NAME_____

Show work and write clearly.

1. Let $h(x) = \sqrt{5 - \sqrt{x}}$. Find $h^{-1}(x)$. State the domain and range for $h(x)$ and $h^{-1}(x)$.
ANS: Using a graphing calculator and the trace function, the domain and range can be determined. The domain is [0, 25] and the range is [0, √5]. Therefore, the domain of the inverse is [0, √5] and the range of the inverse is [0, 25]. The inverse of the function is found with the following steps:

$$y = \sqrt{5 - \sqrt{x}} \Rightarrow x = \sqrt{5 - \sqrt{y}} \Rightarrow x^2 = 5 - \sqrt{y} \Rightarrow 5 - x^2 = \sqrt{y} \Rightarrow \left(5 - x^2\right)^2 = y = h^{-1}(x)$$

2. a. Assume $f(x)$ is even, complete the table below:

x	−3	−2	−1	0	1	2	3
$f(x)$							

ANS: Many answers are possible.
b. Assume $f(x)$ is odd, complete the table below:

x	−3	−2	−1	0	1	2	3
$f(x)$							

ANS: Many answers are possible.
c. Can a function be both odd and even? If so, then assume $f(x)$ is both even and odd and complete the table below. If not, then explain.

x	−3	−2	−1	0	1	2	3
$f(x)$							

ANS: $f(x) = 0$ is both an even and odd function.

3. Find functions f and g such that $h = f \circ g$.
a. $h(x) = 3(\sin x)^2 + 4\sin x$
ANS: $f(x) = 3x^2 + 4x$ and $g(x) = \sin x$.

b. $h(x) = \dfrac{\tan x}{3 + \tan x}$

**ANS:** $f(x) = \dfrac{x}{3 + x}$ and $g(x) = \tan x$.

4. Generally, the more fertilizer that is used, the better the yield of crop. However, if too much fertilizer is applied, the crops become poisoned, and the yield goes down rapidly. Sketch a possible graph showing the yield of the crop as a function of the amount of fertilizer.
ANS: Many answers are possible. At the maximum of the graph, the graph must have a decreasing (with large negative slope) section.

5. a. Find constants A, B, C and k such that the function $f(x) = A \cdot B^{kx} + C$ satisfies all four of the following conditions:

- $f(x)$ is an increasing function,
- $f(x) < 0$ for $x < 0$,
- $f(x) > 0$ for $x > 0$, and
- $f(x) < 2$ for all x.

ANS: Many answers are possible. First, it is important to recognize that $f(x)$ is an exponential function. I choose B = 2 (there are other possibilities). The graph of 2^x looks like:
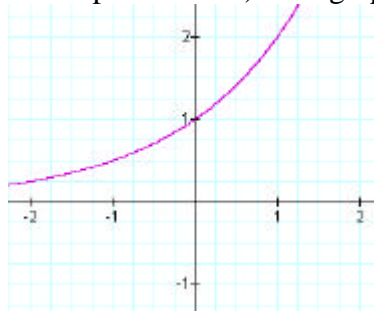
This is an increasing function whose values increase past 2 (because it is concave up). The only other type of increasing function that works here will be one that is concave down. So, let's try to flip this graph over the x-axis. This new function is -2^x, which looks like:
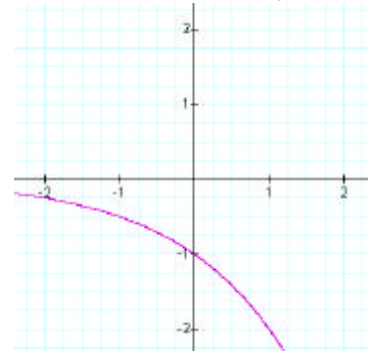
However, this is a decreasing function. So, let's flip it over the y-axis. This new function is -2^{-x}, which looks like:
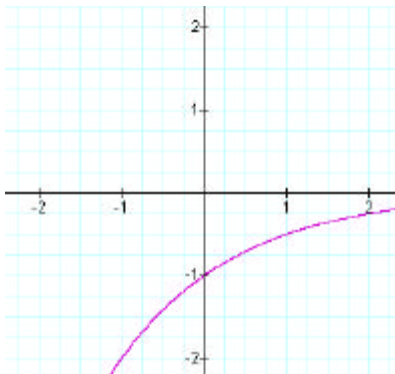
The function does not cross the x-axis. Now we have the first condition met. The second condition states that when $x < 0$, the function is below the x-axis. This condition is met. The third condition states that when $x < 0$, the function is below the x-axis. Our function does not meet this condition. So, we need to shift this function. The easier shift is a vertical shift. So, let's vertically

shift this function by 2 (the function still meets the last condition). Our function is now $-2^{-x} + 2$, which looks like:
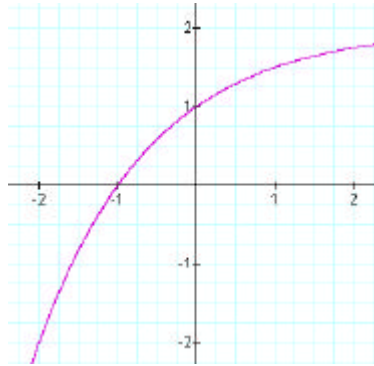
Now the second condition is not met. So, shift the function 1 unit to the right. $f(x) = -2^{-(x-1)} + 2$:
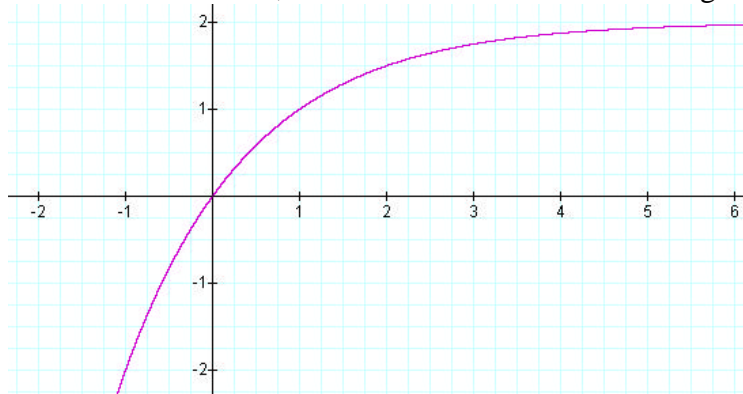
Now, all the conditions are met.

b. Write the equation of the function that is obtained by shifting $f(x)$ two units to the left.
ANS: $f(x) = -2^{-(x+1)} + 2$:

6. Find the domain and range of $f(x) = \dfrac{5}{3 - \cos 2x}$.

ANS: Use a graphing calculator and the zoom and trace functions. D $= (-\infty, \infty)$ and R $= [1.25, 2.5]$.

7. Solve the following algebraically:
a. $\ln(3x + 8) = \ln(2x + 2) + \ln(x - 2)$
ANS: $\ln(3x + 8) = \ln(2x + 2)(x - 2)$ by the product property of logs.

$\quad$ $3x + 8 = (2x + 2)(x - 2) = 2x^2 - 2x - 4$ by composing both sides with e^x.

$\quad$ This implies $2x^2 - 5x - 12 = 0$. There are various methods to find the solutions to this quadratic, which are $x = -3/2$ and 4. However, $x = -3/2$ is an extraneous solution. Try plugging this in for x in the original equation and note what happens.

b. $2e^{3x} = 4e^{5x}$

ANS: $2e^{3x} = 4e^{5x} \Rightarrow \ln 2e^{3x} = \ln 4e^{5x} \Rightarrow \ln 2 + \ln e^{3x} = \ln 4 + \ln e^{5x}$ by the product property of logs. This simplifies to $\ln 2 + 3x = \ln 4 + 5x$. Rearranging, we get $\ln 2 - \ln 4 = 2x$.

Simplifying, we get $\dfrac{-\ln 2}{2} = x$.

8. Find the exact value of each expression:

a. $\log_{1.5} \dfrac{27}{8}$

ANS: $\log_{1.5}\left(\dfrac{3}{2}\right)^3 = 3$.

b. $\log_{0.3} \dfrac{100}{9}$

ANS: $\log_{0.3}(0.3)^{-2} = -2$.

9. Let $f(x) = \dfrac{e^x + e^{-x}}{2}$ and $g(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$. What are the domains of $f + g, fg, f/g$?

ANS: Use a graphing calculator to find the domain of $f(x)$ and $g(x)$. $D_f = (-\infty, \infty)$ and $D_g = [1, \infty)$. Then, the domains of $f + g, fg$ are $(-\infty, \infty) \cap [1, \infty) = [1, \infty)$. The domain of f/g is found the same way, but $g(x)$ cannot be 0. Use a calculator to find when $g(x) = 0$ and we find that $g(1) = 0$. So, the domain of f/g is: $(-\infty, \infty) \cap (1, \infty) = (1, \infty)$.

10. The graph below shows the temperature of a room during a summer day as a function of time, starting at midnight.
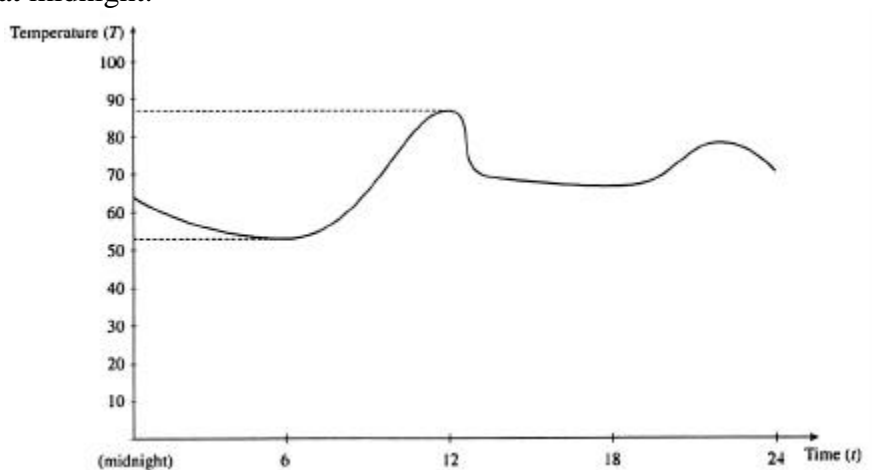
a. Evaluate f(noon) and f(6 p.m.). State the range of f.
ANS: f(noon) $= 88$ and f(6 p.m.) $= 65$. $R_f = (53, 88)$. Other answers are possible.

b. Where is f increasing? Decreasing?
ANS: f increases on $(6, 12) \cup (19, 22)$ and f decreases on $(0, 6) \cup (12, 19) \cup (22, 24)$.
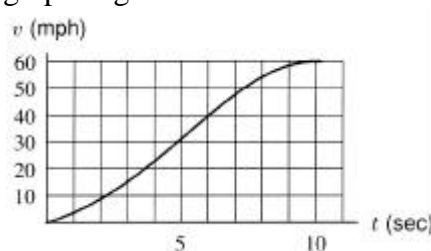
c. Give a possible explanation for what happened at noon.
ANS: There are several possible explanations.

d. Give a possible explanation why *f* attains its minimum value at 6 a.m.
ANS: There are several possible explanations.

11. Let *f* be the function whose graph is given below.

a. Estimate the value of *f*(4).
ANS: *f*(4) = 22. Other answers are possible.

b. Estimate the value(s) of *x* such that *f*(*x*) = 40.
ANS: *f*(*x*) = 40 when *x* = 6, i.e., *f*(6) = 40. Other answers are possible.

c. On what interval is *f* increasing? Decreasing?
ANS: *f* increases on [0, 10]. The function does not decrease. Other answers are possible.

d. Is *f* one-to-one? Explain.
ANS: Yes, since the function passes the horizontal line test.

e. What is the domain and range of f^{-1}?
ANS: From the graph, we can find the domain and range of the function. We switch these to obtain the domain and range of the inverse. So, D = [0, 60] and R = [0, 10].

f. Estimate the value(s) of $f^{-1}(8)$.
ANS: $f^{-1}(8) \approx 1.7$.