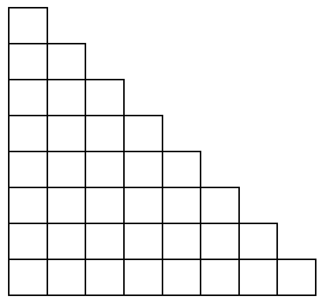
Section 3.1 Problem 19, Jackie Ruff

We are all familiar with Gauss' method for adding the first *n* consecutive integers. We write the numbers in reverse under the original sum,

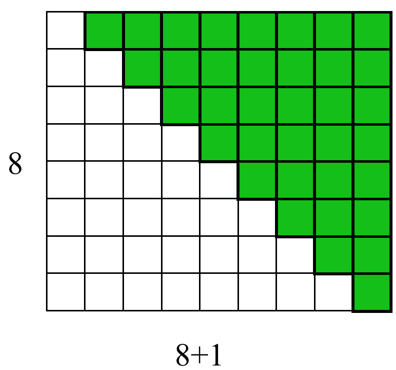
$$\begin{array}{l} 1 + 2 + 3 + \ldots + (n-1) + n \\ n + (n-1) + (n-2) + \ldots + 2 + 1 \end{array}$$

and notice that the sums are all the same, (n+1), and that there are n sums. So, we multiply n (n+1). However, we have now added the numbers twice, so we must divide by 2, giving us the formula $S(n) = \dfrac{n(n+1)}{2}$.

A geometric representation of the sum of the first 8 integers is given at the right. The process of "writing them in reverse" is done geometrically below by rotating the shape 180 degrees.



$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$$



Now, each row represents a sum, 1 + 8, 2 + 7 and so on. So, Gauss' method is still in place. However, the geometric representation has the added quality of creating a rectangle that is 8 by (8+1). But, again the area of the rectangle is twice the sum of the original n integers, so we must again divide by 2. So, this is an area model of the sum of the first n consecutive integers,

$S(n) = \dfrac{n(n+1)}{2}$.