Word Problem Solving Tasks in Textbooks and Their Relation to Student Performance

YAN PING XIN
Purdue University

ABSTRACT The author examined the potential influence of learning opportunities provided in 1 U.S. and 1 Chinese mathematics textbook series on students’ problem-solving performance. Also, the author studied learning opportunities provided in the textbooks by analyzing word problem distribution across various problem types, as well as the potential influence of learning opportunities on students’ ability to solve arithmetic word problems, by determining student success rate (i.e., item difficulty measure) in relation to word problem distribution in adopted textbooks. Results indicated a different pattern with respect to word problem distribution in U.S. and Chinese textbooks. The relation between adopted textbook word problem task presentation and student success in solving problems suggests that the ability of U.S. participants to solve certain problem types better than other problem types may be related directly to the design of U.S. textbooks (e.g., unbalanced word problem distribution).

Keywords: influence of learning opportunities, middle school students with learning disabilities, problem solving, U.S. and Chinese mathematics textbooks

Successful problem solvers seek and find underlying structural information (e.g., problem schemata), whereas unsuccessful problem solvers tend to focus on the surface features of problems, making it difficult for them to transfer their learning to a wide range of structurally similar problems (Silver & Marshall, 1990). In sum, successful problem solvers possess problem schemata that guide the encoding and retrieval of problem information (e.g., Mayer, 1982), and their problem solving is built on a conceptual model of the problem situation (Hegarty, Mayer, & Monk, 1995; Jonassen, 2003).

In addition to learner characteristics, the problem or task features contribute to the relative ease or difficulty in solving problems. Task variables refer to factors associated with the nature of the problem and may include mathematical content and structure (e.g., various problem types), non-mathematical problem context (e.g., various cover stories), and problem syntax (Kulm, 1979; Lester, 1983). Task variables are objective factors that consider an array of problem types with varying difficulty levels (Riley, Greeno, & Heller, 1983). Although some problem types are more difficult (e.g., compare) than others (e.g., combine), “the ease with which children solve a particular problem varies according to the semantic structure of the problem, the position of the unknown quantity, and the precise way in which the problem is worded” (Stigler, Fuson, Ham, & Kim, 1986, p. 154).

In particular, researchers (e.g., Cawley, Parmar, Foley, Salmon, & Roy, 2001; Lewis, 1989; Xin, 2003a) have indicated that U.S. elementary and middle school students with learning disabilities or difficulties, in particular, experience difficulty solving word problems that involve “inconsistent/indirect language,” in which the use of the “key word” (e.g., times) does not cue the operation (i.e., multiplication; e.g., Cawley et al., 2001; Mayer, 1999). For example,
in the problem, “Larry made 12 baskets in the basketball game last night. He made 3 times as many baskets as Tom. How many baskets did Tom make?” it appears that the word “times” implies multiplication. However, division, rather than multiplication, is the choice of operation to solve the problem correctly. That is, an inconsistency exists between the cue word (i.e., times) and the correct process to solve the problem. In contrast, in the problem, “Tom made 4 baskets in the basketball game. Larry made 3 times as many baskets as Tom. How many baskets did Larry make?” the word times is consistent with multiplication as the correct process to solve the problem.

Researchers have revealed that students experience more difficulties with the “indirect/inconsistent” language problems than problems with extraneous information (Parmar, Cawley, & Frazier, 1996). When encountering problems with inconsistent language, many U.S. students were likely to commit a reversal error. That is, they incorrectly applied multiplication (e.g., $12 \times 3 = 36$) rather than division ($12 \div 3 = 4$). These students continuously made the same mistake as those detected over 30 years ago (Cawley et al.).

Previous literature suggests that students have difficulty solving problems with inconsistent language because they have a preference for a certain type of story construction (e.g., problems with consistent language); students have a schema in which only those preferred story constructions fit. As such, students understand and solve the preferred problems with relative ease. When students encounter a problem with inconsistent language, they must reorganize the information presented. During the reorganization or representation of information, many students with learning problems experience difficulty, and their problem solving is prone to errors (e.g., Lewis, 1989, pp. 522–523, Lewis & Mayer, 1987).

Rather than attributing students’ difficulty in solving inconsistent language problem types to their preference for certain problem structures, Parmar et al. (1996) hypothesized that one of the reasons for this difficulty is that “students encounter a language schema with which they are unfamiliar” and with which school curriculum and instruction provide “little time for analyses and examination” (p. 416). Obviously, factors other than the task may also contribute to students’ difficulties in word problem solving. Specifically, if the curriculum and instruction do not attend to varying problem structures during teacher-mediated instruction, students may not have opportunities to solve the inconsistent language problem type. Furthermore, if the instruction emphasizes using cue words (e.g., times indicates multiplication) for deciphering the process for solving the problem, opportunities for students to engage in critical thinking and reasoning are minimal (Parmar et al.). As a result, students will likely make numerous errors when they encounter inconsistent language problems (Parmar et al.). Evidently, providing problem-solving opportunities that emphasize mathematical thinking and reasoning is critical for teaching conceptual understanding of fundamental mathematics.

A key focus of The National Council of Teachers of Mathematics Standards (NCTM, 2000) is conceptual understanding and reasoning rather than rule-driven memorization (Maccini & Gagnon, 2002; NCTM, 2000). According to the mathematics standards, students should receive word problems and opportunities that encourage various types of reasoning. Providing opportunities to solve a variety of word problems encourages analytic thinking and reasoning. Researchers have indicated that solving a range of seemingly different, but structurally similar, problems (i.e., variant problem features) will likely promote schema knowledge and development of generalizable problem-solving skills (Chen, 1999; Mayer & Hegarty, 1996). However, traditional mathematics problem-solving instruction frequently does not provide such experience to students, especially those with disabilities (e.g., Parmar et al., 1996). Rather, with traditional instruction, teachers tend to focus on teaching simple memorization of rules, cue words, or specific steps (e.g., Jitendra et al., 2005; Parmar et al; Woodward, Monroe, & Baxter, 2001).

U.S. mathematics curriculum and instruction has been criticized in the existing literature not only for being less challenging but also for not allowing students to develop an in-depth understanding of mathematics concepts and relations (National Education Goals Panel, 1997). Researchers have conducted many cross-cultural comparison studies that compared student performance (e.g., Becker, Sawada, & Shimizu, 1998; Cai, 2000; Tajika, Mayer, Stanley, & Sims, 1997; The Third International Mathematics and Science Study [TIMSS], 1995, 1999 or curriculum and instruction (e.g., Jiang, 1995; Li, 2000; Mayer, Sims, & Tajika, 1995; Samimy & Liu, 1997; Stigler et al., 1986; TIMSS Video Studies, 1999, Hiebert et al., 2003). The work of Stigler et al. is important because it elucidates the variation in content presentation in U.S. and Soviet elementary mathematics textbooks. Specifically, the authors compared the presentation of addition and subtraction word problems in four American and Soviet elementary mathematics textbooks. Results indicated that “distribution of word problems across various problems types” in U.S. textbook series was not as diversified and balanced as was the Soviet textbook series (p. 153). Most of the problems in U.S. textbook series were simple one-step problems or those that students solved easily. In contrast, the Soviet textbook series showed more diversity across different word problem types and included more complex two-step problems.

Despite numerous international comparison studies, researchers need to produce direct research evidence to examine “influential factors on specific areas of mathematics competencies” rather than focus on overall performance or curriculum comparisons (Wang & Lin, 2005, pp. 4, 10). To date, no researchers have examined the influence of curriculum or textbook design features (e.g., opportunities for students to solve various problem types) on students’ problem-solving performance. One way to examine that
influence is to evaluate students’ ability to solve various problem types by determining item difficulty (i.e., success rate) in relation to the learning and practice opportunities provided in their textbooks. Cross-cultural curriculum comparison would be a preferred context to study the influence because existing literature indicates that traditional American mathematics textbook series strongly resemble each other (e.g., Jitendra et al., 2005; Stigler et al., 1986). Cross-cultural curriculum comparison provides unique opportunities for one to examine and compare the influence of different curricula on student performance.

My purpose in this study was to compare multiplication and division word problem distribution across various problems types in U.S. and Chinese textbooks and its potential influence on student performance. I compared the success rate of U.S. and Chinese middle school students in solving various problem types (I did not intend to make a cause-and-effect claim in determining factors contributing to U.S. and Chinese students’ possible different performance). I selected middle school students because of the noticeable decline of U.S. middle school mathematics performance when compared with elementary mathematics performance (Perie, Grigg, & Dion, 2005). Middle school mathematics deserves particular attention because mathematics instruction during these years lays the foundation for learning algebra and for later success in all areas of advanced mathematics. Furthermore, I chose Chinese students and their curriculum as a comparison because numerous cross-national studies reported distinct differences in U.S. and Chinese students’ performance (e.g., Cai, 2000) and curriculum (e.g., Jiang, 1995; Li, 2000).

Specific research questions for this study follow:
1. What was the success rate in solving the various multiplication and division problem types by a sample of U.S. and Chinese middle school students with learning difficulties?
2. How were the various word problem types distributed in the adopted U.S. and Chinese textbooks?

I hypothesized that there are differences between U.S. and Chinese textbooks series in how word problems are distributed across various problem types and that these differences account for the variations in U.S. and Chinese student problem-solving performance.

Method

Participants

I chose students with learning disabilities or difficulties as participants because existing research shows that these students experience difficulties solving problems of varying structures (e.g., Cawley et al., 2001; Parmar et al., 1996). Fifty-seven students (Grades 6–8) from one urban public middle school in northeast U.S. and 54 middle school students (Grades 6–8) from one urban public middle school in east China participated in the study. I sampled participating schools according to the mathematics textbooks that they used. Specifically, the participating U.S. school used one of the most commonly adopted mathematics textbook series published by Scott-Foresman-Addison-Wesley (Charles et al., 1998). In contrast, the Chinese school used the unified mathematics textbook series designed for developed regions in China (Shanghai Elementary and Secondary School Curriculum Reform Committee [SESSCRC], 1994, 1995, 1996, 1997).

I chose the two participating schools by matching demographic features such as relative socioeconomic development levels in the country, geographic locations, and similarity and diversity of parent vocations. Academically, student performance in the U.S. school ranked somewhat below the average according to its state standards-based assessment. Student mathematics performance at the Chinese school ranked within the bottom quartile on the basis of the metropolitan area’s middle school entry examination.

Table 1 shows demographic information for all the participants. The 57 U.S. students received learning support special education services because they were identified by Pennsylvania special education eligibility criteria as having learning disabilities (i.e., normal or above-normal intelligence but exhibiting severe discrepancy between achievement and intellectual ability). Those students represented about 13% of the total school student population. In particular, the majority of participating students (92%) were either mainstreamed in the regular classrooms (without special education teachers’ presence) or in full inclusion (i.e., with special education teachers’ presence in the regular classroom to provide support). Few students (8%) were in the self-contained classroom.

The 54 Chinese participants, comprising about 13% of the student school population, represented those who fell within the bottom quartile (i.e., 25th percentile) in each grade, according to school academic ranks in mathematics. I used that criterion as, “typically, the 25th percentile is used in studies of learning disabilities” (Jordan & Hanich, 2003, p. 214) because in China, students with learning

<table>
<thead>
<tr>
<th>Variable</th>
<th>U.S.</th>
<th>Chinese</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td>35</td>
<td>28</td>
<td>63</td>
</tr>
<tr>
<td>Female</td>
<td>22</td>
<td>26</td>
<td>48</td>
</tr>
<tr>
<td>Grade</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>19</td>
<td>13</td>
<td>32</td>
</tr>
<tr>
<td>7</td>
<td>13</td>
<td>23</td>
<td>36</td>
</tr>
<tr>
<td>8</td>
<td>25</td>
<td>18</td>
<td>43</td>
</tr>
<tr>
<td>Total</td>
<td>57</td>
<td>54</td>
<td>111</td>
</tr>
</tbody>
</table>

|TABLE 1. Word Problem Solving Study Participant Information|
disabilities were not labeled and they were included in the regular education classrooms.

To examine the equivalency of gender and grade distributions in U.S. and Chinese samples, I performed chi-square analyses. Results indicated no statistically significant difference in gender, $\chi^2(3, N = 111) = 1.032$, $p = .794$, or in grade, $\chi^2(5, N = 111) = 4.971$, $p = .419$.

Test Material and Administration

I assessed the participants on a 16-item word problem-solving test (Xin, 2003b; Xin, Jitendra, Deatline-Buchman, 2005). I designed the test in alignment with the new NCTM standards and emphasized varying construction of word problems to assess conceptual understanding of mathematical relations in word problem solving. Specifically, I systematically varied the construction of each word problem item in reference to the structure of specific problem schemata and the unknown position in a problem (Cawley & Parmar, 2003; Marshall, 1995; Van de Walle, 2004). The test consisted of a range of multiplication and division word problems involving two distinguishable problem types, multiplicative compare (MC) and vary problems (Marshall).

Table 2 shows sample problems of each problem type; MC problems varied with respect to the unknown information. That is, the unknown could be the compared quantity (MC compared), referent quantity (MC referent), or the scalar (i.e., the quantity that indicates a multiple or partial relation when comparing the two quantities; MC scalar).

<table>
<thead>
<tr>
<th>Problem type</th>
<th>Sample problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplicative comparison (MC) problems</td>
<td></td>
</tr>
<tr>
<td>MC compared-I</td>
<td>Luann has nine pictures to put in her photo album. Andrew has 3 times as many pictures as Luann. How many pictures does Andrew have?</td>
</tr>
<tr>
<td>MC compared-F</td>
<td>Julia made 28 cupcakes for celebrating the 100th school day. Patty made 1/4 as many cupcakes as Julia did. How many cupcakes did Patty make?</td>
</tr>
<tr>
<td>MC referent-I</td>
<td>Liz has eight Barbie dolls. She has 4 times as many Barbie dolls as her friend Beth. How many Barbie dolls does Beth have?</td>
</tr>
<tr>
<td>MC referent-F</td>
<td>Larry made six baskets in the basketball game last night. He made 1/3 as many baskets as Tom. How many baskets did Tom make?</td>
</tr>
<tr>
<td>MC scalar</td>
<td>Ann has 5 green color pencils and 30 red color pencils. How many times as many red color pencils as green color pencils does Ann have?</td>
</tr>
<tr>
<td>Vary problems</td>
<td></td>
</tr>
<tr>
<td>Rate times a quantity</td>
<td>Ms. Penn bought three cases of candy bars for the PTA meeting held last week. Each case of candy bars contains 12 Kit Kat bars. How many Kit Kat candy bars did Ms. Penn buy in all?</td>
</tr>
<tr>
<td>Fair share/partition</td>
<td>A building has a fire escape with a total of 90 steps across six floors. If each floor has the same number of steps, how many steps are there for each floor?</td>
</tr>
<tr>
<td>Measurement division</td>
<td>The Joy Company packs 48 cans of tomatoes in each crate. How many crates will the company need to pack 720 cans of tomatoes?</td>
</tr>
<tr>
<td>Proportion</td>
<td>Lee and Sienna were responsible for making drinks for the party last weekend. They used 3 lemons for every two quarts of lemonade. If they bought 12 lemons, how many quarts of lemonade could they make?</td>
</tr>
</tbody>
</table>

Note. I = integer (relational statement illustrates a multiple relation); F = fraction (relational statement illustrates a partial relation).
lems ranged from rate times a quantity (\(R \times Q\)), fare share or partition, measurement division, to proportion problem types. In summary, as the position of the unknown in a problem systematically varied across problems, the 16-item test presented consistent or traditional and inconsistent or nontraditional arithmetic word problem types (Cawley & Parmar, 1994; Lewis, 1989).

Testing Procedures

Classroom instructors conducted testing in groups during regularly scheduled class time. The instructors used scripted directions to administer the test to all participants. The instructors had students read each problem and encouraged them to do their best. Students received assistance if they had difficulty reading words on the test. Instructions also required students to show their complete work. The instructors gave no feedback regarding the accuracy of students' solution or work. All students had sufficient time to complete the test. The U.S. participants took the English version of the test, and the Chinese participants took the Chinese translation of the same test.

Text Translation

I used an English back translation process to ensure the equivalency of the English and Chinese version of the test. Specifically, an individual who was fluent in English and Chinese translated the original English version of the test into Chinese. Then a second individual who was also literate in English and Chinese translated the Chinese version of the test back into English. A third person compared the back-translated English version to the original English version and found that the back-translated English version was equivalent to the original English version, including the order in which numerator information was presented in the word problem. For instance, in proportion problem, if the conditional statement (i.e., the “IF” statement) in the problem followed the action “THEN” statement (i.e., the) in the English version, the Chinese version would maintain the same sequence. After the back translation, I slightly edited the names of characters or units of measure in word problems in the Chinese version of the test to make them culturally appropriate and understandable to Chinese students.

Scoring and Performance Analyses

I used correct answer or correct mathematics sentence or equation for solutions as the criterion for evaluating whether students provided a correct solution to a specific problem item. Specifically, the criterion was met if (a) the mathematics sentence or equation and the answer to the problem were correct, (b) only the correct answer was provided, or (c) the mathematics sentence or equation was correct but the answer was incorrect because of calculation errors. I used that scoring system because the interest was on students' conceptual understanding of mathematics word problem solving rather than on the mechanics of calculations.

My research assistants and I scored each student’s performance on the test. We totaled the number of participants in each group (i.e., U.S. and Chinese) who met the correct solution criterion. We then divided the number by the total number of participants in the specific group. The resulting percentage or success rate was an index measuring item difficulty. In that case, the larger the success rate, the easier the item. We calculated the item difficulty index for each test item to indicate success rate in solving a range of multiplication and division word problem types.

Textbook Analysis Procedures

Textbook selection. I used the Scott-Foresman-Addison-Wesley (Charles et al., 1998) Mathematics textbook series (Grades 3 to 6) and the unified mathematics textbook series (SESSCRC, 1995, 1996, 1997, for the analyses. My rationale was that the U.S. and Chinese participants used the assigned textbooks from third grade when they started to learn multiplication and division word problem solving, through sixth grade, the current grade level of the youngest participants in this study.

Coding word problem. We used the same word problem classification framework as that used to develop the test for student performance assessment (see Table 2) to examine the distribution of word problems across the various problem types in the U.S. and Chinese mathematics textbooks. Specifically, we initially identified all multiplication and division word problems in the lesson and practice (including “Review and Practice”) sections of the textbooks (student version) pertinent to arithmetic operations; as such, we did not examine lessons on geometry, probability, or statistics. We considered as word problems any questions or problems stated in words, including those with information presented in a table or graphic format.

Next, we coded all MC problems into one of three variations (i.e., MC compared, MC referent, and MC scalar) and all vary problems into one of three variations (i.e., rate times a quantity, fare share or partition, or measurement division and proportion; see Table 2). For problems with multiple subproblems, we coded and counted each subproblem toward the total counting of problems for each type. For problems that required multisteps, we applied a code for each substep and counted all coded substeps toward the final counting of problems of a specific type. A substantial portion of word problems in the Chinese textbooks involved multisteps; each step featured differential problem schemata. For instance, “It costs 168 Yuan to buy four pairs of leather shoes. It costs 7 Yuan to buy one pair of fabric shoes. How many times as much does it cost for a pair of leather shoes as for a pair of fabric shoes?” (SESSCRC, 1996, p. 113). The problem required students to (a) solve for unit price for the leather shoes (coded as the “unit price unknown” or “fair share or partition” type, which
required an operation of simple division), then (b) solve for the unknown scalar when comparing the price of leather to fabric shoes (coded as the MC scalar). In contrast, we counted a problem that required repeated application of the same operation (e.g., If settlers planned to go from South Pass to Fort Boise and travel 8 hr per day at 3 mph for 20 days, would they arrive at the Fort?" [distance from South Pass to Fort Boise, 465 miles, was given in the table provided]; Charles et al., 1998, p. 119) as one problem with the same code, that is, rate times a quantity.

**Word problem presentation analysis.** For either the MC or vary problems, we added up the total number of problems coded for each variation (e.g., MC compared, MC referent, and MC scalar) of a type (e.g., MC) and divided that number by the total number of MC or vary problems identified, which yielded a percentage as the indicator of word problem distribution across various MC or vary problem types. That measure presents information on whether the textbooks provided students with balanced opportunities to solve various problem types to facilitate conceptual understanding of mathematical relations in the MC or vary problem schemata.

**Interrater Reliability**

A research assistant who was bilingual in English and Chinese scored all students’ responses with an answer key following a 2-hr training session with me. To assess reliability of scoring, I (also bilingual) rescored 35% of the tests. We computed interrater reliability by dividing the number of agreements by the number of agreements and disagreements and multiplying by 100%. Interrater reliability for student word problem solving test performance was 100%.

I provided a 4-hr initial training session for two independent coders who coded all word problems found in the textbooks. The coders used a table presenting multiple samples of each problem type as an anchor for coding six problems. In addition, throughout the coding process, the coders set up meetings with me whenever they experienced difficulty in coding a specific problem. One research assistant who was not bilingual identified and coded word problems in the U.S. textbooks (Scott-Foresman-Addison-Wesley, Grades 3 to 6). Another bilingual research assistant identified and coded word problems in the Chinese textbooks (SESSCRC, Grades 3 through 6) and word problems in the U.S. third-grade textbook (Scott-Foresman-Addison-Wesley). The purpose of overlapping coding was to ensure that both research assistants coded the problems consistently. Overall, reliability between the two independent coders for the U.S. third-grade mathematics textbook was 97%. Furthermore, I independently coded 60% of U.S. and 60% of Chinese textbook word problems; reliability between my coding and that of each coder was 91% and 95%.

**Results**

Before reporting results on item difficulty and word problem distribution analyses, researchers might report the two student groups’ overall performance on the 16-item word problem solving test. The result in this study indicated that the Chinese sample (M = 59%, SD = 19.9) significantly outperformed U.S. samples (M = 21%, SD = 18.7) in solving variously constructed multiplication and division word problems (mean difference = 38%), F(1, 110) = 108.9, p < .01. In addition, we found that the attempt rate (number of problems attempted divided by total number of problems in the test) in the Chinese groups ranged from 87% to 100%, with a mean of 99% or a median of 100%. In contrast, the attempting rate for the U.S. group ranged from 19% to 100%, with a mean of 88% or a median of 100%.

**Word Problem Solving Performance: Item Difficulty Analysis**

What was the item difficulty or success rate when a sample of U.S. and Chinese middle school students with learning difficulties solved various multiplication and division problem types? Performance comparison revealed similar and dissimilar patterns (see Table 3). For U.S. and Chinese participants, the order of item difficulty (easiest to hardest) for the MC problem types was MC compared-I, MC referent-I, MC compared-F, MC scalar, and MC referent-F. The results indicated that U.S. and Chinese participants

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**TABLE 3. Percentage of U.S. and Chinese Students Who Met Correct Solution Criterion in Solving Problem Types**

<table>
<thead>
<tr>
<th>Students</th>
<th>MC compare-I</th>
<th>MC compare-F</th>
<th>MC referent-I</th>
<th>MC referent-F</th>
<th>MC-scalar</th>
<th>R × Q</th>
<th>Fair share or measurement division</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>52.5</td>
<td>15</td>
<td>20</td>
<td>9</td>
<td>13</td>
<td>53</td>
<td>45.5</td>
<td>25</td>
</tr>
<tr>
<td>Chinese</td>
<td>79.5</td>
<td>59.5</td>
<td>74.5</td>
<td>42.5</td>
<td>58.5</td>
<td>98</td>
<td>85</td>
<td>54</td>
</tr>
</tbody>
</table>

Discrepancy | 27 | 44.5 | 54.5 | 33.5 | 45.5 | 45   | 39.5                              | 29         |

Note. MC = multiplicative compare; R × Q = rate times a quantity; I = integer (relational statement illustrates a multiple relation); F = fraction (relational statement illustrates a partial relation).
experienced relatively more difficulty solving MC problems involving a partial relation (i.e., fraction) when compared with ones involving a multiple relation (i.e., integer). If the factor of integer or fraction was partialed out, MC compared was the easiest problem type, whereas MC-Referent was relatively more difficult to solve. For vary problem types, the order of item difficulty (easiest to hardest) for the U.S. and Chinese samples was rate times a quantity (R × Q), fair share or measurement division, and proportion.

When I examined degree of item difficulty across the U.S. and Chinese groups, a different pattern emerged. Table 3 shows that the largest discrepancy across the two groups in terms of item difficulty was found for the MC referent-I (55 percentage points) and the least discrepancy was found for MC compared-I (27 percentage points) type. Specifically, for the U.S. sample, 53% of all participating students correctly solved the MC compared-I problem type but only 20% met the correct solution criterion for the MC referent-I problems. In contrast, 79.5% and 74.5% of the Chinese sample correctly solved the MC compared-I and MC referent-I problems, respectively. In addition, I observed a relatively small discrepancy (29 percentage points) between the U.S. and Chinese sample when solving proportion problems.

When mixing the two problem types (i.e., MC and vary), the order of item difficulty (easiest to hardest) for the U.S. sample was R × Q, MC compared-I, fair share or measurement division, proportion, MC referent-I, MC compared-F, MC scalar, and MC referent-F. Order of item difficulty for the Chinese sample, however, was R × Q, fair share or measurement division, MC compared-I, MC referent-I, MC compared-F, MC scalar, proportion, and MC referent-F.

**Word Problem Distribution Across Various Types in Adopted Textbooks**

How were various word problem types distributed or what were the learning and practice opportunities provided in the adopted U.S. and Chinese textbooks for students to solve the various problem types? The results of the U.S. and Chinese textbooks comparison revealed similar and dissimilar patterns (see Figure 1). Specifically, the distribution of word problems across a range of vary problem types was similar in the Chinese and U.S. textbooks. That is, the research assistants and I found relatively more problems to be the R × Q, fair share or measurement division types, and fewer problems to be proportion types. However, we observed a different distribution pattern for the MC problem type. In the U.S. textbooks, 63% of all comparison problems were MC compared, but only 14% were the MC referent type. In contrast, in the Chinese textbooks, 41% of all comparison problems were MC compared and 33% were the MC referent.

Table 4 shows U.S. and Chinese students’ success rate in solving different types of word problems, along with word problem distribution across these types in adopted textbooks. One fourth of MC scalar problem types presented in the U.S.
textbooks were ones such as, “Use data file to answer: Which planet has three times as many small moons as large moons?” To some degree, problems such as those require only selective response mode rather than constructive response as to that required in student-performance assessment.

Discussion

Word Problem Solving Performance: Item Difficulty Analysis

Results of item difficulty analyses indicated that although U.S. and Chinese students had more difficulty solving problems involving fractions than those involving integers (a typical developmental sequence, Van de Walle, 2004), the U.S. students seemed to experience much more difficulty than did their Chinese counterparts solving the MC referent type (i.e., problems involving inconsistent language). The difference was more obvious when students were solving MC problems involving no fractions. When fractions were involved, the difference was not as large, which might be attributed to the difficulty of dealing with fractions for U.S. and Chinese students.

That result echoes a subtle pattern that I discovered when I examined item difficulty across the two problem types. Specifically, the Chinese participants did relatively well solving simple multiplication (i.e., R × Q, 98% and MC compared-I, 79.5%) and division (e.g., fair share or measurement division, 85% and MC referent-I, 74.5%) problems involving integers, with success rates (i.e., the item difficulty index) above 74%. However, Chinese students had more difficulty solving problems involving fractions (e.g., MC compared-F, 59.5%; MC scalar, 58.5%; MC referent-F, 42.5%) or multisteps (e.g., proportion, 54%); success rates all below 60%. In contrast, the U.S. participants showed a different pattern. Compared with problems involving fractions or multisteps (success rate < 25%), U.S. students similarly (to their Chinese counterparts) did relatively well solving simple multiplication (i.e., R × Q, 53%; MC compared-I, 53%) and division (i.e., fair share or measurement division problems, 46%) problems, with success rates above 46%. However the MC Referent-I problem type was an exception. Only 20% of the participants met the correct solution criterion when solving the MC referent-I problems, although this problem type requires only a simple operation of division and involves no fractions.

In summary, U.S. participants experienced more difficulties solving the MC referent-I type than did Chinese participants. The results support existing research findings (e.g., Lewis, 1989) that U.S. students (especially those with learning problems) experience substantial difficulty solving word problems with inconsistent language (e.g., MC referent) compared with traditional types (e.g., MC compared). However, the results of this study did not reveal the same pattern with the Chinese counterpart.

When assessing students’ problem-solving process, the research assistant and I found that all Chinese students solved the problem by setting up one or two mathematics sentences based on the algorithm. None of the students used picture representations to help with the solution. In contrast, many students in the U.S. sample relied substantially on visual representations (diagrams or pictures of objects, tic marks) to help solve the problem. For instance, when solving measurement division problems such as the one presented in Appendix A, several U.S. participants relied on numerical visual representation of set size or unit rate and repeated addition or skip counting (i.e., repeatedly added up 30s or skip counted by 30 until reaching 270) to determine how many gallons of gas are needed to drive 270 miles (given the unit rate, 30 miles per 1 gallon of gas). In contrast, most Chinese students used a mathematics sentence ($270 \div 30 = 9$) to solve the unknown. When solving MC referent problems such as the one shown in Appendix B, 1 U.S. student tried to draw a picture with partitions for the answer but was not successful; the other student attempted direct use of an algorithm but made a reversal error in setting up the mathematics sentence. In contrast, the 2 Chinese students (see Appendix B) wrote a mathematics sentence for solving the problem.

Evidently, the U.S. participants used more visual representation in various forms to assist with problem solving than did Chinese participants. In contrast, the Chinese students simply created a mathematics sentence for the solution. That finding is consistent with previous cross-national comparison studies (e.g., Cai, 2001) because “U.S. students frequently used visual or pictorial representation, while Chi-

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**TABLE 4. Student Problem-Solving Success Rate (%) Versus Problem Distributions (%) Across Different Types in Adopted Textbooks**

<table>
<thead>
<tr>
<th>Participants</th>
<th>MC-compared</th>
<th>MC-referent</th>
<th>MC-scalar</th>
<th>Rate times a quantity</th>
<th>Fair share or measurement division</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. students</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Success rate distribution</td>
<td>33.5</td>
<td>14.5</td>
<td>13</td>
<td>53</td>
<td>45.5</td>
<td>25</td>
</tr>
<tr>
<td>Chinese students</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Success rate distribution</td>
<td>69.5</td>
<td>58.5</td>
<td>58.5</td>
<td>98</td>
<td>85</td>
<td>54</td>
</tr>
</tbody>
</table>

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Chinese students were more likely to use symbolic (arithmetic or algebraic) representations” (Cai, 2001, p. 404). Using picture mapping or skip counting (either count up or count down) as observed in the U.S. students’ problem solving may help solve problems with relatively simple relations or small numbers; however, when the numbers in the problem become larger or involve fractions, students’ drawings and computation may become cumbersome and prone to errors. At other times, the drawings may be confusing and fail to work. Extant literature in mathematics problem solving indicated that weak problem solvers “relied more on concrete or memory images that led not only to ineffective but inefficient solutions” (Van Garderen & Montague, 2003, p. 247). Conceptual understanding of mathematical relations in problem solving goes beyond concrete representation to use symbols to generalize arithmetic operations (Curiro & Schwartz, 1997) and to promote generalizable problem-solving skills (Cooper & Sweller, 1987).
APPENDIX B
Examples of Solving a Multiplicative Compare Referent-Fraction Problem by the U.S. and Chinese Participants

Problem: Howard read 12 books in the summer reading program. He read \( \frac{3}{4} \) as many books as his friend Tony. How many books did Tony read?

U.S. Participant 1

\[
\begin{align*}
12/1 & \quad 3/4 \\
\frac{12}{1} & \times \frac{3}{4} = \frac{36}{4} \\
& = 9
\end{align*}
\]

ANSWER: 9 books

U.S. Participant 2

\[
\begin{align*}
12 & \div 3 \times 4 \\
& = 4 \times 4 \\
& = 16 \\
\text{Tony Read 36 books}
\end{align*}
\]

Chinese Participant 1

\[
\begin{align*}
12 & \div 3 \times 4 \\
& = 4 \times 4 \\
& = 16 \\
\text{小童读了16本书。} (Tony read 16 books.)
\end{align*}
\]

Chinese Participant 2

\[
\begin{align*}
12 & \div \frac{3}{4} \\
& = 12 \div 0.75 \\
& = 16 \\
\text{小童读了16本书。} (Tony read 16 books.)
\end{align*}
\]

Word Problem Distribution Across Various Types in Adopted Textbooks

I performed textbook word problem distribution analyses to seek insight into the potential influence of the intended curriculum or textbook features (specifically, learning opportunities for students to solve various problem types) on student performance. The textbook analyses echoed similar and dissimilar patterns found in students’ performance analyses. Although we found a similar distribution pattern across a range of vary problem types, the research assistant and I observed a different pattern for the MC problem type. Specifically, the results of this study indicated that Chinese textbooks provided students with relatively balanced opportunities to solve various comparison problems (e.g., 41% of word problems were MC compared vs. 33% of MC referent). However, the U.S. textbooks showed relatively unbalanced word problem presentation. The majority (63%) of problems involved the MC compared (i.e., the consistent language type) and considerably
fewer (14%) problems constituted the MC referent type (i.e., inconsistent language type).

More important, among the word problems identified, we found that the Chinese textbooks required students with ongoing opportunities to solve all variations of a problem type with the same contextual story. Following is one example found in the Chinese third-grade mathematics textbook (SESSCRC, 1995, p. 41):

1. There are 24 red balls and 8 blue balls. How many times as many red balls as blue balls?
2. There are 24 red balls. There are three times as many red balls as blue balls. How many blue balls are there?
3. There are 24 red balls. There are three times as many blue balls as red balls; how many blue balls are there?

As shown in the above problem set, the Chinese textbook provided students with opportunities to solve exhaustive variations (the first problem asks for the scalar, the second problem asks for the referent quantity, the third problem asks for the compared quantity) of a problem type (i.e., the multiplicative compare problem type). Through systematically manipulating word problem construction and the position of the unknown quantity, the textbooks elaborated the connection between multiplication and division within an overarching problem schema, in this case, multiplicative compare problem schema. In the course of solving problems with various constructions, students eventually grasped mathematical relations in the multiplicative compare problem structure.

In addition, we found that the Chinese textbook required students to solve a problem using arithmetic (e.g., number model or sentence) and algebraic approaches (an equation with a letter representing the unknown variable). Furthermore, the Chinese textbooks asked students to modify a multiplication problem (e.g., an $A \times B$ problem, “The school bought 5 soccer balls; each costs 10 yuan. What was the total cost?”), for instance, to two division problems (i.e., fair share and measurement division problem structure) or modify a division problem (e.g., a fair share problem, “There are 48 students. If they are divided into 4 equal groups, how many students will be in each group?”) to a multiplication problem ([Grade 3] SESSCRC, 1995, p. 36). These textbook or curriculum features focus on shaping problem schemata through various problem-solving opportunities. Nevertheless, they did not appear in the U.S. textbooks analyzed in this study.

**Potential Influence of Word Problem Solving Opportunities on Student Performance and Implications**

The cross-cultural performance analyses that I conducted offered diagnostic information of how a sample of U.S. and Chinese students performed differently to solve multiplication and division word problems. Linking performance assessment to curriculum analyses provided insights into how curriculum or textbook features may have influenced students’ performance.

Wang and Lin (2005) reported that “mathematics learning is a culturally scripted activity whose outcome is a function of interrelated factors and environments” (p. 10). Different performance profiles across U.S. and Chinese students may be caused by cultural (including student self-concept and expectations), language, and teaching-related factors (Wang & Lin). However, the relation between different patterns in student performance and the corresponding differences in word problem presentations in adopted textbooks seems to indicate that school curricula may have a role in shaping students’ preference for certain problem types (see Table 4). Having difficulty solving certain word problem types or activating a specific problem schema to represent and solve a problem (e.g., Riley et al., 1983) may be related to the failure of textbooks to provide sufficient opportunities for students to solve a range of problems to facilitate generalizable problem-solving skills.

The problem sets found in the Chinese book (see the example provided in the preceding section on adopted textbooks) are contrast examples, in which the pedagogical idea of shaping problem schemata is evident. Unfortunately, such problem sets were absent in the U.S. textbooks that I analyzed. According to Stigler et al. (1986), “availability of a problem schema is a joint function of problem characteristics, frequency of exposure, and characteristics of the instructional environment (e.g., textbook presentation and instructional strategies)” p. 168). Researchers need to further explore how students develop necessary schemata for representing and solving problems and how curriculum and instruction can be better designed to facilitate problem schema development.

An examination of some popular U.S. school mathematics curricular programs (e.g., Addison-Wesley Scott-Foresman, Houghton Mifflin, McGraw Hill) reveals that most teaching or practice problems provided in textbooks are similarly structured. Using the MC problem type as an example, teachers often ask students to find the quantity associated with the compared set. For example, in the following problem, “Elaine collected 7 Pokemon cards. Mia collected five times as many cards as Elaine. How many cards did Mia collect?” the first sentence gives information about the referent set. The second sentence refers to the relational statement. The last part of the problem asks for the quantity that corresponds to the compared set. When the problem is so structured, the word “times” in the relational statement will always indicate multiplication as the choice of operation to solve the problem.

Evidently, the typical U.S. school mathematics curriculum in the U.S. does not often emphasize teaching and assessing conceptual understanding by varying the surface structure of problems or the position of the unknown in the problem. Therefore, one implication for practice is that the mathematics curriculum should provide students with opportunities to solve many types of problems (e.g., varied surface appearance but similar underlying structure) to ensure that students grasp the underlying structure of the problem. Cawley et al. (2001) argued that “Word problems
can be constructed to include any dimension of language comprehension or information processing desired without being based on computation" (p. 325). For example, comparison problems should allow students to find not only the compared quantity but also the referent quantity and the scalar (i.e., the quantity expressed in the relational statement).

The findings of this study have critical practical implications in guiding teaching practice and curriculum design efforts. The results highlight the importance of teachers purposefully constructing word problems in direct instruction and practice to ensure that students are taught or exposed to the "whole universe" of a problem type to facilitate generalizable problem-solving skills. My findings will guide further in-depth qualitative cross-cultural curriculum studies, including instructional delivery, so that researchers can identify empirically validated effective instructional features or models. However, I did not examine methods in which instruction was actually delivered in U.S. and Chinese classrooms. Direct observation of classroom teaching will help triangulate the information found in textbook analyses.

Conclusion

I examined word problem distribution across various types in one U.S. and one Chinese mathematics textbook series (part of intended curriculum) and its relation to students' success rate solving various problem types. Chinese textbooks provide students with relatively balanced opportunities to solve various comparison problems, whereas the U.S. textbooks showed an unbalanced distribution of MC compared (63%—consistent language) and MC referent (14%—inconsistent language). The parallel pattern found across students' performance profile and word problem distribution in adopted textbooks indicates that the curriculum feature of U.S. textbooks (e.g., unbalanced word problem distribution) may have a role in shaping U.S. students' ability to solve certain problem types better than others. In contrast, Chinese textbooks provide students with systematic opportunities to solve a range of purposefully constructed word problems. The intent in curriculum design in the Chinese textbooks should facilitate conceptual understanding of problem structure or schema acquisition, which is the primary component of skilled problem-solving performance (Sweller, Chandler, Tierney, & Cooper, 1990).

To conclude, story problems pose difficulties for many students because of the complexity of the problem-solving process (Jonassen, 2003; Schurter, 2002). The task variable, such as semantic structure of a word problem, may determine the difficulty level of problem solving. Nevertheless, an instructional environment in which problem-solving skills are developed may change students' ability to tackle difficult tasks, and, therefore, make a difficult task become an easy one. The way that curriculum (e.g., textbooks) and instruction provide students with opportunities to solve word problems that illustrate the range of problem types is essential to conceptual understanding (Sweller et al., 1990).

NOTE

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