

Reflections on Computers in School Mathematics: CAMP +30¹

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In the Sixties, the Computer-Assisted Mathematics Project (CAMP) at the University of Minnesota High School was a pioneering effort to explore student programming applications of the computer in school mathematics. Over the ensuing three decades, we have witnessed dramatic changes in the capabilities and potentials of the technology. What has occurred in the usages of computers in school mathematics? How has computing technology impacted on the teaching and learning of school mathematics? What have we learned?

For the past thirty-five years, I have participated in an amazing journey within my career as a mathematics educator. This personal journey has involved a continuing search for, and understanding of, significant ways to use the computer in my teaching of mathematics and in my role as a researcher and teacher educator. In my early review of research on computers in mathematics education (Hatfield, 1969) [coincidentally, presented at the April 1969 Annual Meeting of the NCTM in Minneapolis], much of the focus was upon CAI (computer-assisted instruction) and its associated CMI (computer-managed instruction) with only a few examples of student programming emphases. Looking back, I recall there was great interest and anticipation over the potentials, mostly then visions of a future where the computer would be ubiquitous and omnipresent! How has that envisioned future unfolded, to become the past three decades?

In this discussion, I will trace at least some of that journey with a goal of providing insights into the broader issues and impacts of this technology, which has already revolutionized our world and which promises to involve even greater

transformations of our human activities in the future. By examining this history², my goal is to illuminate what I perceive to be the underlying premises and assumptions, the major qualities of these computer usages, and the fundamental issues involved. Perhaps this reflection will stimulate us to ponder more deeply what we are doing, and to shape our choices related to applications of computing technologies in ways that improve mathematical education of our students.

The emergence and growth of computers in mathematics education is a complex set of overlapping and interacting developments and events. There are many ways to sort these. As a broad organizer, I will adopt Taylor's (1981) trichotomy in which the computer is seen as "tutor" (simulating a kind of teacher), as "tutee" (being "taught" by the user via constructed programs), or as "tool" (where the user executes pre-stored software to do computationally-based work). After briefly reviewing the history within each of these domains, I will provide an analysis to examine the themes and issues that appear to cut across and within these computer usages. I will summarize with some indications of what I see to be the major problems and challenges for computers in mathematical education, yet before us.

Computer as "Tutee"

In this category of uses of the computer, the machine provides the context for mathematics students and teachers to write, test, refine, extend, and apply their own procedures or algorithms. The broader goals may vary, ranging from teaching computer science concepts, to training programmers, per se, to developing graphics artists, to creating simulations (such as games), to a focus on the mathematical concepts and processes involved. In the following, I recall some of the very earliest examples of student programming, tracing subsequent developments and variations.

CAMP--Student programming in BASIC

During the winter months of my initial year of teaching school mathematics at Richfield High School in 1963, I enrolled in a special evening workshop at Control Data Corporation (CDC), taught by Dr. Robert ("Doc") Smith. Along with more than 75 other interested Twin Cities area mathematics teachers, I was introduced to simple FORTRAN programming via key-punched data cards. Despite the many input/output challenges and format errors, I was hooked!

After I joined Donovan Johnson and David Johnson, along with two other newly appointed teachers, Dale LaFrenz and Tom Kieren, at University of Minnesota High School in September 1963, we began to explore a brand-new programming language. BASIC had just been made available on the Dartmouth College (Hanover NH) time-sharing computer (itself a major breakthrough). In 1964, under a General Electric Foundation grant we began a two-year experiment with classes of grade 7, 9 and 11 students at U-High. At that time, Pamela W. Katzman and John Walther joined our faculty as well as the CAMP research and curriculum development team.³

In our CAMP research, we investigated the possible role of the computer as a programmable tool in school mathematics. Our philosophical view⁴ emphasized a focus:

- on students using the computer as a problem-solving tool,
- on students controlling the machine rather than vice-versa (as in the contrasting “CAI” of the day),
- on students designing and improving their own algorithms,
- on students using programs and output as a context for modeling and deepening their conceptual and procedural understandings of their mathematics, and
- on students investigating mathematics in a way that might promote their insight and discovery.

We saw school mathematics as a context for students who, because of the nature of the subject matter, must actively engage in a problematic approach, and who, through such approaches, could learn to be generative and self-directed. Indeed, we were mainly believers in the so-called “new math,” seeing the value of learning fundamental structures in coming to understand mathematics as a system “that made sense.” At the same time, we balanced abstract theory with a need to operationalize (apply) new ideas---in the form of computer procedures that would model the theory and allow the potential for extensions of the theory. Within the curriculum advocacies of the Sixties, we also believed in the “new pedagogy” (see esp. Bruner, 1960; Dienes, 1960; Skemp, 1971) that saw students as active learners (rather than passive receptors of information to be memorized and practiced) and teachers as stimulators and guides (rather than sources of information or “telling”). We saw student programming uses of the computer as a context for engaging students in the construction of meaning---building-up understandings, skills, and appreciations by using powerful tools they could control. We chose the BASIC language because of its ease to learn and use, which avoided diverting the focus to an emphasis on computer science or programming, per se.

Among the results of our research, we were able to report certain general and specific benefits to students (Hatfield & Kieren, 1972) and we were able to provide an existence model of the detailed infusion of student programming into school mathematics topics (see published CAMP textbooks, 1968-70). Along with several other mathematics teachers who were exploring student programming activities in school and collegiate mathematics in this early period (Andree, 1967; Bitter, 1970; Dorn, 1970; Foster, 1972; Nevison, 1970), we witnessed the positive results for many students in their conceptual understanding, their development of mathematical procedures, their interest in approaching mathematics as a problem-solving activity, and in their excitement and attraction to using the computer. For some students, the task of constructing computer programs may not have made their learning of concepts or procedures easier, and they may have been as confused or disinterested with using the computer as without---of course, such technology is never a panacea for promoting high quality mathematical experiences.

Turtle Geometry and Logo

By the late Sixties, work on developing an even simpler, more child-oriented computer programming language had resulted in the early versions of Logo (Feurzeig et al, 1969). The focus on young children's creations in controlling the mechanical "turtle" was soon accompanied by mathematicians' interest in such egocentrically-oriented constructions as representing a new theoretical geometry (Abelson & diSessa, 1980). With the appearance of the first Apple microcomputers, versions of Logo appeared that operated within the graphics environment by drawing traces of an icon "turtle's" paths. While the period of 1965-80 belonged to BASIC enthusiasts, the decade of the Eighties saw Logo become a worldwide, multi-age phenomenon.

Launching Logo into popularity as its primary advocate, Papert (1980) provided an important conceptual framework (appealing to developmental psychology and a constructivist epistemology) for its use in schools, and he quickly became the "guru" for a tidal wave of renewed interest in student programming. Perhaps more than any other perspective, Papert's notions of a child-centered, "microworld" environment had broad appeal to those looking for computer applications to mathematics learning that were controlled by the student. Logo proved to be easy enough for even very young children to enter and "play," yet powerful enough to offer capabilities such as mathematical recursion and modularized structures which could be called as procedures (Noss, 1985; Olive, 1985). Internationally, advocates of Logo intensely support its use, yet it is unclear how extensively it is currently used (at least one of the most widely used versions is no longer available in today's Windows or Macintosh operating systems).

Rather coincident with the development of Logo were the urgent calls for reform of computer programming languages and operating systems by computer scientists--- which had the effect of ensuring that Logo did include some of the called-for features. It had become clear that early symbolic languages (e.g., FORTRAN, COBOL, ALGOL) lacked the necessary structural qualities to ensure that anyone (including its author!) would be able to "debug" the massive procedures that were being written (such as for airline reservations). The development of Pascal, C+, and other structured programming systems followed (but, alas, too late for many situations, as found in the current "millennium crisis" involving the imminent computer errors related to miscoding the year to "00"). BASIC's developers, Kemeny and Kurtz, provided an update of the "old favorite" (which through the Apple and IBM PCs had become the built-in programming language on all microcomputers) in their release of True BASIC in 1983, which sought to offer most of the modern features of structured languages.

As the demand for computer programmers to work in the explosive commercial and governmental applications of computers grew in the Seventies, we saw an Advanced Placement precollege computer science course introduced by the College Board. One consequence was to attract many high quality school mathematics teachers, who were typically the only available local faculty who might be trained to teach the course.

Algorithmics & informatics

Particularly in Europe, the perspective of “informatics” was promoted from the mid-Seventies. Within this perspective, students would develop a broader knowledge related to computing technologies, which would include attention to certain computer science concepts, computer programming (not necessarily trained in any specific language, but to learn more basic and fundamental ideas or methods, such as a theory of data structures), and information processing science. The connections to school mathematics varied, but this was often seen as the context where attention to at least certain elements of informatics, such as discrete mathematical approaches, would best occur (Johnson & Tinsley, 1978; Johnson & Lovis, 1987; Cornu & Ralston, 1992).

Within the ranks of professional mathematicians, we can find a more substantial criticism and debate arising over the role of student programming and an emphasis upon algorithmics (eg., Mauer, 1984, 1985 vs. Halmos, 1985; Koblitz, 1996 vs. Dubinsky & Noss, 1996; Wu, 1996). This debate centers on differing perspectives about the nature of mathematical education using computers, especially dealing with issues of the role of proof and learning to construct proofs.

Computer as “Tutor”

This approach may well be the very first way that a computer was used in education (Smallwood, 1962). As I reported in my early review (Hatfield, 1969), the development of computer systems for simulating “teaching” was immediately widespread in the form of CAI (computer-assisted instruction) strategies. Prior to the microcomputer, such systems were rather large mainframe, multi-terminal, time-sharing approaches. The instructional goals were upon mostly low level outcomes resulting from drill and practice of skills or “facts” to be made habituated or immediately recalled. The curricular designs were fostered under the premises of neo-behaviorism where behavioral objectives, task analyses, hierarchies of skills, and performance standards led to the design of tightly controlled, randomly generated sequences of stimulus-response experiences for students.

With the continuing infusion of federal monies to support large R&D laboratories at universities (eg., Suppes at Stanford, Bitzer at Illinois, Glaser at Pittsburg, Hansen at Florida State, Mitzel at Penn State) and in the military (eg., National Science Foundation Center at Ft. Gordon, GA), there occurred a massive effort to design such systems that could be shown to be more effective in instruction than regular teaching. Overall, in school mathematics we saw positive results could be achieved by students participating in regular, systematic practice, and with some forms of tutorial level instruction.

This pursuit of the “will-o-wisp” replacement for a teacher has continued, and it is still possible to find the remnants of this movement in certain commercial systems (eg., Computer Curriculum Corporation, Josten’s Learning Lab, Learning Logic System) which still exist in many schools across the nation. Instructional technology theory has been refined, but remains essentially unchanged with respect to the necessity to analyze formalized ‘knowledge systems’ into fragmented bits which are ‘fed’ to the learner by the

computer system. While there may be incremental improvements in the basis for doing this, there still remains little clear evidence from research on mathematics learning and teaching that can be built into these systems.

For many mathematics educators, however, the paradigm of the child being “taught” and controlled by the machine has been essentially rejected. With the reality of the microcomputer, and the call for a focus on mathematical problem solving (NCTM, 1980), the stage was set for a different kind of computer experience.

Computer as “Tool”

This concept of the computer in the school was originally focused on the ways that the teacher might use it for supporting the management of instruction, such as with data bases of student performance data resulting from CAI sessions. Other such applications by teachers might involve computer “gradebooks,” or computer-generated “worksheets” to be printed, or more general use of word processors for producing curriculum materials or other communiqués. Yet another stream of innovations involved teachers using the computer as an information storage and retrieval system for supporting and enriching the curriculum.

In mathematics education, teachers who could develop their own computer programs could create demonstrations of certain numerical concepts or procedures---here, the focus was on the mathematical output, not on the computer algorithm or student programming, per se. Later, with more sophisticated graphics available on microcomputers, such demonstrations could include graphical figures (but many of us will recall how challenging the writing of such programs could be). Not until we began to see the concept of the computer shift to that of a productivity tool in business (with the first spreadsheet system early in the Eighties) did we begin to see glimmers of a new view of school mathematics tools.

Computer Algebra

One early vision was for a software system that would perform algebra---a symbolic manipulator that functioned as a model of the operations of algebra. While serving as a visiting professor at Teachers College in 1973-74, I recall being visited by a programmer from a Manhattan computer center who demonstrated an amazing program he was developing---an algebraic “solver” that could progressively produce algebraic simplifications from your successive commands (compared to the previous numerical approximations to solutions of equations produced from numerical methods modeled by a computer algorithm). In his program, the user had to examine each result, decide what to try next, enter the appropriate command (which appeared to be an entirely new language to be learned), and wait (sometimes quite awhile) for the machine to produce and print a result (often in an unrecognizable format)! Of course, if you entered a directive that made the expression or equation ‘worse,’ it blindly complied!

Perhaps the earliest version of such a system that did gain considerable use was mu-Math (Wilf, 1982). We all may remember how challenging it was to encode correctly the mathematical operations one desired (a parallel to the early computer programming languages which demanded great precision in formatting and syntax?). Its successors (eg., Algebra Xpresser, Mathematica, DERIVE, Maple), along with the emergence of increasingly sophisticated versions built into powerful graphics calculators, have clearly established a user-friendly environment for performing algebraic (including calculus and geometry, as in Roanes-Lozano, 1996) operations. Of course, here we have a new construct for working---the “mathematical notebook.” Here, the vision is that the student will produce an integrated report of the mathematical solution process, along with a narrative discussion of her/his thinking which documents not only the “product” but the “process” of reasoning used by the student. Many of the important emphases in the NCTM Standards (1989), including problem solving and communication, can be promoted through students experiencing mathematics in such an environment.

Spreadsheets

Many mathematics educators at all levels have embraced the use of modern, general purpose spreadsheets as contexts for encouraging even young students to generate investigations and problem solving (eg., Abramovich & Nabors, 1997). In learning how to use a spreadsheet, a student sees how to encode an intuitive concept of variable by naming a cell (using a column letter and a row number). With the “fill down” command, we see how this encoding is repeated automatically to produce a set of such labels, all of which obey the same column letter but which vary across the rows. Such a set of expressions can be used to generate a set of numbers (eg., the counting numbers as “counters”---enter “1” in A1, enter “=A1+1” in A2, “fill down”) and then be seen to define a transformation of a these successively (eg., enter “=2*A1+1” in B1, “fill down”---to produce the odd numbers “3, 5, 7, ...”). Thus, through “hands on,” quite concrete actions the student can construct and investigate generative algebraic operations easily. Experience in such an environment would appear to promote significant forms of algebraic reasoning. Our mathematics education journals, curriculum resources, and mainstream textbooks now include increasingly serious and rich attention to infusing spreadsheet applications. To guide and support such usages, we need research on student learning in these contexts.

It is interesting to note, that while interest and attention to student programming usages have waned, within the spreadsheet context students are encountering some aspects of the notions of algorithmics. This occurs when students develop procedural actions that generalize across a class of examples. Further, within the available functions are many logical commands for writing instructions completely analogous to earlier computer programs. Such embedded overlaps among the “tutee, tutor, tool” categories blur any attempts at making these sharp distinctions.

Further, we are beginning to see the ‘next stage’ of designs and usages with the construct of the generic spreadsheet. Specialized spreadsheets (eg., DataScope, designed by Konold et al, 1997), designed as “microworlds” (ala’ Papert) for mathematics

students, provide many features especially tailored to the more specific mathematical domain under study. In such a system, the fundamental nature of a spreadsheet is retained while eliminating many of the most esoteric or advanced operations--focusing on those features which would assist in helping students work with fundamental concepts. Efforts to model the concepts or underlying mathematical operations in ways that promote conceptual understanding are emphasized.

Specialized Tools

This notion of adapting generic designs of computer tools to embody features more applicable or accessible to learning and teaching mathematics has produced some of our most significant pedagogical and curricular tools. The Geometric Supposer (Schwartz et al, 1993) operated on the Apple II microcomputers in the early Eighties, providing a rich environment for exploring and proving geometric theorems. In separate developments, both The Geometer's Sketchpad and Cabri Geometry capitalized on the more powerful Macintosh windows-oriented interface to provide a dynamic ("click and drag"), constructive geometric world. To many, these tools represent the best state-of-the-art context for featuring the richness of both metric and non-metric geometric conceptual development, including transformations, dilations, and certain algebraic graphical representations. Currently under development at Key Curriculum Press is the Fathom Dynamic Statistics toolware, promising to provide for data analysis and statistics what GSP has provided for geometry--but interfacing seamlessly to the World Wide Web for easy access to data bases worldwide.

"Microworlds"

In many ways, these specialized tools are exemplifications of the "microworlds" construct advocated by Papert (1980, 1993). Many mathematics education researchers have sought to investigate the design and use of such environments (Kaput & Thompson, 1994). One pervasive notion is the application of "multiple-linked representations"---building upon Dienes (1960) theoretical principle of multiple embodiments, where rich conceptual understanding are posited to result from seeing the ideas re-presented in differing, but related, ways. For example, in a computer microworld we might see the use of tables, equations, graphs, and figural models. Since these are programmed to be "hot linked," a change of values made in one representation automatically results in appropriate changes in all others. Again, while we have increasingly interesting and more powerful examples of such software, we still lack clear research evidence of the efficacy of these environments for mathematics learning.

The Internet & Web

We are witnessing today the explosive growth of the global use of the Internet and the World Wide Web. Among other interpretations, these are informational and communication technological computer-based tools. As we are swept along with the broader social and cultural impacts of this environment, it is not clear how such tools will change school mathematics. Some possibilities can be suggested.

- As a communications tool, the “walls” of the classroom and school are opened up to contact with the “outside” world. Teachers and students can connect with others, as in the Math Forum or to an expanding set of relevant homepages on the Web. Participation in interactive “chat rooms” can give rise to a new form of involvement by students and teachers.
- As an information storage and retrieval tool, Web resources can be used to support an enriched school mathematics curriculum which includes real, data-based applications. Students and teachers can search for information and data related to an ongoing classroom investigation. Especially with tools designed to support it (eg., Fathom Dynamic Statistics), the actual data (eg., from the U.S. census) could be ‘downloaded’ for analysis, interpretation, and use in the classroom.
- As an ‘electronic classroom,’ entire courses can be mounted and offered through a homepage on the Web. One of our faculty, Dr. James W. Wilson, has developed an exemplary model of Web-based courses (see <http://jwilson.coe.uga.edu>) which he teaches. Complete course syllabi, including all of the mathematical problems posed, are available to anyone---and his site has more than 10,000 “hits” per month! More significantly, the students post their solutions (“notebooks”) on his homepage---so, after several offerings of the course the participating teachers have developed a rich array of resource material to share. Further, numerous precollege students have found this material, engaged in solving the problems, and submitted reports of their work to be made available on the same Web site. More recently, he has helped the teacher participants to develop their own homepages where they and their students are working in similar patterns of productive activity---and these are linked to Wilson’s page (and, thus, to each other)! Thus, a growing “web” or network of collaborates, including professors, teachers, and students, is flourishing in the finest tradition of scholarship.
- As an ‘electronic seminar,’ participants (including teachers and their students) can engage in real-time interactions with other classrooms, with university scholars, with political leaders, with workers who can interact about their uses of mathematics, even with parents who wish to ‘visit and observe’ their child’s school mathematical activities. A couple of examples---in 1996 we conducted a doctoral dissertation defense which included a faculty member from her office in Indianapolis, connected through the Internet with distance conferencing software. In May 1997 we sponsored an Internet-based colloquium featuring Professor Alan Schoenfeld (visiting from University of California, Berkeley) as he presented a colloquium to our faculty and students which was simultaneously viewed worldwide by nine other sites via distance conferencing software---with several sites sending email questions and comments to our Athens classroom!
- As an ‘electronic laboratory,’ researchers can be interfaced to schools to conduct joint, even globally-based, school mathematics and science investigations and projects. Researchers, interested in collaborating regularly with classroom teachers in studies of learning and teaching mathematics, can use distance conferencing to plan treatments, observe classroom activity, ‘interview’ individuals. Teacher educators can serve as a ‘distance teacher’ or mentor, stimulate trials of innovative problematic situations or curriculum materials, engage teachers individually or in groups as part of

staff development activities, or help teachers break down their isolation by connecting with other teachers with similar interests or needs.

Synthesizing the First 30 Years: Themes and Issues

To be sure, it is complex and risky to attempt a simplification of something as broad and varied as three decades of activity with computers in mathematics education! But, having been an early and continuous participant in our search for how we might use this marvelous technological tool to improve school mathematics, I've repeatedly been engaged with many colleagues in trying to make sense of each period of activity and each new innovation as it occurred. Here, I'll try, one more time.

The following discussion is organized around several dimensions (or domains) for sorting the various phenomena---the events, developments, factors, and variables that are embodied in computers in school mathematics. For each dimension below, I've tried to address some of the themes and issues of this 30-year history.

Philosophies and purposes

The guiding perspectives for school mathematics teachers and students using computers have remained mostly intact and coherent across time, despite great changes in many of the supporting elements of hardware and software. For the most part, these reflect the more fundamental beliefs held about the nature of school mathematics and its learning and teaching.

That is, in the earliest applications of computers to mathematics education we find dramatic contrasts in the underlying epistemologies, philosophies, and goals. On one 'end' of a continuum are the views of those who see mathematics as a finished, closed system of concepts, procedures and skills to be modeled (by authority) and imitated (by novice)---CAI, tutorial systems). At the other 'end' are the views of those who see mathematics as an idiosyncratic construction resulting from problematic situations posed as explorations and investigations where the daily problem is to build-up meaning and understanding through personal actions and socially developed interpretation and reasoning---CAMP, Papert's Logo & microworlds, GSP and other tools. This contrast (essentially in the extent of student control, initiative, and generativity) can be found early and continuously in this history, despite the emergence of new hardware or forms of access.

I consider this contrast in the fundamental philosophical bases to be a major theme in school mathematics with the most critical issues involved. There is a pervasive tension in our field, exemplified by these very different concepts of our goals and purposes. To me, this contrast is anchored in very different notions of mathematics, per se. What is mathematics? What does it mean to know or to do mathematics? What should be the nature of school mathematical experiences? Why should students have

mathematical experiences---just what impacts and consequences do we intend? What should be the nature of one's mathematical knowledge? In particular, what is the balance of memorization and habituated skills and procedures versus the use of tools to retrieve and solve?

In the realm of computer-based technologies, we must confront these issues. This is the case, because these tools increasingly force us to question the need for requiring students to develop the kinds of mathematical competence that was needed in the past (when such tools were not available). Additionally, we must ponder the 'trajectory' of technological change.

Changes

The reality of technologically-driven change is omnipresent. Toffler's notion of 'future shock' is the crisis of coping with rapid change, and the fact that the rate of change is accelerating. Mathematics and mathematics education are especially impacted, as the technological changes bring to us increasingly powerful contexts for engaging in mathematical work.

Across these decades, we have witnessed amazing changes in the power and nature of the technology (hardware) and its applications (software). The development of the particular applications, the infusion of the power into curricula, the impact upon teaching practices, the shifts in the nature of learning activities---for the most part, none of these have kept pace with the changes in the technology. This entropy ('lag of the seasons') is due to many complex factors, but we know that central to it is the knowledge and will of the teacher. While there may be other systemic, even infrastructural, factors that mitigate against rapid adoption and implementation, the fundamental agent for change in the classroom is the teacher. Across these decades, it has been a constant struggle for teachers to learn about computer applications to their teaching.

Perhaps even more critical are the challenges of adjusting our pedagogical beliefs to a shifting concept of what our students might be doing as they experience school mathematics in ways that reflect new technologies. In the early Seventies, the first inexpensive handheld calculators caused many to ponder---what is its role, and what changes in our expectations for learning arithmetic are needed? With the recent powerful algebraic, geometric, and calculus tools, even collegiate mathematics has become a set of complex issues about learning and teaching traditional materials with traditional goals and expectations. The issues of change, and adaptation to change, remain fundamental concerns of mathematics educators.

Interacting with what is occurring within our profession are the broader impacts of technology on society. Despite home computers and Internet access, parents typically do not understand needed changes in our school mathematics goals and curricula, or in our teaching approaches, or in their children's learning. Parental expectations (that their child's mathematical experiences should be like those they recall from their own schooling) often serve to impede us from making needed changes---despite the fact that

parents will agree that the rest of their world must adapt to new technologies. The issues of helping parents to cope with changes in school mathematics have remained central across these decades.

As we continue to confront change, and all of the challenges of ‘keeping up,’ perhaps the most profound challenge is in the concern for a school mathematical education that prepares one for a future that we simply cannot predict or describe very well at all. What, exactly, should be the nature of a mathematical education today where computational and informational tools are likely to be even more powerful and ubiquitous in the future? What kinds of problems to be solved, and problem-solving abilities, would the students of today need in the next millennium?

Myths and realities

One pervading characteristic is the rhetoric of promise---what we could do with computers, if... From the earliest days, the “mythology of innovation” (eg., Oettinger, 1969) has characterized our thinking about possible innovations with computers in mathematics education. We speculated that through programming our students would be transformed into generative problem solvers, actively engaged in constructing, testing, and refining powerful algorithms. The reality was surely always much less. To be sure, those of us who experienced such usages with our students can testify to impressive examples, but many of the challenges for our students’ learning were still present in the programming situations.

The myths of individualization envisioned in CAI or computer-based tutorials were basic to the rationales for developing these expensive, unwieldy systems. The reality of how poorly real individualization was accomplished pervades the relative lack of success of such systems. The real fact is that we still know so very little about the complexities of a child constructing their own mathematical knowledge, that any feeble attempt to model an insightful, sensitive teacher is certain to fail

When we consider computer environments where we shift greater control and responsibility to the student, we may find greater positive impact. Here, again, we must be cautious about balancing the many complexities of the best teaching---given the most powerful tools imaginable, most students will not ‘go off and invent mathematics’ by themselves. The role of the teacher in stimulating and guiding productive mathematical activity is still central---and this has remained so across these decades of computer work.

Nature of computer uses and users

Strikingly, there appears to be considerable consistency in the nature of the uses and the users of computers in school mathematics education across these decades. While the qualities and details of the applications have changed quite dramatically, the essential trichotomy suggested by Taylor remains. Perhaps the shift toward de-emphasizing the “tutor” and “tutee” uses with an almost explosive emphasis on “tool” is the most

important change to recognize. And, as we explore the role of the Internet and Web, it is likely that this focus will become even more dominant.

Even so, this does not greatly simplify the issues related to users. These tools require us to confront issues related to the fundamental nature of the learning tasks, of the roles of the student and teacher, and of the underlying purposes and goals of a sound mathematical education. Again, given our resistance to change, our lack of clear understanding of mathematical learning, and our inability to predict the future, what can and should we do today to infuse significant uses of computers? How do we help teachers to adapt and adjust to become significant users of computers in school mathematics---given the general and pervasive resistance of many teachers to incorporating any type of modern technology into their teaching (eg., Jackson, 1970)?

Problems and Challenges Before Us: The Next 30 Years?

It is interesting to ask: what, of today's picture of computers in school mathematics, would I have predicted in 1965? I think I would have predicted

- an increasing role for student programming (with more powerful yet simpler to use languages),
- massive tutorial systems capable of adaptive decisions reflecting individual needs and characteristics,
- more pervasive use of practicing with sophisticated computer systems,
- more expansive, yet curricular-based, interactive information storage and retrieval systems designed for students, and
- mathematical 'notebook' software (either on large time-sharing systems or in 'notebook' handheld computers) which integrated programming, word processing, and information retrieval to allow a student to work on comprehensive project-oriented problem solving.

I would not have predicted spreadsheets, GSP, algebraic manipulators, or the Internet, per se. In the main, I think I would have been quite 'off the mark' in my predictions.

Rather than risk even a 'peek in the crystal ball,' I would propose several questions to stimulate possible thinking about our choices, and then offer two scenarios. In both, I'm assuming a backdrop of continued technological innovation and development in which computers will become even more powerful, miniaturized, inexpensive, user-friendly, enhanced, and omnipresent.

1. As I noted above, the most basic questions for me is---what should school mathematics be? This implies that we must consider the basic goals and purposes, the qualities of mathematical experience, and what we believe about mathematics, per se. How do we balance our traditional views of skills and algorithms with an ever greater need to understand and apply?
2. Given the ways that computers can support mathematical activity, what is the role of the mathematics teacher? How must teachers' beliefs and attitudes change? How do

we support teachers to make changes in their practices to reflect the new and changing goals and tools?

3. How must school, as a place, change to allow students and teachers to use computers in ways that transform and enhance mathematical education?
4. What broader issues of equity of access and competence must be addressed?
5. How must the role of the printed textbook change---and what are the roles of publishers, per se, in a future where electronic curricular resources may dominate? How do the economic traditions in education adjust and transform?
6. How do we fill the growing knowledge gap from research on learning and teaching mathematics, when the very nature of the learning and teaching process may shift in ways that make irrelevant the modicum of research we have?
7. How might the emerging home access via the Internet and Web completely change (and even eliminate) the need for school to be a community center?

Depending on how we address these and other questions, I can imagine at least two possible scenarios over the next three decades.

In one, the perceived failures of the schools becoming increasingly a crisis, leading more and more parents to flee to a 'new' alternative: privatized commercial (storefront) 'schools' operated by enterprising teams of 'master' teachers who 'hang their shingle' and capitalize on enriched computer-enhanced environments. With dedicated hard work, these teachers and students can demonstrate amazing results---solid learning (balancing reasonable skills with powerful exhibitions of student productions using enriched computer tools) and positive student attitudes and feelings! More than likely, daily operations would not find all students coming to the 'place' but rather more connectivity from home serving as the primary mode of interactions and work. Coming to the storefront would be for those occasions when a group needs to meet in person, or when face-to-face assessments or mentoring is needed. The mathematics 'curriculum' of the school will surely be more linked to the interests of the individual student, interfaced to other areas of life (eg., other curricular areas, leisure, hobbies, part-time employment, etc). Such 'schools' might grow in number, but not much in size, for 'too big to be successful' is quickly realized. As free enterprise entities, these operations will be subjected to all of the factors and effects of the marketplace---poor performers close down, high performers might become 'bought up' by larger corporations, competition, advertising, rising costs, capital improvement needs, etc. As 'pay as you go' businesses, they may be the havens of the wealthy (unless politicians do enact some system of vouchers to allow use of public funds).

In this scenario what is happening back in the community 'schoolhouse?' Is it likely that the unmotivated, disadvantaged, failing, less capable students will continue to go there (because they must, or wish to, or have no real choices)? If so, can we imagine

the kind of school mathematics curriculum that will be advocated for them? And, given an increasing pressure to be accountable with public monies, can we foresee the extent and type of testing that will dominate the choices of teachers and students? Can we easily see the final, convulsive demise of the place called ‘public school?’

An alternative scenario places the public school at the center of an enriched, more enhanced, broadly-based institution for educating. Many of the desirable qualities of the commercial scenario above can be envisioned as developing in the public school system, propelling its operations on a wave of impacts from computer-enhanced environments. With home connectivity, the school could also become less a ‘prison’ or ‘factory’ model, and more a cultural centerpiece in the neighborhood where students and teachers assemble as needed. With school connectivity to universities, museums, archives, libraries, workplaces---nearby, nationally, even globally---we could imagine students becoming much more motivated to engage in learning activities. Yet, interesting problematic (relevant) situations must still be posed as challenges, and teachers must mentor and advise in ways that stimulate and support students in their activities. Without the essential relationships that have always characterized the teacher and student bond, it is not easy to imagine students who will somehow become productive learners!

We have many choices. With continued computer innovations, we will have new options for engaging our students. Will we have the will? Will we have the vision? Will we allow the changes? Will we be a part of the solution to the problem of an improved school mathematics, or will we be a part of the problem? Can we find the wisdom?

“The road to wisdom?-----Well, it’s plain and simple to express:

Err
and err
and err again
but less
and less
and less.”

Piet Hein.

Notes

Note 1. Prepared for presentation at the 1998 Spring Conference of the Minnesota Council of Teachers of Mathematics.

Note 2. Since I am basing my presentation on the past, I wish to clarify several points related to my approach. First, I am not a historian, and I do not claim that this discussion has resulted from historical research, whose methodology requires an exhaustive search involving (where feasible) primary documents. Rather, this is a “personal history” in which idiosyncratic perspectives, interpretations, and biases are freely represented! I do believe that we badly need careful historical research in the field of mathematics education, as we seem to ignore the lessons of the past when we “make apparent discoveries” and “commit similar errors” while we again “re-invent the wheel.”

Note 3. Current known information about individuals:

Dr. Donovan A. Johnson, Professor Emeritus, University of Minnesota.

Dr. David C. Johnson, Shell Professor of Mathematics Education, Kings College,
University of London

Dr. Dale E. LaFrenz (past President of MECC), President of QTech Systems, Inc.,
Minneapolis

Dr. Thomas E. Kieren, Professor Emeritus, University of Alberta

Dr. Pamela W. Katzman, University of Wisconsin-River Falls

Note 4. Summarized from “Philosophy of CAMP,” in
Hatfield, L. & Johnson, D. (1969). CAMP First Course, Teacher’s commentary.
Glenview IL: Scott, Foresman & Company, pp. 1-3.

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