuss how students use informal strategies and situational knowledge when solving story problems, as well how they engage in non-coordinative reasoning where situation-based reasoning is disconnected from symbol-based reasoning and other problem-solving actions. Results suggest that if contextualization is going to provide students with access to algebraic ideas, supports need to be put in place for students to make connections between formal algebraic representation, informal arithmetic-based reasoning, and situational knowledge.

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The importance of mathematics instruction including contexts relevant to students' lives and experiences is widely acknowledged (Common Core State Standards Initiative, 2010; National Council of Teachers of Mathematics, 2000, 2006, 2009), including internationally (Palm, 2009; Xin, 2009). In mathematics education, story problems, or word problems, have been heavily researched as a mode of contextualization, which is not surprising given that they are a large part of curriculum, instruction, and assessment from kindergarten to undergraduate mathematics courses (Jonassen, 2003). A recent national survey of 743 Algebra I teachers compiled for the National Mathematics Advisory Panel showed that “solving word problems” was cited as the most serious deficiency of incoming students (Loveless, Fennel, Williams, Ball, & Banfield, 2008). This suggests that issues surrounding story problems are of primary concern to educators today, and that story problems are considered essential to the transition from arithmetic to algebra.

In the case of algebra, there are two views of the way in which contextualized problems and mathematics formalism should be juxtaposed. In the *symbol precedence view* (Nathan & Petrosino, 2003), algebraic symbolism should be presented first, and story problems are then used as a way to apply these formalisms. This is based on the idea that symbolic problems bypass English-language comprehension demands, and are easier to solve because they do not require the additional step of translating words to symbols (Koedinger & Nathan, 2004). Consistent with a symbol precedence perspective is the notion that the primary purpose of story problems is solving the “transfer problem”: by giving students contextualized problems in addition to abstract problems, they will be better prepared to face the demands of using mathematics in everyday situations and in the workplace.

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A competing position is the *verbal precedence view*. From this perspective, verbal skills develop before symbol manipulation skills, and thus instruction on story problems should be presented before symbolic equations (Nathan & Petrosino, 2003). This work hypothesizes that "early in the acquisition of a formal skill, students can succeed with grounded representations by using informal strategies that do not require abstract formalisms" (Koedinger, Alibali, & Nathan, 2008, p. 370). Consistent with the verbal precedence work is the idea that contexts may provide accessibility or scaffolding for students, with concrete and familiar situations providing a bridge between what the students know and the abstract mathematics they are trying to learn (Boaler, 1994). Studies have shown that while many teachers and textbooks subscribe to symbol precedence views, students' performance better corresponds to a verbal precedence model (Koedinger & Nathan, 2004; Nathan & Koedinger, 2000a, 2000b; Nathan & Petrosino, 2003; Nathan, Long, & Alibali, 2002).

Research on arithmetic story problems has called into question whether the common justifications behind either of these two models are appropriate. The situated cognition perspective, as discussed by Greeno (2006), asserts that intelligent behavior takes place in complex social systems that include learners, teachers, curriculum materials, and the physical environment, as well as representational, material, and conceptual resources. In this framework, "school mathematics" represents its own social system whose norms, standards, and practices are distinct from problem solving in other contexts. From a situated cognition perspective, mathematical representations such as symbolism are interpretive conventions embedded in social activity and intended to be used as tools to promote participation (Brown, Collins, & Dugid, 1989; Greeno & Hall, 1997). However, the use of symbolism in school algebra tasks often does not resemble authentic uses of representation; indeed, in many school mathematics tasks, including story problems, symbolism is an end in and of itself (Reusser & Stebler, 1997). A body of research has demonstrated that problem solving in school mathematics differs from the applied problem solving of professionals and practitioners (Hoyles, Noss, & Pozzi, 2001; Lave & Wenger, 1991; Masingila, Davidenko, & Prus-Wisniowska, 1996; Saxe, 1988; Taylor, 2005), that students rarely apply everyday knowledge to stereotyped, oversimplified school-based tasks like story problems (Baranes, Perry, & Stigler, 1989; Greer, 1997; Palm, 2008; Reusser & Stebler, 1997; Xin, 2009), and that such application can actually disrupt problem solving (Boaler, 1994; Cooper & Harries, 2009; Inoue, 2005; Kazemi, 2002; Roth, 1996).

As suggested in the preceding discussion, there are a number of different justifications why contextualized problems should be included in mathematics instruction, including arguments for their utility in areas outside of mathematics, for the importance of developing critical citizenship skills, and for fostering creativity, self-reliance, and confidence through problem-solving (Blum & Niss, 1991). However, here we primarily focus on the idea that contextualization can promote access to mathematical learning, which has been explained in several different ways in the mathematics education literature. First, as suggested by the verbal precedence work, verbal skills develop prior to symbolic skills, and thus verbal contexts may allow entry into problem solving through the use of informal, invented strategies (Carraher, Carraher, & Schliemann, 1987; Koedinger & Nathan, 2004; Nathan & Koedinger, 2000b, 2000c). Second, contextualized problems may leverage what students already know about quantities (Baranes et al., 1989; Carpenter, Fennema, & Franke, 1996; Carraher et al., 1987), operating on quantities (Carraher, Schliemann, Brizuela, & Earnest, 2006), and relationships between quantities (Chazan, 1999; Kaput, 2000; Lampert, 2001). Third, and related to the prior two points, concrete representations like story problems may support intuitive understanding, acting as a perceptual scaffold to ground abstract concepts that might be otherwise difficult to grasp (Goldstone & Son, 2005). Finally, relevant contexts may promote interest and motivation (Anand & Ross, 1987; Cordova & Lepper, 1996; Davis-Dorsey, Ross, & Morrison, 1991), which mediate attention and persistence (Durik & Harackiewicz, 2007; Renninger & Wozinak, 1985).

The present study critically examines the idea that contextualization promotes access to mathematical ideas in algebra. The primary focus is on "traditional story problems," which we define as relatively short and closed-ended problems embedded in contexts that reference objects, people, and events from the world – i.e., "real world" contexts. We investigate whether findings from situated studies of arithmetic story problems are applicable to algebra learning, and look at new considerations that arise through the interaction of story contexts with symbolic representations of variable quantities.

1. Literature review

1.1. Research on arithmetic story problems

Arithmetic story problems came to the attention of many researchers in math education following the results of the 1983 National Assessment of Educational Progress (NAEP). This assessment revealed that while U.S. students were able to solve routine, one-step story problems, they had difficulty with non-routine problems that required novel approaches or careful analysis of the story situation (Carpenter, Matthews, Lindquist, & Silver, 1984). Highlighted was a division story problem given to 13-year olds: "An army bus holds 36 soldiers. If 1,128 soldiers are being bussed to their training site, how many buses are needed?" (p. 491). Results showed that 29% of students included the remainder of the division problem in their answer, even though it makes no sense in the context of the story, and another 18% ignored the remainder rather than including the additional needed bus. Based on the NAEP results, it was concluded that many U.S. students had not developed problem-solving skills and "attempt to apply mechanically some mathematical calculation to whatever numbers are given in a problem" (p. 490).

Concurrent research on students solving simple arithmetic problems found that slight variations in problem wording resulted in children using different strategies (Carpenter & Moser, 1984). Further, young children have issues with text comprehension in arithmetic story problems, and students' mistakes often represent correct answers to misinterpreted stories (Cummins, Kintsch, Reusser, & Weimer, 1988). Elementary students may or may not use their knowledge of everyday situations (i.e., "real world" knowledge) when solving story problems, and activation can depend on how the situational context interacts with the numbers given in the problem; for instance, monetary units like 25 cents or time units like 15 min are easier to work with when contextualized (Baranes et al., 1989).

Recent research has found that when given arithmetic story problems that do not have enough information to be solved or that require "practical considerations," students largely adhere to the norms of schooling, making the assumption that all story problems are of a stereotyped nature and have a direct computational answer based on the given numbers (Greer, 1997; Palm, 2008; Reusser & Stebler, 1997; Xin, 2009). An example of such an "impossible" problem is: "Martin's best time to run 100 m is 10.00 s. How long will it take him to run 10,000 m?" (Palm, 2008). As Reusser and Stebler (1997) write,

> As illustrated by data from our studies, most students perceived word problem solving as a puzzle-like activity with no grounding in factual real-world structures and with no relation to a goal-directed, more authentic activity of mathematization or realistic mathematical modeling. (p. 323)

Many unrealistic responses students give to arithmetic story problems represent unanticipated but valid interpretations of story contexts based on their everyday experiences and diverse sense-making activities (Inoue, 2005). Other students conform to "sociomathematical norms" (Cobb & Bowers, 1999), aspects of school activity specific to mathematics learning, which suggest that meaningful application of real world knowledge to stereotypical word problems is unproductive, and that focusing on direct calculation approaches is most sensible. Similarly, when solving multiple choice story problems, elementary students may focus on the answer choices rather than on making sense of the situation, and although students may draw on situational knowledge when their connection with the context is strong, this knowledge can interfere with reasoning and cause students to make assumptions that are incorrect (Kazemi, 2002).

1.2. Research on context personalization

Another line of research has investigated the benefits of personalizing story problems to individual students' interests and experiences. This research is especially relevant to the role of contextualization in providing access, since personalized problems are often designed to leverage student prior knowledge or to enhance motivation. One study found that personalizing instruction on order of operations to elementary school students' interests as measured by questionnaires enhanced learning compared to a control condition (Cordova & Lepper, 1996), with similar findings for arithmetic word problems involving addition and subtraction (Anand & Ross, 1987; Davis-Dorsey et al., 1991). However, situational rewording intended to enrich story contexts does not lead always to increased performance (Cummins et al., 1988; Vicente, Orrantia, & Verschaffel, 2007). Personalized contexts may focus students' attention more closely on the situational aspects of the story, allowing students to connect with the task, but can be distracting to students with lower interest in mathematics (Renninger, Ewen, & Lasher, 2002).

There is detailed knowledge in the field of how students think about arithmetic problem solving, how situational knowledge and verbal understanding affects cognition, how the system of school mathematics interacts with problem solving, and how personalization affects learning. However, studies of algebra story problems have traditionally had a substantially different focus.

1.3. Research on algebra story problems

Students' tendency to use arithmetic rather than algebraic approaches to solve algebra story problems has been well-documented (Hall, Kibler, Wenger, & Truxaw, 1989; Koedinger & Nathan, 2004; Stacey & MacGregor, 1999). Students with arithmetic-bound thinking may view variables as nonspecific referents that are not clearly defined, such that one variable could stand for two different quantities. Such students may also view equations as arithmetic formulas or strings of calculations rather than statements about equality, and as a result fail to understand the utility of using an equation to solve a story problem (Stacey & MacGregor, 1999). Clement (1982) found that when writing equations for certain word problems ("There are six times as many students as professors at this university"), students represent the literal action and objects in the story symbolically, instead of formulating statements about equality.

Koedinger and Nathan (2004) showed that high school students are more likely to correctly solve algebra problems written in verbal formats, including story contexts, compared to problems written as symbolic equations. This supported their *verbal facilitation hypothesis*, which states that story scenarios provide accessibility to students because they are written in English rather than in mathematics notation. Limited support was found for the *situation facilitation hypothesis*, or that story scenarios provide accessibility because students are able to call upon situational knowledge to assist them during problem solving. They found that students performed similarly on verbal word equations (operations written out in English) as they did on story problems, but students were significantly more successful at solving both of these problem types than symbolic equations.

Nathan, Kintsch, and Young (1992) proposed a model of algebra story problem comprehension based upon the idea that when solving word problems, students must coordinate three levels of representation: (1) the *textbase*, a propositional representation of the information in the problem, (2) the *situation model*, a mental representation of the relationships, actions, and events in the problem, and (3) the *problem model*, a mental representation of formal algebraic structure of the problem, involving variables and equations. This framework offers leverage in interpreting students' problem-solving practices as they negotiate text, described action, and mathematical formalisms, and will be drawn upon in our analysis of students' story problem-solving practices in the system of school mathematics.

1.4. Objectives

Here we critically investigate the one of the most prevalent justifications for teaching mathematics in context given by stakeholders in education – the idea that contextualization provides access to mathematical ideas. We focus on algebra story problems, examining both their affordances and constraints as contextualized mathematics. The research questions are:

1. How can contextualization support students in the adoption of problem-solving behaviors that leverage situational knowledge and informal ways of reasoning, promoting access to mathematical ideas? How might this support be related to key goals of school algebra instruction?
2. How are the assumed strengths of contextualization for promoting access to mathematical ideas unrealized in students' problem-solving behaviors? How might this be mediated by the system of "school algebra" and normative views of learning symbolic algebraic representation?

Much of the past research on story problems has focused on elementary students solving arithmetic word problems. Here, we focus on high school students solving algebra word problems, positing that there are a number of important differences between these two methods that may have relevance for understanding the efficacy of story problems as contextualized mathematics. First, high school students have stronger verbal skills and more experience with the genre of word problems, and it has been suggested that these distinctions can have implications for problem-solving success (Koedinger & Nathan, 2004; Puchalska & Semadeni, 1987). Second, algebraic concepts may not be used in day-to-day activities as often as simple arithmetic concepts (Patton, Cronin, Bassett, & Koppel, 1997). For example, we asked 48 high school students how they use math in their everyday life, and many responses were arithmetic-based, with the most common example cited relating to adding prices or counting money (38 out of 57 examples generated). Thus it is unclear if everyday knowledge of situations will mediate problem-solving in similar ways in algebra and arithmetic.

Third, algebra students have been introduced to powerful tools for abstraction and initiated into the culture of school algebra, and their choices about how and whether to leverage symbolic tools for representing variable quantities may provide insight into how contextualization interacts with normative views of learning algebra (i.e., Common Core State Standards Initiative, 2010; Sfard & Linchevski, 1994). Fourth, in the algebra story problems presented in this study, students were asked to either write or interpret symbolic equations with variable quantities, a task not commonly given in studies of arithmetic story problems (e.g., Baranes et al., 1989; Cummins et al., 1988; Vicente et al., 2007). While functions can certainly be conceptualized arithmetically as a string of computations on missing numbers (Stacey & MacGregor, 1999), such symbolization tasks give particular insight into how contextualization may support or be at odds with the traditional goals of algebra.

2. Method

2.1. School and classroom contexts

The participants attended a high school in a large, urban district in the Texas. The school's student population was 65% Hispanic, 22% White, 11% African-American, and 1% Asian/Pacific Islander, with almost 2000 total students. The student population was 58% economically disadvantaged, 13% Limited English Proficient, and 74% "At Risk." The year prior to the study, the school had been rated "Academically Unacceptable" in mathematics under the guidelines of *No Child Left Behind*, with only 51% of students in the 9th grade passing the state standardized mathematics exam. This was not the school's first unacceptable rating, and the school was under pressure to improve mathematics scores. The school received a second "Academically Unacceptable" rating in mathematics for the year of the study.

Students participating in this study were recruited from the regular-level 9th grade Algebra I classes of a single teacher, who was in her fourth year of teaching algebra at the school site. The Algebra I classes of the participating teacher usually consisted of short lectures and note-taking, followed by students completing worksheets at their desks. The worksheets contained short, closed-ended problems often targeted to standardized test preparation, and often framed as traditional story problems. The district had previously been using a curriculum that integrated some reform-based ideas, however due to federal and state accountability pressures, a committee of teachers selected a "back-to-the-basics" textbook series for adoption, and ninth grade coursework increasingly became focused on below grade level standards review and standardized test preparation.

Table 1
Five types of algebra problems given to students during interviews.

Problem type	Example
Normal story problem	Some early Native Americans used clam shells called Wampum as a form of currency. Tagawininto, a Native American, had 80 wampum shells, and spends 6 of them every day. (a) How many shells did Tagawininto have after 10 days? (b) How many shells did he have after a week? (c) Write an algebra rule that represents this situation using symbols. (d) After how many days did he have 8 shells?
Normal story problem with equation	Some early Native Americans used clam shells called Wampum as a form of currency. Tagawininto, a Native American, has a number of wampum shells given by $y = 80 - 6x$, where x is the number of days that have passed. (a) How many shells did Tagawininto have after 10 days? (b) How many shells did he have after a week? (c) After how many days did he have 8 shells? (d) What does the 80 represent in this situation? What does the 6 represent?
Personalized story problem	You are playing your favorite war game on the Xbox 360. When you started playing today, there were 80 enemies left in the locust horde. You kill an average of 6 enemies every minute. (a) How many enemies are left after 10 minutes? (b) How many enemies are left after 7 minutes? (c) Write an algebra rule that represents this situation using symbols. (d) If there are only 8 enemies left, how long have you been playing today?
Generic story problem	You have 80 objects, and lose 6 every day. (a) How many objects will you have after 10 days? (b) How many objects will you have in a week? (c) Write an algebra rule that represents this situation using symbols. (d) After how many days will you have 8 objects?
Abstract problem	$y = 80 - 6x$ If $x = 10$, what is y? If $x = 7$, what is y? If $y = 8$, what is x? Write a story that could go along with the equation $y = 80 - 6x$.

2.2. Participants

Seventy-four Algebra I students classified as being in Grade 9 or above were recruited for the study. Students were asked during class by one of the researchers if they would be willing to participate in an interview where they would solve algebra problems while being audio recorded for a small stipend. Parental consent was obtained for 39 students (52.7%). Due to time constraints, student mobility, and student absenteeism, 29 of the 39 students participated in an entrance interview, and 24 of these 29 students participated in a problem-solving interview, conducted on a different day. Of the 24 students that participated in the problem-solving interview, 13 (54%) were Hispanic, 8 (33%) were white, and 3 (13%) were African-American; this distribution is close to the school distribution given earlier. Of the 24 students, 14 (58%) were male, and 10 (42%) were female, compared to 53% male and 47% female at the school. Nineteen (79%) of the 24 students who participated in the study were eligible for free or reduced lunch, used as a proxy for low socioeconomic status, compared to 75% of all ninth grade students at the school. Fifteen of the 24 participants (62.5%) passed the state standardized mathematics exam in the year of the study, compared to 62% of all ninth grade students at the school.

2.3. Study design

Each student first participated in a 10–15 minute semi-structured, face-to-face entrance interview that was audio-recorded. The interviewer asked a series of questions related to how the student used math in their everyday life, where they see and have to deal with numbers, and what types of activities and hobbies they are interested in. After the entrance interview had been conducted, a set of 4 or 5 algebra problems on linear functions was written for each student. Two of the problems were personalized according to how the student described using mathematics in their everyday life, while the other problems were either normal story problems on linear functions from a previously state-adopted curriculum, versions of these normal problems that included symbolic equations, generic versions of these problems with simplified language and general referents, or completely abstract symbolic equations.

Table 1 shows the five different problem types given to students. In the first two parts of each problem, the student was asked to solve for y given a specific x-value – in the literature these have been referred to as “result unknowns” (Koedinger & Nathan, 2004). The student was then asked to write an algebra rule representing the story, and finally was asked to solve for x given a specific y-value, referred to as “start unknown.” For the problem types in which the student was provided the symbolic equation (the story with equation and abstract problem types), instead of being asked to write an algebra rule, the student was asked to either interpret the parameters (slope and intercept) of the given equation in the context of the story, or write a story that could be modeled by the given equation.

The problems were variations of 14 base story problems from the source Algebra I curriculum, Cognitive Tutor Algebra, which had previously been adopted at the school and was being used in a related study being conducted by the researchers. In the 1980s and 90s Cognitive Tutor was a leader among curricula in contextualizing mathematics using stories designed by algebra teachers to be "personally or culturally relevant to students" (Koedinger, 2001, p. 11). See Appendix A for more information on the problems given to students.

Problems were from the domain of beginning algebra, so most linear functions were of the form "$y = mx + b$." Although some researchers may consider such problems to be on the arithmetic side of the "didactic cut" (Filloy & Rojano, 1989, p. 20) between arithmetic and algebra in that they did not explicitly require students to be able to operate on an unknown (Herscovics & Linchevski, 1994), they were used for several reasons. First, the goal of the study was to explore how relevant, verbal contexts can promote access to mathematical ideas, and research in algebra suggests that verbal advantages are most salient for linear functions with a single reference of the unknown (Koedinger et al., 2008). Second, this was a first year high school algebra class in a school that had been deemed as struggling academically in mathematics, and it was important to select problem types that could show both the resources students brought to bear, and the ways in which they had difficulty making the transition into the type of abstraction valued in first year algebra. Third, although some of the problems could be solved using arithmetic, students had been working with symbol manipulation methods during the school year, and thus it seemed likely that some would use such approaches. Finally, we do not view this didactic cut as being a strict division for algebraic reasoning, and instead take an integrative or transitional perspective, where algebraic reasoning can be developed over time based on students' understanding of arithmetic and their everyday experiences (Carraher et al., 2006; Chazan, 1999; Kaput, 2000; Nathan & Koedinger, 2000b; Nathan, Stephens, Masarik, Alibali, & Koedinger, 2002).

2.4. Designing "relevant" algebra story problems

During each entrance interview, students were asked how they use mathematics in their everyday life. The most common response related to using math when shopping to add prices or count money, a typical arithmetic scenario. The second most common response was that they did not use math apart from schoolwork. Since many students seemed to react negatively to the word "math," we also asked as a follow-up question how students see and deal with numbers in their everyday lives. Although some of the responses had the potential to be considered in terms of rates of change, these questions alone would not have yielded enough information to personalize algebra story contexts to students' experiences.

We employed an approach suggested by Chazan (1999); as an algebra teacher using a traditional curriculum, he describes how he felt that the course consisted of a long list of procedures, with each topic being justified only with respect to future coursework. One method he used to relate algebra to students' lives was to have students "identify the aspects of their experience which could be, at least theoretically, measured, counted, or computed from other quantities" (p. 127). Although symbolic and abstract representations from algebra are not often used in day-to-day activities, Chazan sought to leverage from students' experiences what he considered to be central to the algebraic study of functions – relationships between real world quantities (Kaput, 2000; NCTM, 2000). Other work has similarly explored how the concept of rate of change can be contextualized from students' everyday experiences (Noble, Nemirovsky, Wright, & Tierney, 2001; Wilhelm & Confrey, 2003). Using this approach, we engaged students in discussions during the entrance interview like the one below, where a student describes how numbers are used in one of his favorite video games:

> There's stuff like, this unit has 1000 health and does 100 damage per attack. And then the other units have they might have10,000 health and they might do 20 damage per attack. If I have them attack each other, who will win?

While most responses were not of this quality, discussions with most students yielded at least two everyday situations that they may think about in terms of rate of change. Many of these scenarios ended up relating to students' experiences with technologies such as computers, video games, and TV.

2.5. Methods of analysis

Once a set of problems had been developed based on the entrance interview, students participated in a second interview lasting 30 min to 1 h, which was also audio-recorded. Students were instructed to solve their problems while thinking aloud, explaining to the interviewer what they were doing, and recording their work on paper. Students were given a calculator to use and instructed that the interviewer would not be able to offer them assistance.

The 24 interviews were transcribed in the NVivo Qualitative Analysis software, and put in blocks such that one block of the transcript was one student working one part of one problem, or a student answering an interviewer question. If a student came back to a problem part after moving on, this was considered a new block. Overall, the transcribed problem-solving interviews contained 488 blocks in which students were working problem parts, and 164 blocks where students were answering interviewer questions. Students' written work was integrated with the corresponding problem-solving block. One audio file was destroyed shortly after the interview, and another interview was cut off on the student's final problem when the recorder ran out of power. In both cases student work and interviewer notes allowed the lost interview problems to be used in the analysis in a limited manner.

Each block of the transcriptions was coded with hierarchical coding categories, including what base problem the student was working on, what problem type (see Table 1), and what problem part (result unknown, write equation, start unknown, interpret parameters, write story, answering interviewer question). A set of problem-solving coding categories was identified from the data using the constant comparative method (Glaser & Strauss, 1967). These categories included whether the student arrived at an intended or unintended answer, what strategies were used and what mistakes were made, evidence that situational knowledge was explicitly being used by the student, issues with interpretation of stories, students' use of non-coordinative methods that included a clear bypass of forming a situation model based on the story, and the creation of symbolic equations disconnected from how students solved other parts of the problem. Student solutions were classified as "intended" and "unintended" versus "correct" and "incorrect" because there was evidence of sound reasoning about story scenarios that led students to solutions different than those intended by the original problem authors. Two coders (authors on this paper) coded the categories in a sample of 7 of the 24 interviews, and obtained kappa values for each category from 0.79 to 0.96, which is substantial to almost-perfect agreement (Landis & Koch, 1977). The researchers coded independently, and then resolved any discrepancies; reliability was recorded before the researchers discussed the interviews.

Students obtained the intended answer for approximately half of the problem parts posed to them, with abstract and normal problems being most difficult (45% and 44% success rate, respectively), and personalized, story with equation, and generic problems being easiest (60%, 61%, and 59% success rate, respectively).[1] Result unknown parts had the highest success rate (61%), followed by start unknowns (45%), writing an algebraic equation (42%), interpreting parameters (21%), and writing stories (10%).[2]

3. Results and discussion

The coding of the interview data revealed three processes for story problem solving that seemed central to issues of contextualization in algebra. First, we found that students sometimes struggled to interpret given story contexts and symbolic equations, and to reason about the problem's actions and relationships in the way the problem authors intended. Second, we found that students brought diverse informal and situation-based problem-solving resources to bear on the solving of algebra story problems, and that these practices were sometimes inconsistent with normative views of algebra as involving symbol manipulation. Third, we found that students' situation models and problem models were often not well-coordinated, sometimes resulting in computational or symbol manipulation approaches that bypassed situational understanding. The results are organized around discussing and explicating these three central issues relating to contextualization in algebra story problems. Personalization as a special case of contextualization is woven into the discussion in each section.

3.1. Processes of verbal and symbolic interpretation

3.1.1. Verbal interpretation of story contexts

Koedinger and Nathan (2004) acknowledge under the verbal facilitation hypothesis that algebra students "have by now mostly mastered the English comprehension knowledge needed for matched verbally stated problems" (p. 138). However, we noticed that our participants were consistently having a difficult time with comprehension of stories. We coded instances where students expressed uncertainty about specific semantics of the story, instances where they stated an unintended inference based on the verbal language of the story, and instances where the student said they did not understand a word in the story. Table 2 gives two examples of issues that students had with verbal interpretation. In the first case, the student not understanding what the word "initial" meant caused him to ignore the slope term and use the intercept as the slope. In the second example, the student concluded based on the wording of the story that the intercept term was not significant in the story context. These examples show how verbal issues can manifest themselves in algebra story problems, where students reason in terms of variable quantities, as well as functional parameters like rate of change and intercept.

Issues with verbal interpretation were found in 80 of 500 applicable blocks (16% of the time). Verbal interpretation issues occurred at similar rates even when problems had been personalized. Out of the 24 students interviewed, 22 expressed at least 1 issue with verbal interpretation, and up to 7 issues occurred during a single interview. If a student had an issue with verbal interpretation in a problem block, their chance of getting the intended answer to that problem part was 29%, compared to a 51% overall success rate. These results suggest that for many students, story problems do add significant verbal comprehension demands, even at the high school level, and that these verbal issues impact problem-solving success. This seems to challenge the notion that contextualization in the form of story problems is always effective for promoting access to mathematical ideas.

Many traditional story problems, like the second problem in Table 2, may be unclearly or ambiguously worded from the student's perspective. However, the problems in Table 2 and all of the other base story problems used were from a

[1] These percentages only include result unknown and start unknowns problem parts, since these were the only problem parts consistent across all problem types. $N = 33$ abstract, 47 generic, 152 personalized, 61 story with equation, and 74 normal problem-solving blocks.

[2] $N = 155$ result unknown, 92 write equation, 110 start unknown, 19 interpret parameters, and 10 write story problem-solving blocks.

Table 2
Examples of students' issues with verbal interpretation from interviews.

Description	Problem being worked	Interview transcript
Vocabulary issue	Some rental cars have mobile phones installed. In one car, the cost of making a call from the mobile telephone is \$1.25 per minute with an initial fee of \$2.50. If a call cost a total of twenty dollars, how many minutes did the call last?	I: OK, um, do you know what this word here means? S: No. I: OK, "initial"? OK. And what about the 1.25? How come you didn't use that one? S: The 1.25 is, like, the . . . per minute, how much it costs per minute. (long pause) Per minute, but I think that "initial" is plus tax, or the whole thing together.
Start position confusion	A huge mirror for a telescope is being moved by a truck with 13 axles and 50 tires, from Erie, Pennsylvania to Raleigh, North Carolina. The truck averages 15 miles per hour and has already traveled 60 miles. In three more hours, how many miles will the truck have traveled?	I: Can you tell me um what's up with that 60 there? S: You already traveled that much I: Mhmm. S: So I would have to add on this? I: Do you think that's something that needs to be taken into account in these, or do you think it's not? S: When I read it, cause it's just like saying he . . . he went just 15 miles per hour, but he's already just traveled 60. So it's just like checking in, kind of.

previously state-adopted curriculum – such ambiguous problems may represent the reality how traditional story problems are implemented in instruction[3] (Gerofsky, 1996). Some researchers have challenged the assumption that *any* story problem could be an unbiased and unambiguous portrayal of complex, situated activity (Frankenstein, 2009; Roth, 1996). Since the goal of this study was to investigate the efficacy of traditional story problems for providing access, it is important to note that such ambiguity in story contexts is part of the normal school algebra system, and students' interpretation issues were often caused by the rich, verbal resources they brought to bear being different than the intentions of the problem authors.

3.1.2. Interpretation of symbolic equations

Two problem types were used where students were given symbolic representations – normal story problems with equations, and completely abstract problems (see Table 1). While no response errors occurred 40% of the time when students were presented with completely abstract symbolic problems, no response errors only occurred 11% of the time when students were given story with equation problems. Out of the ten students in the study that were given both story with equation and abstract problem types, three of these students refused to work their abstract problem, at least initially, but were willing to work a story with equation problem.

One of the three students, "Carl," was first presented with the problem "The price of installing wall-to-wall carpet in your house is given by $y = 12.95x$, where x is the number of yards of carpet." Carl successfully solved both result unknowns and the start unknown problem part, and gave a reasonable interpretation that the parameter 12.95 could represent "how much it is by the yard." However, two problems later Carl was presented with the problem "$y = 2x$" and was asked to solve for y if x was equal to 3. The following conversation occurred:

> Carl: Oooh . . . if x equals 3, what is y? I don't like these problems. (mumbling) I don't know how to do these problems.
>
> Interviewer: Okay. Can you tell me what that means there (points to equation)? Or what you think it might mean?
>
> Carl: I don't know. *Y* equals 2*x* . . . what do you mean like, what does it mean?
>
> Interviewer: Just when you see that, what do you think about?
>
> Carl: Ummm . . .
>
> Interviewer: Think that you just don't know?
>
> Carl: Mmhmm.
>
> Interviewer: So you you think you can solve any of these, or no?
>
> Carl: Ummm . . . Probably not.

What is significant here is that there is no way to solve the normal story with equation problem type correctly without dealing with the symbolic equation in the exact same way it must be dealt with in an abstract problem. There is no redundant information relevant to solving the problem that the story scenario adds, and thus there is no way to solve the problem while ignoring the symbolic equation. To our knowledge, story problems that include equations have not been part of many studies, and may be a useful bridge between students' informal understanding of verbal scenarios and their ability to interpret and use symbolic expressions with variable quantities. We found higher success rates, higher response rates, and a greater variety

[3] See Palm (2008) for a framework to assess the "authenticity" of school story problems.

of strategies for story with equation problems than abstract problems. This novel result lends support to the idea that the concrete, verbal contexts in story problems may provide students with access to algebraic formalisms. However no response rates were still higher for story problems with equations than for normal, personalized, or generic problems, suggesting that expressing the story's relationships using a symbolic equation added conceptual difficulty.

One might assume that students' higher success with the story with equation problem type is because they were able to form a situational understanding of the scenario that supported problem solving and the formation of a problem model. However, of the 19 students that were asked to interpret the parameters in the equation that accompanied a story with equation problem, only 4 gave responses that demonstrated a clear understanding of how the equation related to the given story. Students struggled to interpret the slope term as rate of change; for instance given the scenario "The total distance the explorers have traveled is given by the equation $y = 20x$, where x is the number of days they've been traveling," one student responded that the 20 represented distance, rather than interpreting 20 as a relationship between distance and time. Sometimes new quantities were invented; another student when asked to interpret the parameters from the scenario "The distance a jet has flown in miles is given by the equation $y = 1500x + 500$ where x is the number of hours the jet has been flying," said that 1500 was how high the jet was.

Overall when asked to interpret parameters, the responses did not support the notion that embedding the equation in a story scenario enabled students to form coherent and meaningful situation models that supported problem solving. Students struggled to interpret story scenarios, both in purely verbal format, and with imbedded equations expressing the relationships between quantities in the story context. The prevalence of interpretation issues seems to undermine the idea that story problems leverage student knowledge of relationships and quantities, and thus that verbal framings provide students with access to mathematical ideas. The arithmetic word problem literature has shown how students struggle to interpret verbal scenarios relating to performing different operations, and here we extend that work by showing how interpretation issues arise as students confront linear functions with variable quantities.

3.2. *Use of informal and situation-based reasoning*

Students participating in this study struggled with school algebra, as evidenced by their relatively low passage rates on the state standardized exam. However, we found that participants brought many diverse sense-making resources to bear on the solving of story problems. The idea of "situation facilitation," or that students can use their knowledge of the real world to directly help them solve story problems, has been central to previous studies of algebra problem solving (e.g., Koedinger & Nathan, 2004; Nathan et al., 1992). Here, we frame situation facilitation as students' ability to productively use their knowledge of different situations when solving of story problems, in a way that goes beyond what is explicitly stated in the problem text. For example, high school students have rich and diverse resources for situational reasoning from their experiences in out-of-school environments like home or the workplace, and also may leverage situational knowledge from their experiences within various school subjects, including mathematics. This situational knowledge can give students resources for reasoning about mathematically relevant aspects of algebra story problems, such as dealing with quantities and change. During an end-of-year interview, the classroom teacher accentuated the importance of story contexts to provide students with a "connection" to the mathematics content. When asked why she used traditional story problems so extensively in her teaching, she responded, "To try and get that connection. To try and make them use that brain and see that this word problem is the same as an equation problem, you're doing the same thing in it."

Results that will be discussed in this section suggest that story problems did in some ways seem to provide students with access to problems. Students used informal, arithmetic-based strategies to solve algebra word problems, particularly on personalized and normal problems. Students also sometimes directly used their situational knowledge to reason about the actions and relationships in algebra story contexts. However, this informal and situational knowledge was not always well-connected to symbolic reasoning in algebra, even though the teacher had accentuated this connection as being an important benefit of story problems for providing access.

3.2.1. *Using situational knowledge when solving story problems*

An important resource that students bring to bear when solving algebra story problems is explicit knowledge about the situations being described in the story contexts, based on their related experiences with quantities and change. In order to begin to examine the idea of situation facilitation, we coded instances where students explicitly generated inferences from their experiences that were not given in the problem text. Overall only 20 such instances were found out of the 500 applicable blocks; 10 of the 24 students used situational knowledge while solving a problem, between 1 and 5 times each per interview.

Table 3 shows examples of students explicitly using situational knowledge to solve algebra story problems. Instances were coded as being productive or unproductive with respect to the "intended" solution path of the problem, i.e., a solution path leading towards an answer that would be valued in a school mathematics context. Of the 20 instances found in the data, 9 were coded as productive and 11 were coded as unproductive. The proportion of use of situational knowledge was generally not higher for personalized problems (3%) than normal problems (2%) or generic problems (5%). Story problems

Table 3
Examples of situational knowledge being explicitly used.

Description	Problem being worked	Interview transcript
Productive use of situational knowledge	You have 80 objects, and lose 6 every day. After how many days will you have 8 objects?	I: Why wouldn't you want to get a decimal for the answer to this one? S: Because you can't lose half an object. **Because if you have a toy and you lost half of it, it doesn't make any sense.**
Unproductive use of situational knowledge	You have 175 friends on MySpace. You get 4 more friends every day. How many total friends will you have in 20 more days?	S: Like I could do like 175 times 20, so I'm like okay. Like okay, see this is how I was thinking, because **I have this dance camp, and my dance class over the summer have to have to have 20 hours**, so I was saying, okay, so, and we have to do it for that 20 hours, so I was thinking, okay, **I can do 2 hours every day for 2 weeks and that comes out to 20 hours**. So I'm like thinking of something how I could do it for this one. So, I'm just like trying to, <> for a second, so I'm just like trying to figure out like, how I could that, for how I did that one. Like 2 times 5 is 10 a week, and then, I was trying to figure it out like that . . .
Unproductive use of situational knowledge	The number of students getting A or B in algebra class is given by the equation $y = .25x$ where x is the total number of students taking algebra. If 40 students earned an A or a B in Algebra last year, how many total students were enrolled?	S: 80 students were enrolled. (pause) (mumbling) I: So how did you get 80 for that one? S: Umm. . . you just times the umm 40 students times 2, **cause there's always a half that doesn't get like the full stuff done**, like umm . . . pretty much, there's so many students and then, it divides umm how many students get an A or a B, and the other students don't get an A or B. So I guess it divides how many A's or B's I have.

< > = inaudible; bold = portions of speech most related to use of situational knowledge.

$20 = 1.25x + 2.50$

3.75

$3.75 \times 8 = 30$

$3.75 \times 7 = 26.25$

$3.75 \times 6 = 22.5$

$3.75 \times 5 = 18.75$

Fig. 1. Example of a trial and error strategy to solve a start unknown.

with equations had a higher rate of use of situational knowledge (10%)[4]. As shown in the previous section, interpretation of the symbolic equations imbedded in these scenarios seemed to sometimes require resources relating to students' situational knowledge. Finally, situational knowledge was usually only explicitly applied when solving start and result unknowns; there was only one instance for a write equation block.

In the first example in Table 3, the student uses the fact that he got a decimal answer for "number of objects" as a signal that he made a mistake. Students' recognition that their answer was unreasonable in the context of the story was the primary way in which knowledge of situations was used productively in problem solving. In the second example, the student attempted to reason with situational knowledge, but it was not helpful; she was trying to see if there was a relationship between a problem she had recently solved in her everyday life about figuring out how many hours of dance she needed to do each day, and the problem about MySpace she was being asked to solve. The two scenarios were incompatible in their structure – her real-life problem about dance hours involved division and taking into account a 5-day week structure, while the problem on MySpace involved a combination of multiplication and addition and was framed simply in terms of days. In the third example in Table 3, the situational knowledge the student applied was also unproductive – the student appeared to reason based on his experience with how grading normally works in algebra classes to determine that half of all students usually get Cs and Ds because they "don't get the full stuff done," despite the symbolic equation that suggested that three-quarters get C's or D's. The student may have decided to leverage this situational knowledge because of the conceptual difficulty of dealing with a symbolic equation.

Overall, the data demonstrate that while students can use situational knowledge to provide access to problems and error catching benefits, explicit use is rare. Later we will discuss how even when such practices were used to catch errors, it was often in the context a *non-coordinative approach* to solving the problem. Results also showed that knowledge of situations can often be unproductive, sometimes suggesting unintended answers.

3.2.2. Students' informal strategies to solve start unknowns

Along with explicitly using their knowledge of situations and quantities when solving algebra story problems, students also used informal, arithmetic-based strategies that may stem from their experiences with computations outside of school, as well as their experiences in elementary and middle grades mathematics classes. In start unknown problem parts, students are asked to find the value of x in a linear function given a specific value of y. Traditionally, this is thought to provide motivation for equation solving; however as mentioned previously, research has shown that students often use arithmetic to solve such problems.

During the interviews, we observed that students sometimes used "trial and error," what we call an "informal" approach to solve a start unknown, where they plugged different values of x into the equation or story, and tried to get the given value of y. A related strategy students used was repeated addition; here students continually add the slope value, trying to reach the given y value. This was similar to a third strategy where students would proportionally "scale up" or "scale down" a previous answer to reach the given y-value. These strategies all involve going forward in a functional relationship, and are tied to the direct action of the story or equation. Fig. 1 shows a student's trial and error approach to solve a start unknown from the story scenario "Some rental cars have mobile phones installed. In one car, the cost of making a call from a mobile telephone is given by $y = 1.25x + 2.50$, where x is the number of minutes used. If a call cost a total of twenty dollars, how many minutes did the call last?" The student uses 3.75 as the slope term, adding the 1.25 and 2.50, and then systematically tries x values of 8, 7, 6, and 5 trying to find a number that can be multiplied by 3.75 to get 20.

Students also used "unwind" approaches to solve start unknowns, where they began with the given y-value, and reversed the slope and intercept arithmetically. These strategies involve systematically going backwards in a functional relationship, reversing operations as is done in equation solving, although here there is no notion of balancing two sides of an equation. Fig. 2 shows a student using an unwind strategy to solve the problem, "You have 175 songs downloaded onto your iPod from Limewire and iTunes. You download 4 more every week. If you have 275 songs, how many weeks have passed?" The student successfully reverses both operations, but makes an arithmetic error.

Five of the 24 students used equation solving, a "formal" approach to a start unknown, performing operations on both sides of a symbolic equation to isolate the x variable (Fig. 3). Overall, an equation-solving strategy was used between 6%

[4] Interpret parameter and write story blocks were not included in this analysis, since these problem parts were designed to require students to use situational knowledge. This analysis was meant to capture spontaneous use.

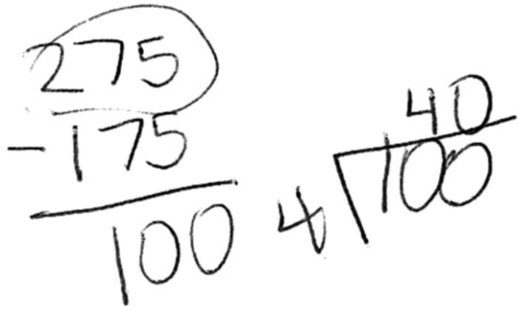


Fig. 2. Example of unwind strategy to solve a start unknown.

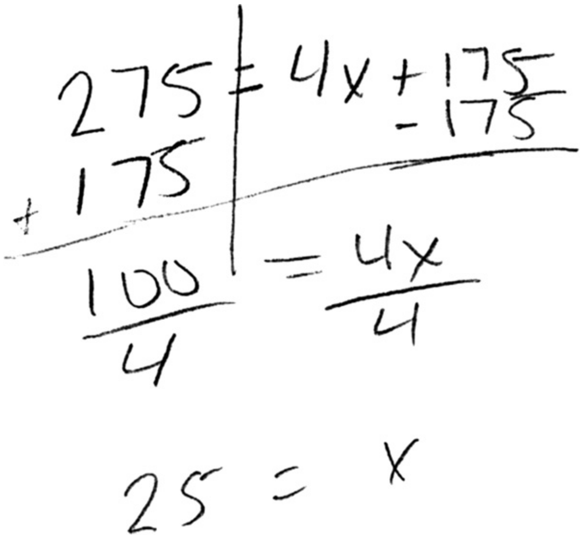


Fig. 3. Example of an equation-solving strategy to solve a start unknown.

and 22% of the time depending on problem type, despite the fact that for each problem students were either provided an equation or asked to write one in the problem part immediately before the start unknown.

There were differences across problem types in the strategies students chose to use. Fig. 4 shows the proportions of strategies used to solve start unknowns that were informal (trial-and-error, repeated addition, proportional), unwind, and

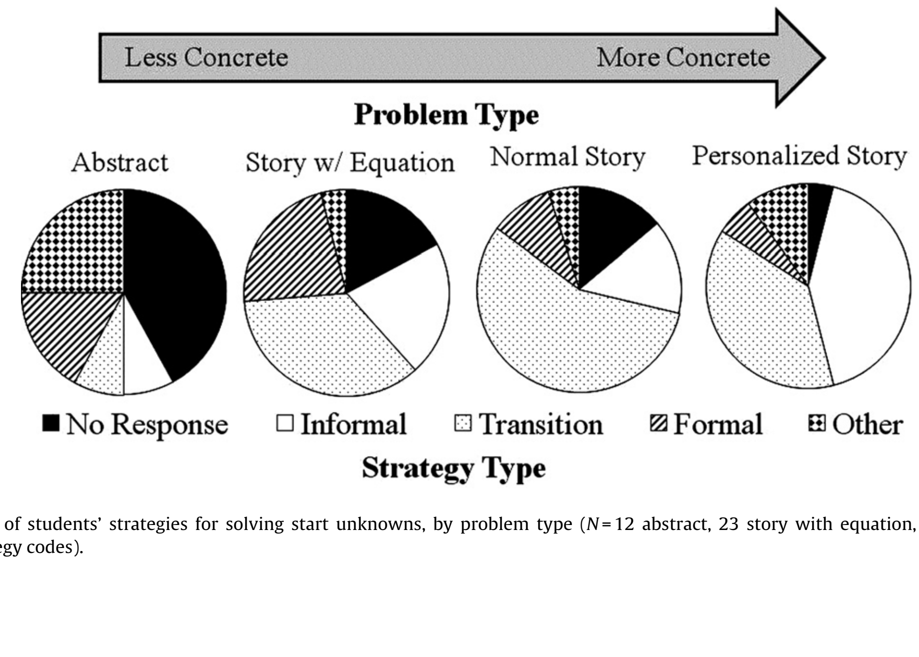


Fig. 4. Prevalence of students' strategies for solving start unknowns, by problem type ($N = 12$ abstract, 23 story with equation, 21 normal story, 50 personalized strategy codes).

formal (equation solving) by problem type. Also included are how many "no response" errors there were and strategies that were classified as "other," which were most often strategies based on errors that were not replicated. The pie charts are organized such that moving from left to right, the concreteness and relevance of the problem context increases; abstract problems are least relevant and concrete, followed by normal story problems with equations, followed by normal story problems, and then personalized problems are the most relevant and concrete.

As can be seen in Fig. 4, moving from more abstract problem types to more concrete and familiar problem types, no response errors (black) show a large decrease. Students' use of informal strategies (white), such as the trial and error approach, becomes more prevalent as problem concreteness increases. Students' use of the unwind strategy (dots) increases up to the normal story problems, then for personalized problems this strategy is overtaken by informal strategies. Use of the formal strategy of equation solving (diagonal lines) is most prevalent in abstract problems and story with equation problems, and its incidence decreases moving to normal and then personalized story problems. This trend was also confirmed by examining patterns of strategy use for individual students; the analysis showed that different levels of problem concreteness caused individual students to change their approach to the problem.

These results suggest that students may choose different strategies based on problem framing, with more relevant and concrete problem contexts eliciting greater use of informal, arithmetic-based reasoning, and more willingness to attempt a solution. This implies that contextualization generally, and personalization specifically, may provide students with access to problems by allowing them to use informal strategies, perhaps stemming from experiences in school arithmetic or out-of-school situations, to reason through the operations needed to solve problems in the algebra classroom. However, these informal strategies seem somewhat at odds with a highly valued practice in the school algebra system – learning to write and manipulate symbolic equations with variable quantities. We now turn to a discussion of this tension.

3.2.3. Contrasting informal and formal strategies

The student work shown previously in Fig. 3 stood out from most of the other solutions given in the study in that it seemed to demonstrate a mastery of equation solving. The story problem the student "Anna" was solving read, "The distance a machine called the Crawler has traveled from its hangar is given by the equation $y = 4x + 175$, where x is the number of seconds the machine has been moving. In how many more seconds will the Crawler reach the launching pad, which is a total of 275 feet from the hangar?" As shown in Fig. 3, Anna generated "$x = 25$" using equation solving, and was then asked immediately afterwards what the "4" could represent in the story situation and what the "175" could represent. Anna's response was that the Crawler could have started at 4 feet, and 175 could be the number of seconds it took the Crawler to move 4 feet. This answer suggests that while Anna obtained the "correct" answer and used perhaps the most valued strategy in the context of high school algebra (Nathan & Koedinger, 2000a), she may lack a fundamental understanding of central algebraic concepts like rate of change and intercept, and how these concepts related to both equation solving and applied scenarios like travel. Current standards in math education have recognized that "In general, if students engage extensively in symbolic manipulation before they develop a solid conceptual foundation for their work they will be unable to do more than mechanical manipulations" (NCTM, 2000, p. 39).

Compare this to the reasoning given below of another student, "Mark." Mark used an unwind strategy to solve the problem, "You have a Verizon cell phone and you have a gift card from Verizon with $7.87 left on it that you plan to use on this month's bill. This month, Verizon is going to charge you $0.23 for each text message you send. At the end of the month, you pay $38.13 of your own money to Verizon. How many text messages did you send?"

> Mark: . . . so there was 38.13, you're gonna add the 7 from the card, plus 7.87 (punching on the calculator) 38.13 plus 7.87 equals to $46, it equals to 46, and now you're going to divide those, 46 divided by 0.23, which would equal to 200 text messages.
>
> Interviewer: Can you tell me why you added the 7.87 there?
>
> Mark: Yeah, because it's telling you at the end of the month you pay 38 of your own money. So that's not including the money that you already used from the card. So, and then it's asking you how many text messages you send, so you need the total amount of money that you used to see how many text messages that you sent.
>
> Interviewer: So why did you divide the 46 by the .23?
>
> Mark: Because 0.23 is what they charge for each message. So you divide the total amount by the charge per message to give you the number of message you sent.

Mark demonstrates a clear understanding of the story scenario and how concepts of rate of change and intercept operate in this context, as well as why and how these operations are reversed when solving a start unknown. There seems to be much potential with such reasoning to connect what students informally understand about story scenarios to formal algebraic manipulations. The classroom teacher identified that students could use informal approaches to solve story problems, "I think the word problem would probably be easier, because they don't necessarily need the equation to do it, and they can just logically think about it to figure it out, if it's a word problem" (Mrs. C, May 15, 2009). However, she, like many algebra teachers, largely taught the course in the context of symbol manipulation strategies.

Table 4
Examples of non-coordinative approaches used by students in the interviews.

Description of approach	Problem being worked	Interview transcript
Student plugs in numbers without intermediate situational reasoning	An object moves at 1500 mph. It has already moved 500 miles. How far will it have moved total 30 minutes from now?	I: Can you tell me what you think this situation is about? Like, what's the story about? S: I have no idea. So, if it moves at fif - one thousand five hundred miles per hour. That means . . . so I think I would just, like, divide 500 and 30. I'm not sure. Alright, so 16. So maybe for that one, 16 miles? I: OK, can you tell me, explain why you divided 500 by 30? S: Umm. . . because I'm not sure exactly what to do, but I think that if you divide the miles by the times then you'll get, so like how much it's already moved by the time you'll get, like, the answer to it?
Student applies well-known schema to problem not fitting that schema	You're buying a new skateboard that is on sale for 25% off. If the skate board costs $44 normally, how much will you save?	S: You're buying a new skateboard that is on sale for $225 off. 25% off. If the skateboard costs 44 normally, how much will you save? For this I need 44 times .25 to figure out, um, the percentage, like how much you take off, and subtract it by 44 . . . That equals 11, so 44 minus 11.50 would be the cost of the skateboard would be 32 dollars and 50 cents.

Some researchers dismiss students' informal arithmetic-based strategies; one study of algebraic reasoning concludes with "Under the guise of teaching algebra, some teachers promote non-algebraic methods because they believe they are easier for students. By doing so they fail to provide opportunities for students to learn more powerful mathematical methods" (Stacey & MacGregor, 1999, p. 164). However, an alternative view is that students' informal, situation-based reasoning can be built upon to provide access to algebraic ideas, similar to research on the development of arithmetic reasoning using Cognitively Guided Instruction (Carpenter & Moser, 1984; Carpenter et al., 1996). Emerging research suggests that leveraging or integrating students' informal and arithmetic reasoning can provide a bridge to algebraic thinking (Carraher et al., 2006; Chazan, 1999; Kaput, 2000; Kieran, 1988; Mark & Koedinger, 1999; Nathan & Koedinger, 2000b, 2000c; Nathan, Long, et al., 2002; Nathan, Stephens, et al., 2002). In the present study, exiting Algebra I students were only able to successfully solve start unknowns 45% of the time in simple linear equations. This suggests that being able to solve a start unknown by any method is an important mathematical strength to be capitalized on. Here, story problems with relevant contexts seemed to have potential to connect to students' ways of making sense of relationships between quantities (Chazan, 1999).

3.3. *Emergent coordinative and non-coordinative approaches to problem-solving*

3.3.1. *Coordinative approaches to solving algebra story problems*

Many students used approaches to solve story problems where they seemed to explicitly link their problem-solving actions to a detailed understanding of the situational context. One example of such reasoning is Mark solving the Verizon cell phone bill problem in Section 3.2.3. Such reasoning is important to any discussion about contextualization, because it suggests that students do successfully coordinate situation models with problem models when solving algebra story problems, allowing situational understanding to support numeric reasoning. The example in Section 3.2.3 is in the context of an informal, arithmetic-based strategy, but we also saw examples of such coordinative approaches in the context of algebraic strategies. For example, a student "Liz" was presented with the scenario "The distance a machine called the Crawler has traveled from its hanger is given by the equation $y = 4x + 175$, where x is the number of seconds the machine has been moving. How far will the Crawler be from the hanger in 20 more seconds?" and spontaneously used the following approach:

I think the $4x$ is how fast it's moving per second, that's how you find the distance. Adding the plus 175, I guess it starts out at 175, I guess in most games you use a factory you build units with, so I guess the 175 is where it started out at. But if I plug in the number 20 seconds, if x equals the number of seconds . . . so 4 times 20 plus 175 equals 255. So that's how I find out our distance.

Liz reasoned about the parameters of the linear equation – slope and intercept – in the context of the story, coordinating situational understanding with the action of placing values into a symbolic equation. Supporting such coordinative reasoning may be central to using contextualization to meet the goals of algebra instruction.

3.3.2. *Non-coordinative approaches to solving result and start unknowns*

In arithmetic story problems, it is well-documented that students sometimes use "direct translation" or "keyword" approaches (Hegarty, Mayer, & Monk, 1995; Jonassen, 2003; Nesher & Teubal, 1975). We observed a number of approaches to solving algebra story problems that we refer to as *non-coordinative*, in which students seemed to be translating from the problem text to a problem model without developing a fully-elaborated situational understanding of the given story scenario (Table 4). Some students plugged in the numbers in the story seemingly at random, using various operations and trying to obtain an answer that "looked right" (see first example in Table 4). We also coded problem solving as non-coordinative if

2) Due to a billing error last month, Amanda has received a $7.87 credit towards next month's cellular phone bill. She pays a flat $0.23 per minute with no additional monthly charge.

a) If 43 minutes are used, calculate the bill for the next month.
b) If 100 minutes are used, calculate the bill for the next month.
c) Write an algebra rule that represents this situation using symbols.
d) After finding that billing error last month, this month Amanda will make sure that her bill is correct. If her bill is for $38.13, how many minutes has she talked?

Fig. 5. Example of problem that student believed to be multiple choice.

the student was applying a well-known schema to a problem, when careful reading of the problem would reveal it did not fit that schema (see second example in Table 4). Around 39% of all unintended answers given during the interviews were coded in blocks containing non-coordinative reasoning. However, 20% of the time students engaging in non-coordinative reasoning in a block still ended up with the intended answer (compared to a 56% overall success rate).

We observed that non-coordinative approaches occurred even when the problem had been personalized. Incidence of non-coordinative approaches was similar for personalized and normal problems (15% and 12% respectively), but was considerably higher for abstract, generic, and story with equation problems (22–27%), suggesting that more relevant, verbal contexts can promote lower use of non-coordinative approaches. However, 13 of the 20 students who used non-coordinative approaches used them on one or both personalized problems they received, with 3 of these students using these approaches *only* on their personalized problems.

3.3.3. Reasons why students may use non-coordinative reasoning

The pattern of non-coordinative reasoning for each of the 20 students was further examined to determine if there were certain "profiles" of students who use these approaches. Eight students used non-coordinative reasoning on only one problem, 6 used non-coordinative reasoning on 2 problems, and 6 used non-coordinative reasoning on 3 or 4 problems. This suggests that students are not simply blindly applying the approach to every problem they solve; rather specific aspects of the problem context or mathematical structure may be cueing use of non-coordinative approaches.

Students' use of non-coordinative approaches seemed to be tied to both issues with verbal interpretation and students' use of situational knowledge. Approximately 27% of all non-coordinative blocks were also coded as instances where the student had an issue with verbal interpretation of the story. Many of the episodes in these blocks suggest that non-coordinative reasoning is something students resort to when the verbal semantics of the story make the formation of a reasonable and understandable situation model difficult.

Similarly, out of the 20 blocks coded as students explicitly using situational knowledge, 11 were also coded as containing non-coordinative reasoning. This relationship seemed to occur for two reasons. First, students would sometimes choose to employ situational knowledge that was not related to the intended situation and relationships presented in the problem text, as in the third row of Table 3. Second, students would take a computational approach, plugging in the given numbers in different orders, and then would use situational knowledge to determine if the final product was reasonable; this was the case in the first row of Table 3, which was considered productive.

Inoue (2005) differentiates the "mindless calculational approach" of direct translation from a "conformist approach" where students suspend sense-making as a result of critically but perhaps unconsciously evaluating their experiences participating in school mathematics. The student in the second row of Table 4 may have been applying a well-known schema for solving a "percent off" school mathematics problem, and as a result may have bypassed carefully reading and making sense of the situation as it was posed. However, an interesting case of how such sociomathematical norms may mediate story problem solving comes from a student "Toby" presented with the problem in Fig. 5.

Although the interviewer did not realize it initially, Toby operated throughout the interview under the assumption that the problems he was being given were multiple choice. The interviewer was understandably confused by Toby's insistence that the answer was "d" in the excerpt below:

> Toby: . . . (a) says if 43 minutes are used, calculate the bill for the next month. And I was just putting in, uh, 43 times .23.
>
> Interviewer: OK, great.
>
> Toby: I came up with 989 so that doesn't work. Or 9.89, so that doesn't quite work.
>
> Interviewer: Why does it not work?
>
> Toby: Because I think it's trying to find, like, how many minutes she can be on the phone to add up to 7.87.And for (a) it says 43 minutes, and I came up with 9.89, so . . . and (b) is wrong because it's even more minutes than 43 minutes . . .

> Toby: . . . I'm gonna go with (d) because it says, like, if her bill was, uh, 38.13 dollars, and it's trying to find, like, how many minutes she's talking and so you just divide it by .23, and I came up with 165.7 minutes. Uh, cuz, the total bill $38.13 and that divided by .23, I just came up with 165.78, so I'm just going to go with (d).

Toby's interview took place one week before the state standardized test for 9th grade mathematics. His school had been "Academically Unacceptable" in mathematics the previous year, and was facing significant sanctions if they did not improve student performance. As a result, algebra students in the classes participating in this study were drilled on multiple choice standardized test-style algebra problems starting the first week of school, with increasing intensity as the state test approached. Toby was one of the more successful students in the class in terms of demonstrating proficiency with algebraic notation; he was able to write a correct general symbolic equation for each story when prompted. The fact that a mathematically competent high school student believed that the problem in Fig. 5 and several other problems he was given were all multiple choice, and attempted to reason through these problems under this assumption, points to the importance of considering the system of school algebra that students are participating in when they solve story problems.

These results suggest that use of non-coordinative reasoning was a significant part of problem solving in this traditional algebra classroom, and that simply personalizing a story problem to students' interests and experiences may not be enough to combat wide use of these approaches. The prevalence of non-coordinative approaches seems to undermine the idea that contextualization, in the form of traditional story problems, provides access to mathematical ideas–if students are not trying to make sense of the stories they are presented with, use of verbal and situational resources to support problem solving seems unlikely. The prevalence of non-coordinative reasoning further suggests that problematic epistemological statements about the knowing and doing of mathematics may be communicated in school through story problems, and causes concern for the degree of conceptual sense-making students engage in around central ideas in algebra as they apply to modeling the world.

When we coded students' use of non-coordinative approaches, we coded only result unknown and start unknown problem blocks. However, we also saw non-coordinative aspects of students' reasoning about symbolic equations they generated. We now turn to a discussion of these cases.

3.3.4. Non-coordinative aspects of student-generated symbolic equations

Research on arithmetic story problems has demonstrated that students sometimes translate directly from the problem text to a symbolic representation consisting of fixed numbers and operation signs, bypassing the intermediate step of conceptually reasoning about the relationships being presented (e.g., Nesher & Teubal, 1975). Here, we point out an interesting parallel between non-coordinative aspects of students' reasoning in symbolic arithmetic notation and in symbolic algebraic notation, where symbols representing variable quantities are introduced.

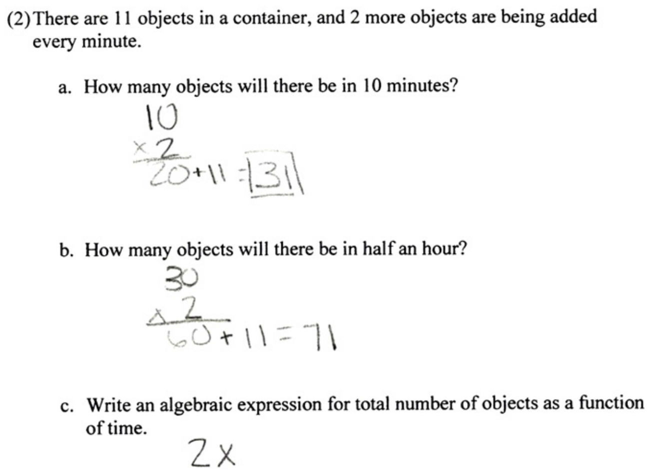
Earlier, we showed how when symbolic representations of functions were provided for students and imbedded within story scenarios, students' reasoning was often tied to these symbols, and seemed disconnected from the quantities in the story. This symbol-based reasoning can also be considered non-coordinative, as students are not giving the symbols situation-based meaning. The other story problems included in the study all asked students to generate a symbolic algebraic representation based on the situational context being presented. For these personalized, normal, and generic story problems, after solving two result unknown problem parts, students were asked to write an algebra rule before solving the start unknown. Students generated a total of 85 symbolic algebra equations from story contexts.

The most common mistake students made when writing an equation or expression was to leave out the intercept term (16 out of 62 mistakes). Some students used the intercept term to solve one or more of their result unknowns, but did not include the intercept term when writing the corresponding equation; an example is shown in the student work in Fig. 6. Some students also made the opposite mistake – they would not use the intercept term when solving result unknowns, but when they wrote the equation, the intercept term would appear.

We coded 34% of all equations written as being clearly disconnected from how students solved the other problem parts. It seemed that writing the equation, or forming an explicit problem model in the way most valued in school algebra, was sometimes disconnected from the situation-based reasoning students used to solve result unknowns and start unknowns. This is similar to a study of 8th grade students that uncovered a disconnect between students' verbal understanding of relationships in story problems, and students' ability to write symbolic equations, prior to instruction (Bardini, Pierce, & Stacey, 2004). In the present study, students did not always seem tied to the belief that the relationships in the story problems had to have consistency from problem part to problem part, suggesting a weak connection between situational reasoning and formal problem-solving procedures. This again suggests that such connections may need to be explicitly attended to for the benefits of contextualization to be meaningful.

3.4. Summary

This article has demonstrated that understanding different dimensions of students' practices when solving contextualized problems – such as processes for interpretation, use of informal and situational knowledge, and coordination of situation and problem models – is central to understanding students' performance and problem-solving. Students often used non-coordinative approaches to solve story problems, where they focused on numbers, symbols, and calculations rather than reasoning about relationships between quantities. These non-coordinative approaches were associated with


(2) There are 11 objects in a container, and 2 more objects are being added every minute.

a. How many objects will there be in 10 minutes?

10
×2
20+11 = $\boxed{31}$

b. How many objects will there be in half an hour?

30
×2
60+11 = 71

c. Write an algebraic expression for total number of objects as a function of time.

2x

Fig. 6. Example of student work where intercept is taken into account in result unknowns, but not equation.

lower performance, and suggest that contextualized scenarios do not always elicit situational sense-making from students. Further, students often struggled to interpret the actions and relationships in story problems in the ways intended by problem authors, and when they had such issues they were less likely to arrive at the intended answer and more likely to use non-coordinative approaches. When students explicitly used their situational knowledge to interpret the quantities and relationships in a story scenario, it was often unproductive with respect to the intended solution path, and was also frequently associated with non-coordinative approaches.

Students used valid, arithmetic-based strategies to solve algebra story problems, which allowed for some degree of success in obtaining correct answers. Informal strategies were often closely tied to reasoning about the story situation, and were more prevalent for more concrete, relevant contexts. However, students may have been missing opportunities to connect these strategies to powerful algebraic ways of reasoning. These practices demonstrate the ways in contextualization mediates problem-solving, suggesting how algebra story problems may have the potential to promote access to mathematical ideas. They also suggest that story problems as contextualized mathematics may have limited usefulness for providing access if appropriate coordination of algebraic representations of functions and informal, situational understanding is not supported.

This study replicated many of the results from research on arithmetic story problems – this body of work has shown that students struggle to interpret stories, students use computational, direct translation approaches, and that applying everyday knowledge to traditional story scenarios can be problematic. Showing that results from arithmetic hold in algebra is of value because it extends our understanding to a new area of mathematics. Algebra is also an important domain because it scales up a great deal in secondary and post-secondary physical science, social science, and mathematics. That findings hold for older students who have been exposed to the culture of school algebra and have had more experience “playing the game” of story problems is interesting, however, working within the context of algebra problem-solving, and focusing on students' understanding of formal and symbolic strategies and equations, we extend previous work in several ways.

First, in arithmetic story problem solving it is much more straightforward that the informal, everyday strategies that contextualized problems seem to encourage can be used to develop more formal algorithms; one example is the Cognitively Guided Instruction program (Carpenter et al., 1996). However, in the context of algebra instruction and working towards the goal of symbolic strategies, connecting informal strategies and situational knowledge to algebraic reasoning requires new ways of thinking about how students use numbers and quantities and understand rate of change, and how arithmetic can bridge to algebraic understanding. We showed data suggesting that more relevant, concrete story contexts may encourage arithmetic over algebraic approaches, providing access to problems for students who struggle with algebra. More research is needed to determine if the access provided by these problems can be leveraged to help teach students how to coordinate situation and problem models, or to generalize arithmetic strategies.

Second, we observed that students used non-coordinative approaches where situational understanding of the story context was disconnected from their formal problem-solving actions. However, the discussion of Mark working on the Verizon cell phone problem shows that simply being able to coordinate situation and problem models may not be enough for success in algebra. Mark's reasoning, while strongly coordinative, remained closely tied to the action of the story, and his informal understanding of arithmetic. Similarly, we saw Carl, a student willing to use a symbolic equation imbedded in a verbal story, but not a symbolic equation presented in isolation. In order to fully realize the benefits of contextualization for promoting access to algebraic thinking, students may need to be able coordinate situational understanding with formal symbolic representation and manipulation strategies. Our data suggests that simply presenting students with traditional story problems does not always directly elicit or support this coordination.

Liz was the most successful participant in terms of getting intended answers during the interview, and was also one of the strongest students in the algebra classes participants were recruited from. We showed Liz coordinating situational reasoning with symbolic strategies in order to successfully solve the Crawler problem[5]; however it is interesting that we only see clear evidence of such coordination in one of the strongest students. This seems to problematize the idea that traditional story contexts can provide struggling students with access to algebraic thinking. This is contrasted with the reasoning of Anna and Toby, who showed some proficiency with symbolic representation or symbol manipulation, but did not coordinate these problem-solving actions with situational understanding. Anna still obtained the intended answer to the problem part using an equation-solving strategy, as shown in her work in Fig. 3, but struggled with most of the other story problems she was given during the interview. Toby's non-coordinative approach to his story problem set caused him to get nearly every problem incorrect.

Taken together, the cases of Mark, Carl, Liz, Anna, and Toby show that using contextualization to promote access to algebraic thinking with traditional story scenarios is problematic, and may need explicit support. Some students, like Anna and Toby, essentially ignored the situational context, focusing on symbol manipulation, while others, like Mark and Carl, reasoned closely to the situation using arithmetic strategies, but ultimately did not move towards formal algebraic representation. In algebra, if contexts are to promote access to central concepts, they ultimately should give meaning to abstract representational systems. Whether this can be achieved by traditional story problems within the system of school algebra, and whether it is likely without strong support for such coordination, is more complex than everyday notions of the benefits of contextualization would suggest.

3.5. Limitations

There are several limitations to the study presented here that should be discussed. First, this study used a population of students from an "Academically Unacceptable" urban school, and it is unclear whether the parallel between arithmetic and algebra word problem findings regarding the prevalence of interpretation, situational knowledge, and non-coordinative issues would be similar among other populations. However, the purpose of this study was to provide insight into how contextualization can provide access to algebraic ideas, so it was essential to the research purposes to study a population that needed this access. As discussed in Cobb and Bowers (1999), findings from situated research must remain grounded in the particulars of the context; however, as they argue, situated approaches are a powerful means for advancing important discussions about educational practice.

Second, the study used an individual interview methodology, which does not incorporate an examination of students' problem solving during instruction, and thus may elicit problem-solving behaviors that would not occur in the classroom. However, it is important to note that this study was both preceded and followed by classroom-based studies of students' story problem solving, and the primary researcher spent a significant amount of time observing in the classroom in order to gain a full understanding of the context. Individual interviews have certain strengths over classroom-based studies that make them part of many educational research programs – they allow for an in-depth account of student thinking where the interviewer can reactively question students about problem-solving behaviors, and they allow problem tasks to be easily individualized – here, in the form of personalization.

Finally, as this study is qualitative in nature, a systematic analysis of how comparable different problem types (normal, personalized, etc.) were in terms of difficulty is beyond the scope of the analysis. Thus the prevalence with which different behaviors like informal strategies or non-coordinative approaches appear across problem types should be interpreted with caution. The trends across different problem types were intended to be exploratory, and need to be verified by quantitative work with tighter experimental designs and larger sample sizes. This study has been complemented by quantitative analyses (Walkington & Maull, 2011) that take into account both student and problem structure. The findings for personalized versus normal problems (the primary comparison of interest to our research program) seem to bear out in these studies. Results show that personalization does support situational understanding and provides access to reasoning about the actions and relationships in algebra story problems.

4. Implications

We conducted an exploratory series of interviews of Algebra I students from a low-performing high school, with the intent of critically examining the efficacy of algebra story problems for providing access to mathematical ideas. We found that algebra story problems had some potentially important affordances – like promoting accessible informal strategies and leveraging situation-based knowledge. However, traditional story problems also seem to have some limitations – such as an incompatibility between verbal and situational knowledge and the intended problem-solving path, and students' tendency to engage in non-coordinative approaches, particularly when formation of a situation model is challenging. Especially problematic was the seemingly large disconnect between students' situation-based reasoning and their use of the symbolic representation and manipulation strategies that are valued outcomes of algebra instruction.

The purpose of a qualitative study of this type is not simply to reach an overall conclusion that story problems are effective or ineffective. Rather, these results describe the ways in which the effects of contextualization are complex, with

[5] This approach was also characteristic of Liz's work on other parts of the Crawler problem, and on other problems.

multiple factors of difficulty and facilitation operating simultaneously, each being specific to the individual cover story of the problem, the underlying mathematical structure, and the individual student and their experiences participating in school and out-of-school activity. When students solve algebra story problems, they may leverage a variety of different practices and resources, including situational knowledge (e.g., knowledge of quantities and change from participation in different social systems), informal and arithmetic-based reasoning (e.g., using informal strategies that apply operations to fixed quantities), and practices from "school algebra" (e.g., symbol manipulation, writing and reading symbolic expressions with variable quantities).

Given the diversity of student experiences, understanding how contextualization impacts learning or problem solving is a complex endeavor. Not only do researchers and teachers lack knowledge of the breadth of the experiences of individual students, it is difficult to anticipate if and how the knowledge of those situations will be connected to a given problem, and whether or not it will be productive in helping students obtain the intended solution. This points to the importance of future research in secondary mathematics leveraging in situ investigation of students' diverse participation practices in different systems of activity, and investigating how these practices can be supportive of the goals of algebra instruction. Only when such connections are better understood can contextualized problems begin to reach their full potential for providing students with access to mathematical ideas that can provide a basis for the meaningful use of algebraic representations.

This call is especially important as this study suggests the issues that can arise when students begin to suspend situational sense-making, which may in part reflect a view of story problem solving as a practice constrained to participation in school algebra. Students engage in non-coordinative reasoning where they formulate mathematical models without making critical connections to the situational context – for instance, they may engage in meaningless symbol manipulation. If algebra story problems are not connected to students' diverse everyday practices in meaningful ways, there is a risk of implementing an algebra instruction that is heavily procedural and disconnected from the participation structures that are often valued in applied formal and informal mathematical activity. This study suggests that problem-solving in school algebra involves a complex interweaving of knowledge and participation practices from a variety of sources, and understanding these relationships may be critical to using contextualization to promote access.

A critique of traditional algebra instruction and a promising approach for helping students to utilize out-of-school experiences and coordinate them with more formal mathematical models is discussed in Chazan (1999, 2000). Rather than attempt to adapt a problem to a single context for all students, Chazan began with the idea of identifying and representing quantities and relationships among quantities, and had students identify situations in their experience for which this practice could be applied. Students learned to coordinate their own knowledge and experiences with more formal mathematical models, including investigating how such situations arise in the work of businesses, laborers, and merchants in their own community. Designing algebra instruction in this way may be a more effective way to accomplish the professed goals of using story problems, i.e., incorporating students' situational knowledge and informal strategies, and providing students with greater access and motivation in learning the subject matter. Furthermore, it acknowledges and values the diversity of students' experience, and the complexity in incorporating it into students' learning and problem solving. However, such approaches are not widespread, especially in low income schools like the one in this study. Researchers and curriculum developers in mathematics education need to grapple with the ways in which such approaches can be designed and implemented on a larger scale.

A recent survey conducted for the National Mathematics Advisory Panel report found that when Algebra I teachers were asked to identify the single most challenging aspect of teaching algebra, the overwhelming response was "working with unmotivated students," and the second most frequent response was "making mathematics accessible and comprehensible to all of my students" (Loveless et al., 2008). Both of these concerns are highly related to issues of contextualization, and the choices being made by education stakeholders about how school mathematics can be framed as "relevant" to students' lives and experiences. This study and the body of work surrounding story problems suggests that educational researchers and practitioners need to move beyond thinking of mathematical activities as "contextualized" or "not contextualized," and instead focus on how different types of contextualization mediate students' participation practices. Thus, it is important that teachers, curriculum developers, assessment designers, and researchers in mathematics education understand both the affordances and constraints of what we term in this paper as "traditional story problems," and proceed with caution and thoughtfulness when integrating such problems into the system of school mathematics.

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Appendix A. Description of problems given to participants

Each participant was given 4 or 5 problems to solve. Problem types were: normal story problems from the source Algebra I curriculum, personalized story problems, normal story problems that included symbolic equations, generic story problems,

Table A1
Linear functions underlying story problems given during interviews.

Linear function	Special factors of ease/difficulty
$y = 20x$	No intercept
$y = (60/30)x$	Unit conversion (60 min = 1 h), no intercept
$y = 0.25x$	Percentage, no intercept
$y = 12.95x$	Decimal, no intercept
$y = 2x + 11$	None
$y = 4x + 175$	None
$y = 15x + 60$	None
$y = 1500x + 500$	Large numbers
$y = 1.25x + 2.5$	Decimal
$y = 10x - 500$	Negative intercept
$y = 80 - 6x$	Negative slope
$y = 0.23x - 7.87$	Negative intercept, decimal
$y = -2.5x - 30$	Negative slope and intercept, decimal
$y = x - 0.25x + 10$	Double reference unknown, percentage

and abstract problems (see Table 1). Each student was given 2 personalized problems, 1 normal problem, and 1–2 problems that were either abstract, normal with equation, or generic. The breakdown is as follows:

- Five students were given 2 personalized, 1 normal, and 1 generic
- Nine students were given 2 personalized, 1 normal, 1 generic, and 1 normal with equation
- Ten students were given 2 personalized, 1 normal, 1 normal with equation, and 1 Abstract

The order in which problems of various types were presented to students was randomized. All problems had 1 of 14 different linear functions underlying them, and each student received a different linear function in each of their problems (Table A1).

The wide variety of linear functions was used so that when students mentioned a personal experience during their entrance interview, there would be a variety of problem structures to match their experiences using numbers and rate of change. The numbers in the problem were occasionally slightly modified to better match students' personal stories, but the changes were made so as not to change problem difficulty (e.g., $y = 17.95x$ instead of $y = 12.95x$; $y = 3x + 11$ instead of $y = 2x + 10$).

Two of the base story problems from the source curriculum contained irrelevant/distractor information (i.e., numbers that were not relevant to the problem's solution). However, none of the students attended to these numbers; it was relatively obvious to students in both cases that these numbers should not be used.

Using a mixed-effects logistic regression model (model characteristics reported in Walkington and Maull (2011)), it was also determined that readability level of normal and personalized problems was not a significant predictor of performance, nor were various background characteristics of students such as gender, race, or ESL status.

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