

**MAT 195 – Spring Quarter 2002**

**Final**

NAME \_\_\_\_\_

**Show work and write clearly.**

1. State the definition of the derivative as a function.
2. Use the definition of the derivative to find the derivative of  $f(x) = \frac{1}{2x + 1}$ .
3. Algebraically evaluate the limit, (if it exists):  $\lim_{x \rightarrow 3} \frac{x^2 - 4x + 4}{x^2 + x - 6}$ . If the limit does not exist, explain why.
4. Algebraically evaluate the limit:  $\lim_{x \rightarrow +\infty} \frac{4 - 3x}{\sqrt{2x^2 - 5}}$ . If the limit does not exist, explain why.
5. Find the derivative:  $y = (\tan^2(x) - x^2)^3$ .
6. Find the derivative:  $\frac{3}{x} - \frac{3}{y} = \sqrt{xy}$ .
7. Find the derivative:  $f(x) = x^3 \sin x \cos x$ .
8. Find the derivative:  $f(x) = \frac{3x^3 + 27}{x + x^2}$ .
9. Find the second derivative:  $f(x) = 3 \sin^2(2x) - 4 \cos^2(2x)$ .
10. Find the equation of the tangent line to  $f(x) = x^{1/3}$  at  $x = 8$ .
11. For what values of  $x$  does the function  $f(x) = 4x^3 - 7x^2 + 2x - 1$  have a horizontal tangent line?
12. A container with square base, vertical sides, and open top is to be made from 1000 ft<sup>2</sup> of material. Find the dimensions of the container with greatest volume.
13. On what interval(s) is  $f(x) = \sqrt[3]{(x - 2)^2}$  continuous? On what interval(s) is  $f(x)$  differentiable? Explain.
14. Find an antiderivative of  $f'(t) = \frac{5t^2 + 7}{t^{4/3}}$ .
15. Calculate the left-hand, right-hand, midpoint and trapezoid sums with 250 subdivisions. Which of these sums are overestimates and which are underestimates? Explain. Estimate the value of the definite integral. Explain.  $\int_{-2}^{-0.5} \left( \sqrt[3]{x} + \frac{1}{\sqrt[3]{x^2}} \right) dx$
16. Find all points of discontinuity for the function  $f(x) = \frac{x - 2}{|x| - 2}$ . Determine whether the discontinuity is removable or not.
17. If  $f(x) = 2x + \ln(x)$ , find  $f^{-1}(2)$ .
18. Express the function  $r(x) = \frac{1}{\sqrt{3 - \sin^2 x}}$  as a composition of four functions.
19. Find the domain and range of  $f(x) = \sqrt{4 - 3x^2}$ .
20. Find all vertical and horizontal asymptotes of  $f(x) = \frac{x^2}{x^2 - 2x - 3}$ .