Math 1431
Summer 2003 - Test \#2 - Answers
Name
You are allowed to use your calculator. Explain all answers- answers with no explanation will receive only partial credit. Use complete sentences. Show how you used the calculator to answer the questions below.

1. (16 points) Let X be the product of the random roll of two fair four-sided dice.
a. Construct the sample space.

ANS:

| $1 \bullet 1=1$ | $2 \bullet 1=2$ | $3 \bullet 1=3$ | $4 \bullet 1=4$ |
| :--- | :--- | :--- | :--- |
| $1 \bullet 2=2$ | $2 \bullet 2=4$ | $3 \bullet 2=6$ | $4 \bullet 2=8$ |
| $1 \bullet 3=3$ | $2 \bullet 3=6$ | $3 \bullet 3=9$ | $4 \cdot 3=12$ |
| $1 \bullet 4=4$ | $2 \bullet 4=8$ | $3 \bullet 4=12$ | $4 \bullet 4=16$ |

$n=16$.
b. $\mathrm{P}(\mathrm{X}$ is odd $)$. ANS: $\mathrm{P}(\mathrm{X}$ is odd $)=4 / 16=1 / 4$.
c. $P(X>6)$. ANS: $P(X>6)=6 / 16=3 / 8$.
d. $P(X \geq 15)$. ANS: $P(X \geq 15)=1 / 16$.
e. $\mathrm{P}(\mathrm{X}=3$ or $\mathrm{X}=$ even $)$. ANS: $\mathrm{P}(\mathrm{X}=3)+\mathrm{P}(\mathrm{X}$ is even $)=2 / 16+12 / 16=14 / 16=7 / 8$.
2. (15 points)
a. What is a sampling distribution?

ANS: The sampling distribution of a statistic is the distribution of values taken by the statistic in all possible samples of size $n$ from the population. In other words, it is the distribution of sample means.
b. How do you construct a sampling distribution?

ANS: Find the mean of a large number of samples of size $n$. Construct a histogram of these means.
c. What is the mean and standard deviation of the sampling distribution?

ANS: The mean of a sampling distribution is close to the population mean, $\mu$ and the standard deviation of a sampling distribution is $\sigma / \sqrt{ } n$ (where $\sigma$ is the population standard deviation).
3. (10 points) Here is the population of scores of 10 students on an exam:

| Student | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Score | 73 | 81 | 70 | 75 | 86 | 78 | 91 | 82 | 83 | 52 |

a. Find a SRS of size 4 . Show work.

ANS: On calculator, use RandInt(1, 10, 4). The results vary. I obtained 5, 8, 1 and 4.
b. What is the mean score of the sample?

ANS: The respective scores for these students are $86,82,73$ and 75 . The average of these scores is 79 .
4. (16 points) The distribution of actual weights of 8 -oz. chocolate bars produced by a certain machine is normal with mean 8.1 ounces and standard deviation 0.1 ounces.
a. If a sample of 5 bars is randomly selected, what is the probability that their average weight is less than 8 ounces? Show work.
ANS: Use NDAREA on calculator with mean 8.1 and standard deviation $0.1 / \sqrt{ } 5$ (since this is a sample with $n=5$ ). The probability that the average weight of the sample is less than 8 ounces is 0.01267 .
b. Is average weight a parameter or a statistic? Explain.

ANS: The average weight of the sample is a statistic whereas the average weight of 8.1 ounces is a parameter (since it measures the population).
c. What is the probability that the weight of a single candy bar is greater than 8 ounces? Explain.

ANS: Use NDAREA on calculator with mean 8.1 and standard deviation 0.1 (since this is not a sample). The probability that the average weight of the sample is greater than 8 ounces is 0.8413 .
d. Is the standard deviation of 0.1 ounces a parameter or a statistic? Explain.

ANS: The standard deviation of 0.1 ounces is a parameter (since it measures the population).
5. (8 points) A student is chosen at random from a certain mathematics course. Here are the probabilities for the student's grade:

| Grade | A | B | C | D | F |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.15 | 0.35 | 0.25 | 0.05 | $?$ |

a. What is the probability of receiving a grade of F in the course? Explain.

ANS: The probabilities of the whole sample space must add up to 1 . Thus, $\mathrm{P}(\mathrm{F})=0.2$.
b. What is the probability of receiving a grade of A or B in the course? Explain.

ANS: The events A and B are disjoint. So, $\mathrm{P}(\mathrm{A}$ or B$)=\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})=0.15+0.35=0.5$.
6. (10 points) A group of college students believe that herbal tea has remarkable restorative powers. To test this theory, they make weekly visits to a local nursing home, visiting with residents, talking to them, and serving them herbal tea. After several months, many of the residents are more cheerful and healthy.
a. Sketch the design.

ANS:

b. What are the experimental units?

ANS: The experimental units are the residents.
c. What are the response variables?

ANS: The response variables are the cheerfulness and healthiness of the residents.
d. What are the factors and their levels?

ANS: There is only one factor, the tea. It has a single level since the study only considers tea or no tea.
e. Identify a possible lurking variable in this study.

ANS: Here's one possible lurking variable: The residents may feel better from having the company of the college students.
7. ( 5 points) In a recent study, a random sample of children in grades two through five showed a significant negative relationship between the amount of homework assigned and student attitudes. Is this an experiment or observational study? Explain.

ANS: This is an observational study since the researchers do not attempt to influence the responses.
8. (5 points) A news release for a diet product company reports: " There's good news for the 65 million Americans currently on a diet." Its study showed that people who lose weight can keep it off. The sample was 20 graduates of the company's program who endorsed the product in commercials. The results of this study are most likely unbiased or biased? Explain.

ANS: This is most likely biased. The subjects are graduates of the company's diet program - if they graduated, they probably lost weight. Thus, they are probably biased towards the product.
9. (5 points) A factory produces plate glass with a mean thickness of 4 mm and a standard deviation of 1.1 mm . A simple random sample of 100 sheets of glass is to be measured and the sample mean thickness of the 100 sheets is to be computed. What would we expect the mean and standard deviation of the sample to be? Explain.

ANS: A sample of size 100 is being considered. According to the sampling distribution, the mean of the sample is close to the mean of the population (i.e., 4 mm ) and the standard deviation is $1.1 / \sqrt{ } 100=1.1 / 10=0.11$.
10. ( 10 points) Consider rolling a fair, 6 -sided die 25 times. Let $x$ denote the sample mean of this sample. The standard deviation of the population is 1.708 .
a. What is the approximate sampling distribution of $x$, and state the name of the theorem which is used to draw this conclusion.
ANS: The sampling distribution is approximately normal by the central limit theorem.
b. What is the probability that for a single roll of the die, the out-come will be le ss than 2.5 ?

ANS: This is the roll of a single dice. Thus, $\mathrm{P}\left(\begin{array}{l}\mathrm{X}<2.5\end{array}\right)=\mathrm{P}\left(\begin{array}{ll}\mathrm{X} & 2\end{array}\right)=2 / 6=1 / 3$.

