The University of Georgia

MATH 7200, Fall 2008

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Midterm Exam

1. Consider a cyclic quadrilateral inscribed in a circle O. Assume that pairs of **opposite** sides are not parallel.

Construct the angle bisectors of opposite sides, either by using your construction from Problem 1.2.22 or by extending the lines of sides outside the circle.

Prove that the bisectors of the angles between **opposite** sides are perpendicular.

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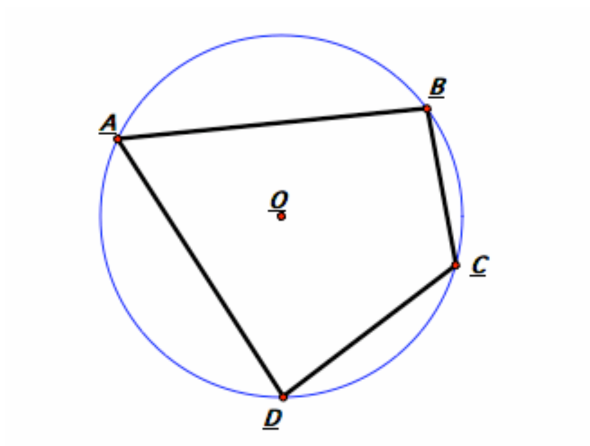

2. Given an acute triangle ABC inscribed in a circle with Orthocententer at H. BE and AD are altitudes of ABC. Extend the altitude AD to the intersection with the circumcircle at K.

a. Prove that BDEA is a cyclic quadrilateral.

b. In class we used used the cyclic quadrilateral HDCE to establish conditions so we could show two triangles were congruent in order to conclude that HD = HK

Here, use cyclic quadrilateral BDEA from part a to establish the conditions to find congruent triangles to use to prove HD = DK.

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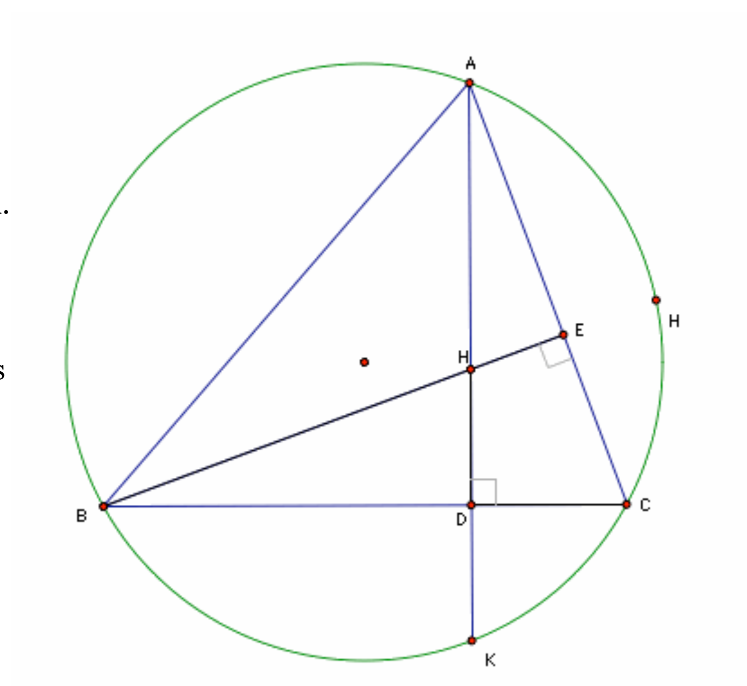


3. Midsegment Theorems

a. State and prove the midsegment theorem for triangles (with appropriate illustrations).

b. State and prove the converse of the midsegment theorem for triangles.

c. State and prove the midsegment theorem for trapezoids.

d. State and prove the converse of the midsegment theorem for trapezoids.

4. A couple of triangle things. We do not have similarity theorems available to us yet. It this problem, you will have a triangle ABC and are asked to prove that the triangles described in part a and part b have angles congruent to the angles of ABC and sides congruent to one-half the length of the corresponding sides of ABC. Appropriate illustrations will be helpful.

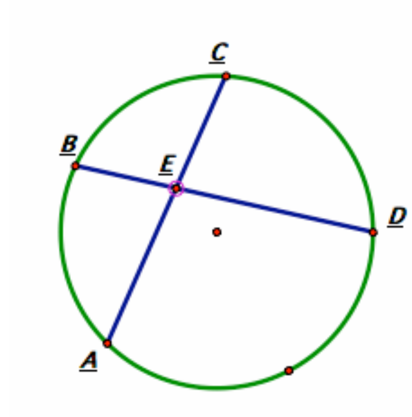
a. Take any triangle ABC and construct its orthocenter H. Construct HA, HB, and HC and find the midpoints R, S, and T respectively, of these three segments to construct triangle RST. Show RST has angles congruent to the angles of ABC and sides one-half the length.

b. Take any triangle ABC and construct the midpoints of its sides, D, E, and F. Prove triangle DEF has angles congruent to the angles of ABC and sides one-half the length.

c. Take any triangle ABC and any point P on the interior of the triangle. Construct segment PA, PB, and PC. Construct the midpoints X, Y, and Z respectively of these segments and construct the triangle XYZ. Compare this triangle to the triangle RST in part a (assuming RST and XYZ are constructed in the same triangle ABC).

5.

a. Show the angle formed by the intersection of two secants on the interior of the circle in terms of the intercepted arcs.



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b. Show the angle formed by the intersection of two secants on the exterior of the circle in terms of the intercepted arcs.

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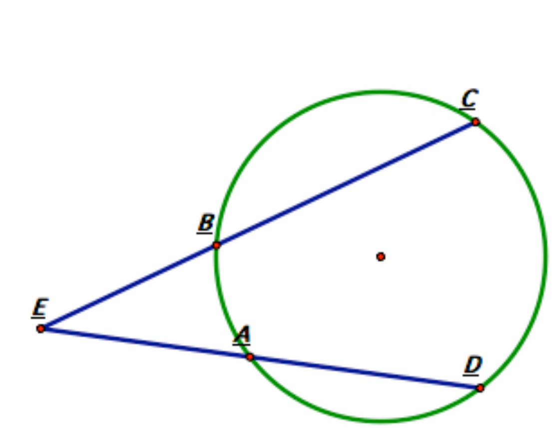

6. In the book and in class we have used the theorem that measure of an angle formed by a tangent and chord is equal to on-half the measure of its intercepted arc to prove the inscribed angle theorem. Part of the problem is to construct appropriate drawings/illustrations

Take the inscribed angle theorem as given and prove the measure of an angle formed by a tangent and chord is equal to on-half the measure of its intercepted arc.

7. In triangle ABC, CN is a median and M is the **midpoint** of that median. We do not have similarity tools available to us. Prove that point P on BC is a trisection point. That is, prove that

$$2PC = BP$$

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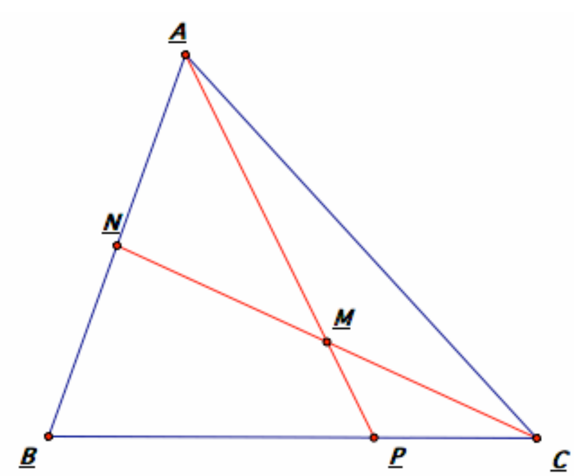

8. Three adjacent squares for a rectangle. Let x, y, and z be the measures of the angles as indicated.

Prove x + y + z = 90 degrees

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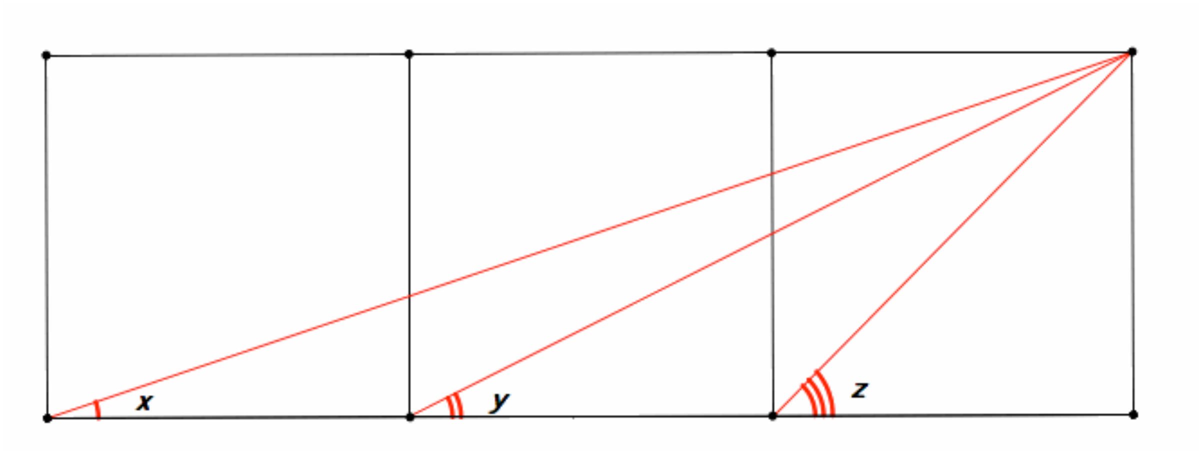


9. Construct a triangle given two of its sides and a median to the third side.

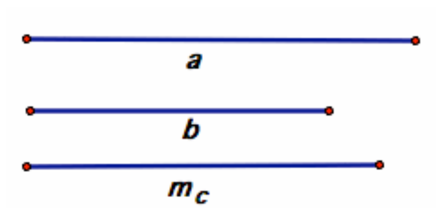
Present your solution in the format of

Investigation

Construction

Proof

that has been used and discussed in the text.



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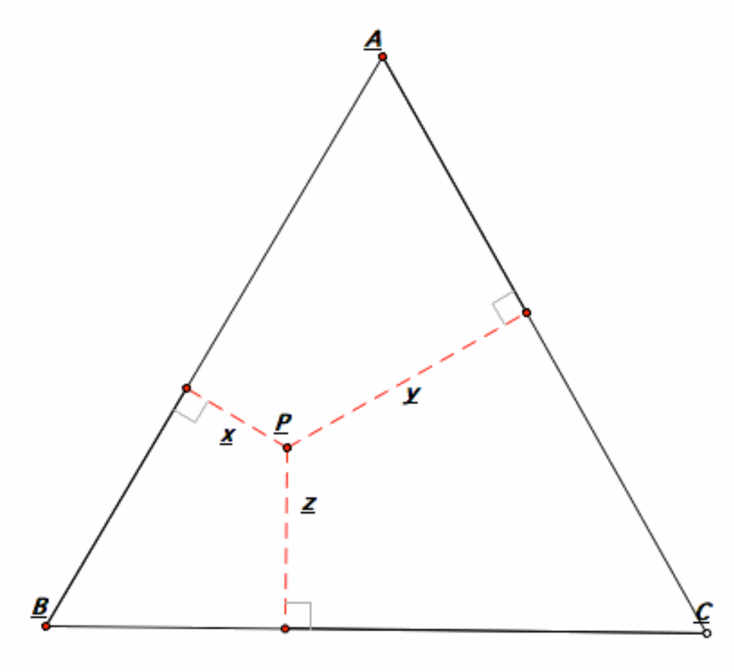
10. Given an **equilateral** triangle ABC. Let P be an arbitrary point on the interior of triangle ABC and x, y, and z be the perpendicular distances to the sides of the triangle. Since triangle ABC is equilateral, the altitude h is the same from each vertex.



Using the tools available to us from the first two chapters, give **two** proofs that

$$x + y + z = h$$

You might wish to label the three points on the sides as D, E, and F and phrase the problem in terms of segments PD, PE, and PF. If the three segments were laid end to end along a line, the total length would be a segment congruent to the altitude.;

[A GSP file is available]