LEARNING IN LESSON STUDY: A PROFESSIONAL DEVELOPMENT MODEL FOR MIDDLE SCHOOL MATHEMATICS TEACHERS

by

SHERRY LOVE HIX

(Under the Direction of James W. Wilson)

ABSTRACT

The purpose of this study was to examine what teachers may learn while participating in lesson study and if certain lesson study experiences can be linked to teacher learning. Two teams of middle school mathematics teachers participated in lesson study and a qualitative research methodology was used to address research questions regarding what teachers learned, what experiences in lesson study may be linked to that learning, and what may be included in the roles of the knowledgeable others and the lesson study facilitator. The lesson study teams, one sixth-grade team and one seventh-grade team, met approximately every two weeks for seven months and completed two cycles of lesson study. They were interviewed before lesson study began, after the first cycle, after the second cycle, and after the process was completed. The teachers showed three threads of growth in what and how they think about mathematics teaching. The teachers were using a reform curriculum, and during lesson study, they developed a clearer view of their role as teacher in a classroom using a reform curriculum. They also showed growth in mathematical knowledge for teaching in the areas defined as common content knowledge, specialized content knowledge, and pedagogical content knowledge. Finally, they claimed to better focus lessons after the experience of lesson study. The features of lesson study that were linked to this learning in this study were the detailed, collaborative planning, anticipating student responses, creating evaluation questions for the public teaching, observing the public teaching, and discussing public teaching with knowledgeable others.

INDEX WORDS: Lesson study, Professional development, Teacher knowledge, Mathematical knowledge for teaching, Teacher learning

LEARNING IN LESSON STUDY: A PROFESSIONAL DEVELOPMENT MODEL FOR MIDDLE SCHOOL MATHEMATICS TEACHERS

by

SHERRY LOVE HIX

B.S., Kennesaw State University, 1991

M.Ed., University of Georgia, 1997

A Dissertation Submitted to the Graduate Faculty of The University of Georgia in Partial Fulfillment of the Requirements for the Degree

DOCTOR OF PHILOSOPHY

ATHENS, GEORGIA

2008

LEARNING IN LESSON STUDY: A PROFESSIONAL DEVELOPMENT MODEL

FOR MIDDLE SCHOOL MATHEMATICS TEACHERS

by

SHERRY LOVE HIX

Major Professor: James W. Wilson

Committee: Denise S. Mewborn
Jeremy Kilpatrick
Sybilla Beckmann

Electronic Version Approved:

Maureen Grasso
Dean of the Graduate School
The University of Georgia
August 2008

DEDICATION

This work is dedicated

to Almighty God,

who in His almightiness, parted the waters so that my path was clear and dry;

to Steven,

the man God created me for,

who has held my hand and walked every step of that path with me,

protecting and loving me as he was created to do;

to Savannah and Ethan,

who have fearlessly believed in me since they were born,

and who unfailingly encourage me with their laughter, their intellect, and their love.

ACKNOWLEDGEMENTS

Arriving at this place, in what my children call "dissertation land," is an incredibly humbling experience. Nothing that I have done here could have been accomplished by me alone. The support I have received in so many different ways is entirely too overwhelming to consider for extended periods of time.

I could not possibly acknowledge by name everyone who has touched me during this time. There are two groups of people who made me want to be a better teacher. I have to begin with my own students, those who looked me in the eye and those who did not. I also owe a tremendous amount of gratitude to my undergraduate and graduate professors who worked with me and believed in me. Among those, I will go so far as to name Chris Schaufele, Nancy Zumoff, Pat Wilson, and Les Steffe. I want to thank my committee for the time they spent helping me develop as a researcher and teacher and colleague. I also want to thank my friends and family who have been with me through this.

Jim Wilson was the first person I met at the University of Georgia in 1990. I do not know if he could hear my knees knocking while I sat in front of him, he never let on if he did. He was my ally from that first day, and there have been many times I wanted to hide behind him so he could protect me in this world of academia, especially when I did not believe I belonged. He always made me think he believed I belonged nowhere else.

Denise Mewborn has been a rock and a voice of reason for me. This by no means makes me special, she is those things in her very nature. She has an amazing gift of seeing and reading between the lines and being able to summarize with wonderful insight for those around her, as necessary. She has been patient with my questions, she has been available whenever I need her, and she believed in me throughout the process.

Sybilla Beckmann is someone I first met through my work with PRISM where I had the opportunity to watch her work with in-service teachers. Her passion for students learning mathematics and her respect for teachers who usher that learning are most reassuring and refreshing characteristics.

Jeremy Kilpatrick has such an amazing, complete, and comprehensive understanding of mathematics education, research, and most anything that contributes to those that I always found myself uncharacteristically terrified when I was around him. He gave me invaluable advice, asked me hard questions, and guided me to think more clearly about many aspects of my work.

Dava Coleman hired me to work for her on the PRISM grant. She trusted me and pushed my thinking from the beginning. She provided thoughtful comments to me, sometimes on a daily basis, about the work I was doing. She is as much responsible for my completing this degree as anyone.

Nancy Vandergrift and Sandy Blount were my co-workers at PRISM. They provided a constant support structure for me that I cannot even begin to describe, not the least of which were their very soft knocks at my door and their very strong prayers.

Tad Watanabe freely gave me his time. In fact, he gave me so much more time than I should have asked. His insight about my work was irreplaceable; I have learned so very much from him.

The teachers who studied with me in this process provided a means for me to grow professionally and personally. I do not think they will ever know how much they gave to me and how much I learned from them during our time together.

I also could not have finished this work without the prayers, support, and encouragement from Danie, Ginger, Kelly, Robyn, Chrissy, Kelli, and Robin. They helped me shoulder the weight and allowed me to lean when I needed to lean.

When my husband and I told his parents we were getting married before either of us were through college, certainly without real jobs, and now that I have children of my own, at an age that seems to be entirely too young, his dad said that he would only give his blessing if I promised to finish school. Little did he know I would spend the next 19 years, on and off, "finishing school." Jimmy and Vernell have supported me at every level of my education, and they have done it with tremendous grace and patience and love.

My sister was my first teacher. She has always been my favorite and certainly the best I have ever known. She is the most talented, wonderful, and amazing teacher, and the most patient and loving sister. I want to be like her. April and Larry are the ones that, given a chance, everyone would choose for a sister and brother. I was just lucky enough to have been granted them.

Daddy has always loved me so deeply and purely and completely that I am certain I would be nothing without him. He instilled in me the traits of thoughtful determination, hard work, and reflection; and he challenged me, even when he did not know it.

"This material is based on work supported by the National Science Foundation under Grant No. EHR-0314953."
In addition to the above statement, every publication of material (including World Wide Web pages) based on or developed under the grant, except research articles or papers appearing in scientific engineering, technical or professional journals, must contain the following disclaimer.
"Any opinions, finding and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation."

TABLE OF CONTENTS

LIST OF TABLES

Page

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

I taught high school mathematics for 8 years. During that time, I developed the reputation of being a good teacher. People, colleagues in the building with me, and others outside the school and district who were part of the community would often comment in conversations about what a good teacher I was. I would usually follow the compliment with some feeble attempt at humility, “Oh, I don’t know about that,” or on good days I might try to give credit to some notion of team, “That’s only because I work in a great system.” But I always walked away from those comments unsatisfied, knowing in my heart that the claimant knew only part of the picture.

The part of the picture apparent to the outside observer had absolutely nothing to do with mathematics and everything to do with my heart, my soul, and my management ability. I love with my whole heart interacting with young minds; I have a desire deep in my soul to better the world around me; and I have a style of management that young people just love to love. Although these facets of my nature may enable a learning environment, they do not enable learning. And knowing this left me dissatisfied with my ability and longing to improve it.

When I was in the classroom, most of what I did I believed to be quite innovative and rich with educational, and especially mathematical, value. Not only did I believe that my ideas had merit, I received repeated reinforcement of this idea in my numerous professional development experiences. The ideas presented were often the very ones I used in my own classroom. My students worked in groups, explored problems in context, presented different solutions to the class, and participated in many other unique practices. I was very willing to try new ideas, and I did. My students performed well on high stakes tests, and they often seemed to gain satisfaction from certain mathematical activities. Then what could have been wrong?

As I reflected deeper about my practice, I became more and more convinced of the hypocrisy of my situation. People, including my students, their parents, my colleagues, and me, believed that I was a good mathematics teacher, yet absolutely no proof existed to prove it to me. The standardized tests that my students took left more than a little to be desired. Scoring well on them was really no indication of being able to do mathematics. And while accolades were nice, the people giving them had never been in my classroom. My students showing a positive disposition toward mathematics was rewarding, but I knew it was not an indication of the big picture. Therefore, all of this came down to one person: me. I was the only one in my classroom who had a big picture of my goals. That left a huge conflict of interest for the evaluator and the person being evaluated. I realized that I could, in fact, be damaging the mathematical knowledge of these students, and no one would ever know.

Upon entering the university setting, I have been afforded the opportunity of reflection that few practicing teachers can experience. I know that when I was teaching I had some very good ideas. Unfortunately, I had lots of poor implementations of those ideas and plenty of bad ideas. I really wanted to figure out what I needed as a classroom teacher, and maybe what other teachers like me needed, in order to improve instruction. Reflecting, I used my own experiences as a teacher and with teachers to mold a more general understanding of "teacher" and the demands attached to that title. With this model, I developed some ideas about what I think may be helpful to some teachers.

Reflecting on my own experience, I would have benefited greatly from the input of colleagues. I needed to be asked "Why?" by people around me that I respected. I needed to be in a situation to dialogue with those peers to hear their suggestions and not be so convinced that my methods were always best. I needed collaboration that forced articulation on my part, and I needed it to be ongoing, not just during my entry into the profession. But accomplishing this type of professionalism and collaboration could be nearly impossible. Certainly, approaching it on an "as needed" informal basis is a method of operation riddled with flaws. My colleagues and I needed a structured way for us to collaborate in order to develop mathematics for ourselves and mathematical opportunities for our students. And we needed to situate this collaboration not in an auditorium with hypothetical situations, but in our own classrooms looking critically into our own practice.

Background

The reform movement in mathematics education has been defined in many ways. For the purposes of this work, the "reform movement" will refer to a student-centered approach to learning mathematics where students engage in mathematical reasoning and mathematical discourse, conjecture and justify, and develop mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001; National Council of Teachers of Mathematics [NCTM], 2000). Of critical importance to this reform movement is the teacher (NCTM, 2000). In order to realize any reform in mathematics education, mathematics teaching must also reform. Requiring classrooms to operate differently, for instance, focused on student learning, requires teachers to operate differently also. This requires new knowledge for and a different professional development of mathematics teachers. It is not only the content of the professional development that must change, however, it is also the model used.

The traditional model of professional development consisting of telling (telling teachers to collaborate, telling them to reflect, or telling them to use reform curriculum) will not produce the desired classroom (Kazemi & Franke, 2003; Mewborn, 2003; Sowder, 2007). Teachers are not passive recipients of knowledge any more than the students they teach (Simon, 1997). The act of telling is often responsible for superficial understanding. And superficial understanding of mathematics education reform would likely yield superficial implementation of the reform. Acknowledging that one right way to proceed through this reform does not exist, professional developers need to equip teachers with a means to explore mathematics teaching in an authentic way, one that may result in generative growth. According to Franke and Kazemi (2001), generative growth may be described as growth that continues after a professional learning experience, not just sustaining what was learned in the experience, but learning that stimulates continued learning and adaptation. The culture that professional developers provide for teachers is as important as, and quite similar to, the culture teachers should provide for students. Lesson study is one form of professional development that introduces teachers to a culture of learning for teachers that is authentic and grounded in the practice of teaching.

Lesson study is a Japanese model of professional development in which teachers collaboratively plan and study live lessons. In one model of this professional development, a group of four to six teachers form the teacher team, also called the planning team. They choose an overarching goal for students, which may be common to other planning teams in the building, if this is a school-wide model. The overarching goal is focused on the whole child and is not necessarily content specific. The planning team will choose a topic for their focus, a unit and unit goals for that topic, and a lesson and lesson goals for that unit. All of these choices are made with an eye on the overarching goal, choosing carefully the topic, unit, and lesson that will assist in the attainment of this overarching goal. The planning team will then begin the planning phase. They will spend a considerable amount of time creating a lesson plan that is descriptive and detailed. One person from the team, called the teaching teacher, will teach the lesson to a group of students. This is called the public teaching or the public lesson. The remaining members of the planning team and other invited guests will observe the lesson, gathering evidence of student learning. After the lesson, the team and the observers will discuss the lesson and the evidence gathered. The observers have the important task of describing what the students actually did in order for the whole group to determine if the lesson goals were achieved. One of the observers, called the knowledgeable other, is invited specifically to speak to the intended mathematics, comparing it to the evidence of student learning of the mathematics and to make connections of the mathematics across grade levels. The planning team will meet afterward to revise the lesson based on the suggestions during the post-lesson discussion. A different member of the planning team will then teach the revised lesson to a different group of students, which will again be observed, discussed, and revised. This completes one cycle of lesson study. This model of lesson study closely aligns with the work of Lewis (2002a), Fernandez (2002), and Yoshida (1999).

Rationale

Rare is the occasion that allows a collaborative team of teachers to make joint decisions about how a lesson actually happens. Rarer still is the opportunity to make those decisions, see them implemented, and critically determine if the decisions were effective in achieving the goals of the lesson. Although the training for preservice mathematics teachers includes discussions of philosophy, of methods, and of curriculum, manipulating this knowledge into teachable lessons that accomplish intended learning goals is left mostly to chance. Activities are often shared, but few lessons are researched for evidence of student learning. Although preservice mathematics teachers have a methods course, a curriculum course, and a mathematics teaching and learning course, each designed to contribute to the preparation of the preservice teachers, professional development for inservice teachers traditionally offers few opportunities for continued development in similar areas that are grounded in the work of teaching. Lesson study may offer such an opportunity for teachers to engage in work about methods, curriculum, and teaching and learning with and during their teaching experience.

In this model of professional development, teachers form a community of learners, centered and focused on examining practice. Lesson study should not be seen as a professional development activity so much as it is a culture (Watanabe, 2002) that enables teachers to develop professionally. Lesson study is a structured process of inquiry for teachers to “shift from ‘teaching as telling’ to ‘teaching for understanding’” (Lewis, 2002b, p. 3), ultimately focusing on student learning and development. Fernandez (2005) claims that this examination embedded in practice allows the teachers to connect the learning with the reality of the classroom.

The practice of teaching should not be defined by activities or strategies of best practice, nor should the sharing of these be considered as development for a teacher. Best practices in the hands of incompetent teachers will not produce student learning. The practice of teaching requires reflection to determine connections between what the teachers believe is taught and what the students reveal as learned. Improving or altering teaching and building new models of teaching, that is, professionally developing teaching, may be facilitated by the active observation and discussion of the teaching act. Lesson study is one form of professional development that provides this opportunity for observation and discussion. As lesson study begins to find its place in the United States, however, adaptations to this Japanese form of professional development are inevitable and necessary. Nationwide collaboration and research are needed to determine the adaptations that enable the success of lesson study in the United States while preserving the integrity of lesson study in general. Additionally, the pathways for teacher learning as well as the practices of those engaging the teachers in lesson study should be elucidated (Fernandez, 2002), building a theory of teacher learning while adapting the practice (Lewis, Perry, & Murata, 2006). Specific exploration of defining characteristics of lesson study should be explicit. Two such characteristics are reported as the contextual live observation of the public teaching and the collaborative community of learners.

Contextual Live Observation. As teachers in lesson study become acclimated to the role of observer in a classroom, one looking for evidence of student learning and not necessarily behavior or misbehavior, they develop “eyes to see students” (Lewis, 2002b, p. 21). This characteristic of teachers is an important one in constructivist theories of learning, which I will elaborate in chapter 2 (van Oers, 1996; Simon, 1997; Steffe, in press). Since the observers for a public teaching in lesson study cannot interact with the students, these observer-teachers learn to hone their listening skills and to see students in a new way. They hear and see the students as the students think and discuss mathematically. In this way, teachers are learning from practice. As teachers learn to listen to students without talking to them, teachers may have an opportunity to gain vital mathematical knowledge for teaching about what sorts of activities engage students in these processes.

Collaborative Community of Learners. Teachers who participate in lesson study often report collaboration as an important aspect of the process (Perry, Lewis, & Akiba, 2002). This collaboration is between teachers and between teachers and "knowledgeable others" (Wang-Iverson & Yoshida, 2005). Perry et al. (2002) report that this collaboration pushes "teachers to come to joint understandings" (p. 12). During lesson study, teachers are not isolated, and the opportunity is granted to engage with perspectives from the outside; that is, from other schools, other districts, or universities. These contributions become an impetus for what the teachers may learn during lesson study. Without the input of outside experts at some level during lesson study, Fernandez (2005) found that collaboration alone did not promote teacher learning as much as it could have and even that it could be unproductive at times.

In summary, characteristics of effective professional development are that the learning is contextual, is supported long term, is based in a community of learners, and provides opportunities for teachers to collaborate with colleagues and other experts to improve their practice (Franke & Kazemi, 2001; Lave, 1996; Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003; Mewborn, 2003). The sum of the aspects described here as those of lesson study clearly aligns with these characteristics. Additionally, these characteristics are also present in the findings of Garet, Porter, Desimone, Birman, and Yoon (2001) as features that contribute to improvements in teacher and classroom change and improved teacher learning. To realize reform in mathematics education, reform is sought in mathematics teaching. Researchers suggest teachers need to learn in ways similar to those of students. Learning should be contextual and done in a community (Featherstone, Pfeiffer, & Smith, 1993). Ball and Cohen (1999) suggest that teachers must learn how to learn in and from practice. They further assert:

> Our argument is not that teachers should become researchers or teacher researchers. Rather, it is that a stance of inquiry should be central to the role of teacher. … Teachers must be actively learning as they teach. The best way to improve both teaching and teacher learning would be to create the capacity for much better learning about teaching as a part of teaching. (p. 11)

In order to achieve reform in mathematics teaching, the professional development of mathematics teachers must be aligned with the characteristics of that desired reform. Lesson study offers an environment of professional development providing purposeful guidance for teachers about *how* to critically examine their practice, occurring *while* teachers are critically examining their practice, specifically within a lesson.

Since lesson study provides such an opportunity for teacher learning (Fernandez, 2005), this study attempted to trace what teachers learned as they participated in lesson study, considering pathways for teacher learning. This is an area in which Lewis, Perry, and Murata (2006) and Fernandez (2002) have requested more research. Fernandez also claims that more should be known and reported about the work of people who lead teachers in these inquiry practices. This study attempted to make transparent the work of those, including both the knowledgeable others and the lesson study facilitator, in order to inform other types of professional development projects that lead teachers to “learn in and from practice” (Ball & Cohen, 1999, p. 10).

Research Questions

For this study, mathematics teachers in a middle school participated in a professional development experience that used lesson study to examine their practice. Lesson study is defined as a cycle in which teachers work collaboratively to plan, teach, revise, and reteach a lesson (Fernandez & Yoshida, 2004; Lewis, 2002a). The knowledgeable other in the study was defined as the outside expert invited to provide specific content knowledge with a broad perspective and to select key ideas to focus the post-lesson discussion (Watanabe & Wang-Iverson, 2005). The purpose of the study was to understand what mathematics teachers may learn about the practice of mathematics teaching as they examine practice while participating in lesson study. The research questions that guided the study are as follows:

- What do mathematics teachers learn when they participate in lesson study?
- What do mathematics teachers consider to be key experiences of lesson study that help them learn?
- How do mathematics teachers describe the role of the knowledgeable other in lesson study?
- How does the role of the knowledgeable other influence the learning of the mathematics teachers?

- What is the role of the lesson study facilitator?

The study attempted to link certain key experiences of lesson study, including the role of the knowledgeable other, with teacher learning. Connecting the particular experiences of lesson study with teacher learning may provide a foundation for successful adaptation of lesson study into the United States, preserving the integrity of the process itself while allowing it to evolve into a new practice in a different culture.

> In the United States, policymakers and university-based researchers frequently ask whether lesson study works. To us, the question “Does lesson study work?” is a lot like the question “Does teaching work?” --- the answer always depends on the details of how it is done. (Lewis, Perry, & Murata, 2006, p. 9)

This study sought to contribute to an understanding of how lesson study is done in a way that contributes to teacher learning.

CHAPTER 2

LITERATURE

In this study, I implemented lesson study with two teams of middle school mathematics teachers to examine what teachers may learn while participating in lesson study and if certain lesson study experiences can be linked to teacher learning. I also wanted to investigate the role of the knowledgeable other and the lesson study facilitator, especially with regard to the influence on teacher learning. Models described by Lewis (2002a), Fernandez (2002), and Yoshida (1999) guided me in the design of lesson study for the study. These models led me to carefully consider literature on the professional development of teachers. And since I focused on what the teachers may learn in the experience, I reviewed literature on how learning occurs and on what teachers may learn in professional development experiences.

This chapter summarizes the related work in research that was relevant to the theoretical framework for the study. I studied characteristics or designs of professional development considered effective, knowledge teachers should develop in these professional development experiences, and models of professional development that could be used to meet these design and knowledge requirements. I have divided the chapter into three categories: professional development design, mathematical knowledge for teaching, and lesson study.

Professional Development Design

I have included literature on both professional development and teacher learning, and even learning in more general terms, in this section. For the purposes of the study, I did not separate learning from professional development since professional development should imply learning. Simon's (1997) definition of professional development includes learning of concepts, skills, and information and development of dispositions, awareness, and sensitivities, allowing for professional development that encompasses the many different aspects of teaching in general. The book *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001) suggests that professional development should be focused on "helping teachers to understand the mathematics they teach, how their students learn that mathematics, and how to facilitate that learning" (p. 398).

Mathematics education research is rich with images of what meaningful professional development should and should not look like, providing descriptions of desired learning environments. Historically, professional development has often been in the form of a workshop designed to give advice, handouts, or best teaching strategies (Ball & Cohen, 1999). Ball (1996) suggests that traditional professional development, not unlike traditional teaching, maintains a stance of answers. She contends that professional development necessary to enable reformed teaching of mathematics should have a "stance of inquiry and critique, a stance of asking and debating" (p. 9).

How People Learn (Bransford, Brown, & Cockling, 2000) shows the perspectives on learning environments that may be used with students and teachers using a graphic.

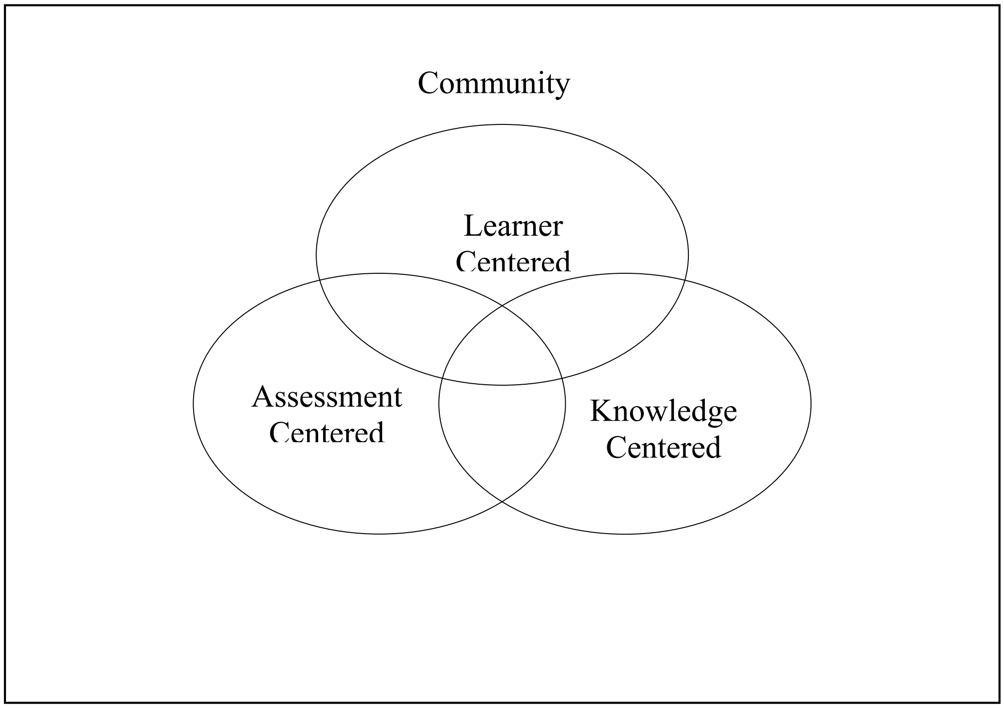

Figure 1. Learning environments (Bransford, Brown, & Cockling, 2000, p. 134)

According to *How People Learn*, a learning environment that is learner-centered (in this case, learners who are teachers, I will call them teacher-learners) uses what the learner brings to the situation to tailor instruction. A knowledge-centered learning environment for teacher-learners carefully accounts for and addresses the specific content to be taught by the teacher-learner. A learning environment centered on assessment designed for teacher-learners provides them an opportunity to "receive feedback from colleagues who observe attempts to implement new ideas in classrooms" (p. 196). Community-centered environments develop communities of practice for collaboration in order to provide opportunities for participants to share experiences, decisions, and learning. The recommendations in the book suggest aligning the perspectives of the learning environment in the region of overlap, where each perspective is influencing the other, since this may potentially accelerate the learning. In considering this general design of a learning environment, I explored characteristics of effective professional development necessary in a program, effects of communities of learners, and learning both for the teacher-learner and for the student.

Characteristics of Effective Professional Development

Within the general learning environment at the intersection of these perspectives, specific characteristics for effective professional development can be implemented. These characteristics describe what is happening within the professional development setting. Professional development should provide teachers the opportunity to collaborate, to build content and pedagogical content knowledge, and to examine practice (Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003; Garet, Porter, Desimone, Birman, & Yoon, 2001). Guskey (1995) adds that professional development should provide follow up and support to the teachers as they are implementing what they learned. Wilson and Berne (1999) make the case that "teacher learning ought not be bound and delivered but rather activated" (p. 194), implying the need to focus on the learners and the construction of knowledge by those learners.

Many researchers focus on reflection as a key component to what the teachers should learn in these learning environments, while building knowledge and examining practice. The notion that teachers should reflect is not a new one, but I want to draw attention to how the literature stresses the need for teachers to learn how to reflect on their teaching. This point is made clear by Ball (1996). While acknowledging that reflection is an important part of teacher learning, she claims that no guidance is given with regard to what teachers should reflect on and how they learn to reflect. Grouws and

Schultz (1996) urge teacher educators to help teachers learn to reflect on their teaching and the lessons they teach on a daily basis.

> Reflecting on pedagogical delivery systems, such as cooperative group learning, peer tutoring, or manipulatives is insufficient. Planning must focus on subject matter as it relates to three important factors: (1) the content itself – the discipline of mathematics and pedagogical content knowledge; (2) knowledge of how one promotes conceptual and operational understanding in students and (3) curriculum knowledge – the framework of mathematical structures at different developmental stages. (p. 456)

Ball and Cohen (1999) implicitly turn to the need for teachers to reflect when they say, "…Much of what [teachers] would have to learn must be learned in and from practice rather than in preparing to practice. Or, perhaps better, they would have to learn, before they taught and while teaching, how to learn in and from practice." Darling-Hammond (1998) emphasizes that teachers should "be able to analyze and reflect on their practice, to assess the effects of their teaching, and to refine and improve their instruction" (p. 8). In examining research on teacher learning, and reflection in particular, Sowder (2007) expresses the need for teachers to "develop the ability and habit of reflecting on practice" (p. 198) and says that strong collegial relationships are necessary in this development. The National Council of Teachers of Mathematics [NCTM] (2000) points to the necessity of reflection in and out of the classroom in order to properly realize the vision of *Principles and Standards for School Mathematics*. Addressing the characteristics of professional development opportunities for teachers is necessary in order for teachers to be equipped to address the new opportunities that NCTM expects them to provide students.

Communities of Learners Focused on Teaching Practice

Learning how to reflect is often described in the context of the teacher's practice and classroom and within communities of learners. Franke and Kazemi (2001) claim that teachers who learn in the context of their own classrooms may experience "generative growth." Cobb, Wood, and Yackel (1990) report that in a research project focused on analyzing second graders' mathematical learning, the classroom became a learning environment for the teacher, unintentionally, simply as a result of the research occurring in the teacher's own practice. In a different project, Stein and Brown (1997) concluded that learning takes place "within the ongoing work practices of the community" of teachers (p. 162), which includes facets of the overall practice of teaching, not just the particular act of teaching. Putnam and Borko (2000) similarly describe teachers' learning "grounded in some aspect of their teaching practice" (p. 12), elaborating on the possibilities presented by these aspects of teaching outside of the act itself. Even the development of conversations within communities of practitioners, about student thinking and the reflections and actions that result from those conversations, influence the practitioners to have similar conversations with their students in the classroom (Featherstone, Pfeiffer, & Smith, 1993). Ball (1993a) reminds readers, using her own practice as an example, that communities of practitioners can be a catalyst for teacher learning but may be quite difficult to establish:

> Another resource worthy of development is the professional community of teachers and the discourse about practice in which teachers might engage. Typically teachers face the problems and dilemmas of their work alone. Isolated from one another, rarely do they have satisfying or helpful opportunities to talk about practice. (p. 395)

Learning

Within these communities of learners, professional development and teaching in general consistent with constructivist perspectives, as Simon (1997) states it, must carefully consider how individuals learn. Constructivist theory is a perspective of learning and knowledge development in which humans, having no way of knowing an objective reality, construct and organize knowledge into an operating system from their experiences with the world around them. When an experience does not fit into that system, it perturbs the system, causing disequilibrium (van Oers, 1996; Simon, 1997; Steffe, in press). The system must be reorganized, or accommodated, which is what Steffe calls learning. Simon (1995) explains it this way: "When what we experience differs from the expected or intended, disequilibrium results and our adaptive (learning) process is triggered. Reflection on successful adaptive operations (reflective abstraction) leads to new or modified concepts" (p. 115). Observers, or mathematics teachers in the case of students as learners, create a scheme to represent what the learner is constructing. From that scheme, mathematics teachers develop "intentional interventions" (Simon, 1997, p. 58) in order to create an opportunity for students to construct mathematics. A constructivist classroom is designed in a way that the teacher acts based on his or her models of the students' mathematics and creates intentional interventions so that students may construct mathematical knowledge.

Teachers in this type of constructivist classroom focus on these models representing children's mathematics and, as Ball (1993b) explains, on interactions that will "foster mathematical understandings" (p. 159). This ability to concentrate on models and interactions is what Ball (1996) calls a "bifocal vision.…[Teachers need to be] both responsible and responsive to students" (p. 180). However, Stigler and Hiebert (1999), using a video analysis of teachers in three different countries, concluded that classrooms in the United States do not reveal this bifocal vision. They found teachers in the United States subscribe to a culture of teaching that appears static. Although elements of reform teaching could be found, like the use of manipulatives, cooperative groups, or technology, Stigler and Hiebert described teachers as viewing the use of the elements almost as the mathematical knowledge, instead of the elements leading students to construct mathematical knowledge, thus defining success by features and activities used rather than what students were learning.

Considering "new models for mathematics teaching consistent with constructivist perspectives," Simon (1997) claims, as stated above, that "mathematics teaching is a particular situation in which intentional interventions are made to promote the construction of powerful mathematical ideas" (p. 58). He uses his Mathematics Teaching Cycle (MTC) to show the tension created by the construction of mathematics by the student and the intentional interventions of the teacher in order to accomplish his or her mathematical goals. Simon describes this notion of teaching as an inquiry process, focused on asking questions and guiding student discussion around those questions. Simon (1995) defines the hypothetical learning trajectory to be the "teacher's prediction as to the path by which learning might proceed. It is hypothetical because the actual learning trajectory is not knowable in advance" (p. 135). Steffe (2004) refers to using this trajectory in his own teaching experiments. Figure 2 shows how Simon (1997) interprets the interactions of the factors that encompass mathematics teaching, the Mathematics Teaching Cycle, abbreviated.

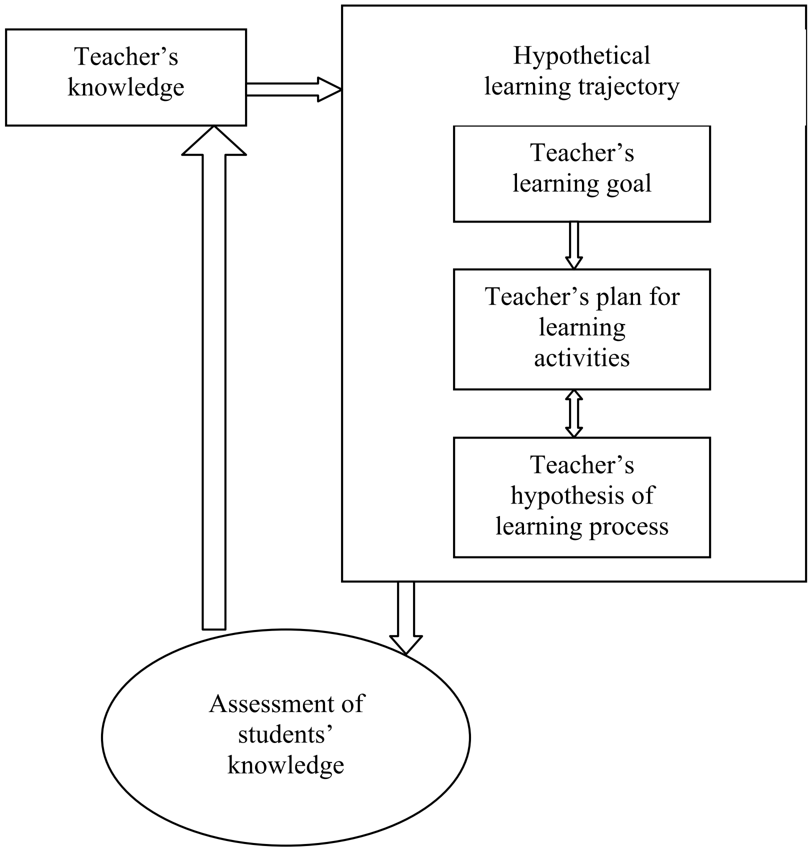

Figure 2: Mathematics teaching cycle, abbreviated (Simon, 1997, p. 136)

Summary

Adding It Up (Kilpatrick, Swafford, & Findell, 2001) describes many professional development experiences as focusing "almost exclusively on activities or methods of teaching and seldom attempt to help teachers develop their own conceptual understanding of the underlying mathematical ideas, what students understand about those ideas, or how they learn them" (p. 381). Instead, the book proposes a different experience for teachers to learn about mathematics teaching.

> Professional development programs [to develop proficient teaching] situate their portrayals of mathematics and children's thinking in contexts directly relevant to the problems teachers face daily in teaching mathematics. This grounding in reality allows knowledge of mathematics and knowledge of students to be connected in ways that make a difference for instruction and for learning. It is not enough, however, for mathematical knowledge and knowledge of students to be connected; both need to be connected to classroom practice. (p. 381)

These notions from the literature moved my thinking from a global understanding of learning environments to particular characteristics important in an effective professional development experience to a focus, for my work, on communities of learners centered on teaching practice and finally to how and what the participants in these communities can learn.

Mathematical Knowledge for Teaching

The knowledge required to be a mathematics teacher has been described in many different ways. Acknowledging that quantifying the knowledge by counting the number of advanced mathematics courses taken or by comparing grade point averages is neither accurate nor helpful for understanding the knowledge, researchers began to examine practice to find other ways to understand it. Shulman (1986) began this examination and research when he divided the knowledge that teachers use into three categories of content knowledge: subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. Marks (1990) used his research to show four major areas of pedagogical content knowledge: subject matter for instructional purposes, students' understanding of the subject matter, media for instruction in the subject matter, and instructional processes for the subject matter. Ball, Lubienski, and Mewborn (2001) explain that researching mathematical knowledge necessary for teaching requires a focus on the knowledge *of* mathematics, that is, "substantive knowledge of mathematics…others may call subject matter knowledge," as well as the knowledge *about* mathematics, which includes "what it means to know and to do mathematics" (p. 444). Ball, Thames, and Phelps (2007) further describe this mathematical knowledge needed by teachers using research based on the question of what good teaching requires. They define mathematical knowledge for teaching as "mathematical knowledge needed to carry out the work of teaching mathematics" (p. 21). In this description of mathematical knowledge for teaching, they considered the categories of content knowledge and pedagogical content knowledge. They divide content knowledge into common content knowledge and specialized content knowledge. They divide pedagogical content knowledge into knowledge of content and students and knowledge of content and teaching. I elaborate these categories in the sections below.

Common Content Knowledge

Educational Testing Services (2005) specifies, in the instructions for the Praxis Content Knowledge Test designed for certification in secondary mathematics education, that "the examinee will be required to understand and work with mathematical concepts, to reason mathematically, to make conjectures, to see patterns, to justify statements using informal logical arguments, and to construct simple proofs (p.1)." Shulman (1986) defines subject matter content knowledge as the ability to know what is true in the content and why it is true. Ball, Thames, and Phelps (2007) define common content knowledge as the "mathematical knowledge and skills used in settings other than teaching" (p. 32). This is the definition and category (common content knowledge) that I use in this report.

Specialized Knowledge for Teaching

This knowledge focuses on a form of mathematical knowledge that is specialized to the profession of teaching, depending entirely on knowledge of mathematics, "needed by teachers for the specific tasks of teaching" (Ball, Thames, & Phelps, 2007, p. 40). Ball, Lubienski, and Mewborn (2001) claim that researchers must refocus on practice and teaching, instead of teachers.

> Teachers must be able to know and use mathematics in practice, not merely do well in courses or answer pedagogically contextualized questions in interviews. This conclusion suggests the need to redefine the problem from one about teachers and what teachers know to one about teaching and what it takes to teach. (p. 451)

This kind of mathematical knowledge is different from the mathematical knowledge that may be held by other professions that use mathematics. These professions often need to compress mathematical understandings in order to attain a level of fluency to move into deeper, more advanced mathematics. Teachers must actually decompress previously learned advanced topics in order to guide students' mathematical development (Ball & Bass, 2003). Ball, Hill, and Bass (2005) state that while "others can avoid dealing with… [the emerging] mathematics [of students], teachers are in the unique position of having to professionally scrutinize, interpret, correct, and extend this knowledge" (p. 17). Ball and Bass (2000) claim that specialized content knowledge is

> a kind of mathematical understanding that is pedagogically useful and ready, not bundled in advance with other considerations of students or learning or pedagogy….An endless barrage of situations – of what we are beginning to understand as mathematical problems to be solved in practice – entails an ongoing use of mathematical knowledge. It is what it takes *mathematically* to manage these routine and nonroutine problems. (p. 88)

They also assert that an important distinction exists "between knowing how to do math and knowing it in ways that enable use in practice" (p. 94). They describe this kind of specialized knowledge necessary for teaching as being difficult to quantify. Ball, Thames, and Phelps (2007) depict this knowledge as the ability to "make features of particular content visible to and learnable by students" (p. 35).

Pedagogical Content Knowledge

Shulman (1986) refers to pedagogical content knowledge as the many ways the subject can be represented or modeled to others in order that others may comprehend the content, linking the content with the pedagogy, bringing a focus of student learning to the content matter. Marks (1990) used his experience with mathematics teachers to refine the definition of pedagogical content knowledge, illustrating that the focus of inquiry into knowledge of a mathematics teacher may lead to valid but different interpretations of that knowledge. He further describes pedagogical content knowledge as having three deviations. First, adapting content knowledge to pedagogical content knowledge requires interpretation on the part of the teacher. Second, adapting general pedagogical knowledge into pedagogical content knowledge requires the teacher to specify a general idea in a particular context. Third, adapting content knowledge and general pedagogical knowledge or previously held pedagogical content knowledge requires the teacher to synthesize these types of knowledge. Grouws and Schultz (1996) define pedagogical content knowledge as "a subset of content knowledge that has particular utility for planning and conducting lessons that facilitate student learning" (p. 444). Ball and Bass (2000) add that "pedagogical content knowledge is a special form of knowledge that bundles mathematical knowledge with knowledge of learners, learning, and pedagogy" (p. 88), as opposed to what they termed specialized content knowledge that is unbundled. Ball, Thames, and Phelps (2007) offer their two categories of knowledge of content and students and knowledge of content and teaching as elaborating Shulman's (1986) definition of pedagogical content knowledge. For this reason, the references in this work will be to the more general notion of pedagogical content knowledge, being defined as Shulman (1986) explained it, "In a word, the ways of representing and formulating the subject that make it comprehensible to others" (p. 9).

The concept of mathematical knowledge for teaching continues to be a not-very-well-defined notion. Many authors provide a laundry list of descriptors as found below:

> Teachers need several different kinds of mathematical knowledge – knowledge about the whole domain; deep, flexible knowledge about curriculum goals and about the important ideas that are central to their grade level; knowledge about the challenges students are likely to encounter in learning these ideas; knowledge about how the ideas can be represented to teach them effectively; and knowledge about how students' understanding can be assessed. (NCTM, 2000, p. 17)

The definitions and descriptions often seem to not contain the full extent of necessary or sufficient conditions. However, the notion is better understood as one looks at practice and what is involved in teaching mathematics.

Hill, Rowan, and Ball (2005) define *mathematical knowledge for teaching* as follows:

> mathematical knowledge used to carry out the work of teaching. Examples of "work of teaching" includes explaining terms and concepts to students, interpreting students' statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing students with examples of mathematical concepts, algorithms, or proofs. (p. 373)

The Hill et al. study suggests, as other studies had before it, that a correlation does not exist between teaching certificate or number of methods courses taken and mathematical knowledge for teaching. It also showed, among the many other important findings, "teachers' mathematical knowledge for teaching positively predicted student gain scores in mathematics achievement during first and third grades" (p. 399). Hill and Ball (2004) used results from the study to show how mathematical knowledge for teaching can be improved through professional development with certain design features, especially the professional development model used in their particular study.

Lesson Study

> If teachers are going to engage in inquiry, they need repeated opportunities to try out ideas and approaches with their students and continuing opportunities to discuss their experiences with specialists in mathematics, staff developers, and other teachers. These opportunities should not be limited to a period of a few weeks or months; instead, they should be part of the ongoing culture of professional practice. (Kilpatrick et al., 2001, p. 399)

Lesson study is a Japanese form of professional development that offers teachers exactly these opportunities described above. According to Lewis (2002a), Fernandez (2002), and Yoshida (1999), in lesson study, teachers work collaboratively on a team, called the teacher team or planning team, to set goals for student learning. They collaboratively plan a lesson that will contribute to the attainment of the goals. A teacher from the planning team teaches the lesson to a group of students with observers present (this is called the public teaching or research lesson). Observers gather evidence of student learning that occurred in the public teaching. They discuss the evidence and other lesson suggestions with the planning team. The lesson is then revised and taught by a different teacher to a different group of students with (ideally) the same observers present to gather evidence and discuss afterwards. Yoshida (2005a) explains that this form of professional development affords an authentic classroom illustration of teaching and learning. He asserts "the ultimate purpose of these activities is to yield new ideas about teaching and learning based upon a better understanding of student thinking" (p. 5).

Stigler and Hiebert (1999) make the claim in simple terms: "If you want to improve teaching, the most effective place to do so is in the context of a classroom lesson" (p. 111). In their book *The Teaching Gap*, they further explain what lesson study is and is not. "Lesson study is a process of improvement that is expected to produce small, incremental improvements in teaching over long periods of time. It is emphatically not a reformlike [*sic*] process" (p. 121). Lewis (2002a) also warns about the slow road to improvement in lesson study:

> Lesson study is not a quick fix, but a slow, steady means for teachers to improve instruction, and to build a school and district culture focused on inquiry and improvement. Perhaps the most surprising threat to lesson study in the US is the enthusiasts who expect to see perfect research lessons and become discouraged or dismissive when they don't. (p. 78)

The intended benefit of lesson study is not a perfect lesson. In fact, the lesson may even be seen as a byproduct of the process. Fernandez (2002) describes lesson study as a "systematic inquiry into teaching practice…which happens to be carried out by examining lessons" (p. 394). Later in the same article, Fernandez emphasizes that one of the challenges to lesson study is that the teachers must make decisions about what is being learned by the teachers themselves. They will ultimately be responsible for how that learning is structured. In other, more traditional, professional development, those decisions and structures are provided by the deliverer of content, which is clearly absent in this process. Lynn Liptak, a principal at a school involved for a number of years in lesson study, states a similar conclusion in Lewis (2002a):

> For too long, professional development time has been allocated to outside experts to "train" teachers rather than given to teachers to reflect collaboratively on their practice. We need to tap outside expertise; we need to improve our content and pedagogical knowledge. But the professional development process needs to occur in the context of our classrooms and be driven as an ongoing activity by professional practitioners. (p. 48)

Lewis, Perry, Hurd, and O'Connell (2006) report on one of the first lesson study groups in the United States, located on the West Coast. The school reported in the study had just begun the sixth year of lesson study at the time the article was published. The teachers show many instances of learning about student thinking, observing students, and curriculum, among other things. The teachers describe the emerging instructional coherence that is present school wide. In addition, state mathematics achievement test scores showed a net increase over 3 years for students who remained at the school, an increase that was more than triple that elsewhere in the district. Lewis et al. claim that a causal connection cannot be made, but that "other obvious explanations (such as changes in student populations served by the school and district) have been ruled out" (p. 7).

Fernandez (2005) studied a group of elementary teachers during the first year of implementation of lesson study in the northeast part of the United States. She explored specifically the teachers' "opportunity to learn," to identify what lesson study has to offer; and also "whether teachers have sufficient subject matter knowledge to make engaging in lesson study a worthwhile endeavor" (p. 268). She identified opportunities to learn pedagogical content knowledge and specialized content knowledge. However, she also found instances of the teachers being confused about the mathematics of the lesson and moving in unproductive directions because of the confusion. But in those cases, the teachers often recognized their own inconsistencies and misunderstandings, even suggesting the need for outside support (see p. 282).

There are many features of lesson study that at first glance may seem confusing, possibly daunting, or even irrelevant. As lesson study begins to find its place in the United States, adaptations are inevitable and necessary, for many reasons, not the least of which is the cultural and educational differences between Japan and the United States. Some researchers (Lewis, Perry, & Hurd, 2004), however, want to carefully consider the adaptations that would enable the success of lesson study in the United States while preserving the integrity of lesson study in general. According to early research (Fernandez & Yoshida, 2004; Lewis, 2002b), detailed lesson plans with anticipated student responses and evaluation questions are critical to the process. Also important to the process are the contributions of the knowledgeable other (Fernandez, 2005; Gill, 2005) and the live observation of the public teaching. Fernandez (2005) explains how certain features of lesson study are likely to be critical to teacher learning:

> Not only does lesson study provide a venue for teachers to talk about the content that they teach in the context of thinking about how to teach this content, but in addition, it also seems critical that this thinking is organized around working on a lesson. Lessons are natural units of teaching that teachers think about on a daily basis…. Moreover, it also seems important that this process requires teachers to discuss their observations of their lesson as taught in real time, with actual students, and on the tail of extensive planning and reflection. (p. 283)

Detailed Lesson Plans

According to many lesson study researchers (Fernandez & Yoshida, 2004; Lewis, 2002a; Stigler & Hiebert, 1999), for numerous reasons the bulk of time in lesson study is spent in the planning of the lesson. The details contained in a lesson plan for the public teaching are typically more involved, in content and number, than would be found in a typical daily lesson plan. Teachers in the planning stage of lesson study try to account for all teacher moves, verbally and nonverbally, and they attempt to articulate all anticipated student responses during the lesson. They create evaluation questions in each segment of the lesson plan to guide the observers, expecting the questions to facilitate the observers' understanding of the team's intentions with regard to student learning.

Decisions are made during planning about examples to be used, including specific choices of numbers in the examples and reasons for those choices, often recorded in the plan to better inform the observers of these choices (Yoshida, 1999). Even the story that the blackboard will tell at the end of the lesson is determined in advance, and pictures are often taken at the end to discuss after the lesson with observers (Yoshida, 2005b). Throughout the planning process, the teachers revisit the goals of the lesson and unit in order to better specify the questions that should reveal student progress during class. The combination of these acts during the planning stage of lesson study engages teachers in mathematics, providing them with opportunities to learn mathematics (Fernandez, 2005), and often uncovers gaps in their own understanding (Lewis, 2002a).

> Lesson study alone does not ensure access to content knowledge. But teachers are likely to build their content knowledge as they study good lessons, anticipate student thinking, discuss student work with colleagues, and call on outside specialists. Lesson study can help educators notice gaps in their own understanding and provide a meaningful, motivating context to remedy them. (p. 31)

Teachers in the United States are often concerned about the amount of time that may be spent developing one lesson in lesson study, which could be a number of months to even a year or more. It is often even somewhat uncomfortable for United States teachers new to lesson study to design the lesson plan for that length of time and as detailed as lesson study produces. Fernandez and Yoshida (2004) interviewed Japanese teachers to ask why they developed such a detailed lesson plan. The teachers responded that anticipating student responses and teacher reactions to those was excellent preparation for the lesson itself, preparing the teachers for the content, possible responses, and ways to direct the student thinking. This planning phase and the details it encompasses are what they contend lay the groundwork for the teacher learning that occurs.

Contributions of the Knowledgeable Other

Reports of lesson study groups in Japan and the United States claim that knowledgeable others are vital to the lesson study process (Watanabe & Wang-Iverson, 2005; Yoshida, 1999). Those authors also suggest that the knowledgeable other be external to the district. If instead the knowledgeable other was an assistant principal, mathematics supervisor, or other district person, the role of this key player could be perceived to be that of an evaluator instead of a learner. In establishing lesson study groups, every role, including that of the knowledgeable other, should be viewed equally and as learners (Fernandez, 2002).

The knowledgeable other interacts in many different ways with the teacher teams. This person may even operate differently in one team than the next, depending on the needs of the different teams. Researchers (Gill, 2005; Watanabe & Wang-Iverson, 2005; Yoshida, 1999), however, do sketch a portrait of what should be common to all knowledgeable others. Specifically, the knowledgeable other provides guidance, a different perspective, and critique about the lesson; provides content knowledge expertise; shares the work of other lesson study groups (Chokshi & Fernandez, 2004; Fernandez, Yoshida, Chokshi, & Cannon, 2001). Watanabe reports in Lewis (2002a) that the knowledgeable other must carefully read the audience of teachers and anticipate what the group is ready to learn.

Live Observation of the Public Teaching

Lewis (2002a) claims that the live observation of the public teaching is at "the heart of lesson study" (p. 43) since the observers are able to "pick up the mutterings and shining eyes of the students" (p. 44) which are important in the gathering of evidence of student learning. "Lesson study focuses not just on student work, but students *working*" (Yoshida, 2005a, p. 16). A Japanese teacher wrote that the public teaching allows the teacher to see his or her own teaching from different perspectives. "A lesson is like a swiftly flowing river; when you're teaching you must make judgments instantly" (Lewis & Tsuchida, 1998, p. 15). Lewis (2002a) explains further that live observations "enable you to slow down the 'swiftly flowing river' of instruction in order to study it" (p. 68).

During the discussion of evidence gathered after the observed public teaching, observers often "notice different things and … the resulting discussion will foster teachers' development" (Lewis, Perry, & Murata, 2006, p. 7). Japanese teachers attribute "developing the eyes to see children" (Lewis, 2002b, p. 21) to the gathering of evidence of the live lesson. Much of this evidence cannot be seen through some other medium like video, and can therefore be elusive if the observer is not keenly alert, since much of the evidence can be nonverbal. These discussions are meant to offer advice and critique in order to improve student learning. Lesson study is focused on student learning, not teaching, which directs comments away from individual teachers to the lesson plan itself (Lewis, 2002b), which contrasts much professional development that many teachers may have experienced in the past, according to Ball (1996):

> Traditionally, professional development and professional forums assume a stance toward practice that concentrates on answers: conveying information, providing ideas, training in skills… Because discussions of teaching sometimes resemble "style shows" more than they do professional interaction, teachers' development of their practice is often a highly individual and idiosyncratic matter. The common view that "each teacher has to find his or her own style" is a direct result of working within a discourse of practice that maintains the individualism and isolation of teaching. This individualism not only makes it difficult to develop any sense of common standards but also makes it difficult to disagree. Masking disagreements hides individual struggles to practice wisely and so removes a good opportunity for learning. Politely refraining from critique and challenge, teachers have no forum for debating and improving their understandings. (p. 504)

The live lesson observation provides a structure for critique and challenge in the safety of lessons planned collaboratively with colleagues.

The structure of the lesson study for the present study was based on the work of these researchers (Lewis, 2002a; Fernandez, 2002). The aspects, characteristics, and experiences described as contributing to teacher learning or essential in the structure of lesson study as described here were preserved in this study.

Summary

Teachers are now charged to provide students with mathematical learning opportunities that have not traditionally been part of the mathematics curriculum. NCTM (2000) proposes a vision for school mathematics that develops students with factual knowledge, procedural proficiency, and conceptual understanding. *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001) describes the need for students to be mathematically proficient, indicating strands that include conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. For teachers to facilitate the development of students with this proficiency, the teachers must receive professional development in a manner that is in alignment with these student criteria. Lesson study offers one model that contains characteristics considered effective in the professional development of teachers.

Lewis (2002a) repeatedly states that "the point of lesson study is not to polish the skills of a few star teachers but to help all teachers progress, in order to reach as many students as possible with successful lessons and a coherent experience" (p. 55). She also reminds readers that "lesson study is often more productive when teachers study existing lessons or approaches" (p. 62), rather than trying to create novel lessons. Mewborn (2003) claims that teacher development models used in Japan, like lesson study, "seem to incorporate components of professional development [that promote change]: long-term, school-based reform in a community of learners with opportunities to grapple with significant mathematical ideas and to consider how students engage with these mathematical ideas" (p. 50).

Lesson study researchers in the United States have made many calls for future research. Lewis, Perry, and Murata (2006) state that one research need is to

> explicate the mechanism by which lesson study results in instructional improvement…. Models that specify the connections between lesson study's observable features and instructional improvement, even in a tentative, emerging fashion can be useful in several ways…. Models may enable innovators to avoid rote implementation of surface features and to adopt a more thoughtful and flexible – less recipe-like—approach to innovation and accompanying research. (p. 5)

Fernandez (2002) calls for research to "make transparent the practice of those who engage teachers in these activities" (p. 404).

In this chapter, I have reported about the literature that molded and guided my decisions in this study. I began with professional development environments, characteristics that promote effectiveness, communities that should exist, and ways the learning may occur. Next, I specified the types of mathematical knowledge for teaching that are reported in this study: common content knowledge, specialized content knowledge, and pedagogical content knowledge. Finally, I referenced the lesson study literature that provided the foundation for the two lesson study teams in this project. As a whole, I used the literature to consider what makes professional development effective for teachers, and what learning may occur in those effective settings, and what specific model could be used to be effective.

CHAPTER 3

METHODOLOGY

Lesson study is a professional learning experience that allows teachers to collaboratively examine their own practice. The purpose of this study was to examine what mathematics teachers may learn about the practice of mathematics teaching while participating in lesson study. In this chapter, I describe the environment and context in which I explored the learning of mathematics teachers, including the participants and their school; the lesson study experience I designed and the reasons I chose to tailor the experience as I did; and how I analyzed the data.

Participant Selection

I was a fulltime employee with a grant from the National Science Foundation called Partnership for Reform in Science and Mathematics (PRISM). I worked as a mathematics specialist with the partner districts in the grant. I facilitated several schools in lesson study as part of my work with PRISM. In planning this study, I wanted to work with middle school mathematics teachers. All the schools in our state were implementing new performance standards in mathematics. These standards were being rolled out year by year. Sixth grade was first, seventh grade followed the next year, and the eighth grade standards were rolled out during the year I was collecting data. All of the middle schools in our region were implementing a new textbook series, the Connected Mathematics Program (CMP), and were in the first few years of that implementation. At the time I was prepared to collect data, I was asked to facilitate lesson study for one of the partner schools in PRISM for sixth- and seventh-grade mathematics teachers in the district where I had last taught as a high school mathematics teacher. This district had been awarded a grant by the state to form partnerships with higher education faculty to improve the content knowledge of mathematics teachers in Grades 3 – 8, and they wanted to couple that grant with the PRISM grant to pool their resources for greater gain. The teachers in the middle school chose to use lesson study as the focus of their professional learning for that grant. After accepting the facilitator position, I purposely selected this site for my study because the site was congruent with my choice of teaching level, and I was familiar with the school and faculty, having previously taught in the district for 4 years.

I collected data from a sixth-grade team and a seventh-grade team from September through March. Each grade level in the middle school had collaborative planning, so I met with each team during their respective planning times. Each grade level team comprised two regular education teachers and two special education teachers. The school used an inclusion model of collaborative teaching for the special education program. Therefore, the special education teacher co-taught the mathematics classes with the regular education teacher. In chapter 4, the term *co-teacher* is used as a generic description for either the regular education teacher or special education teacher in that class. The co-teaching pairs were assigned for the year, and some of these pairs had worked together for multiple years.

Data Collection Methods and Design

Following the suggestions of Lewis (2002a) and Fernandez (2002), the lesson study teams used the following schedule for a complete cycle of lesson study: four to seven planning meetings; first public teaching; post-lesson discussion; revision meeting;

second public teaching; post-lesson discussion. Tables 1 and 2 give the schedule of the professional development experience for the year for each grade level.

Table 1 (Sixth-Grade Schedule for Lesson Study)

Cycle	Planning meetings	1st public teaching	Post-lesson discussion	Revision meeting	2nd public teaching	Post-lesson discussion
1	Sept. – Nov. (7 meetings)	Nov. 29	Nov. 29	Nov. 29	Nov. 30	Nov. 30
2	Jan. – Mar. (4 meetings)	Mar. 13	Mar. 13	Mar. 13	Mar. 14	Mar. 19

Table 2 (Seventh-Grade Schedule for Lesson Study)

Cycle	Planning meetings	1st public teaching	Post-lesson discussion	Revision meeting	2nd public teaching	Post-lesson discussion
1	Sept. – Nov. (7 meetings)	Dec. 6	Dec. 6	Dec. 6	Dec. 7	Dec. 7
2	Jan. – Feb. (4 meetings)	Feb. 28	Feb. 28	Feb. 28	Feb. 29	Feb. 29

The schedules followed the suggestions of Lewis (2002a, p. 42). Very little adaptation of the model was made. However, the suggestions for how to conduct the post-lesson discussion were not followed as completely. Lewis suggests a structured agenda, formal dissemination of evidence, and structured questions from the planning team posed to the audience during the discussion. Our teams did not practice formality at any stage of the lesson study experience. The "plan to guide learning" (Lewis, 2002a, p. 127) was adapted to contain less information and be more informal.

First Cycle of Lesson Study

The teams met as grade-level teams seven times during the first cycle of lesson study for 60 to 70 minutes each during their collaborative planning times. Then, we had the public teaching, usually during a 72-minute class period. The middle school sometimes had an alternative schedule on Fridays for different activities, and if a public teaching happened to be on one of those days, the class period was 65 minutes. Immediately following the public teaching, a post-lesson discussion was held for approximately 60 minutes with the planning team and all observers who attended the meeting. Each public teaching had from one to five additional observers in attendance. On the same day, after the post-lesson discussion, a revision meeting was held for 60 to 70 minutes. The following day was the public teaching for the revised lesson, and a post-lesson discussion was held after the second public teaching. The first cycle had three more planning meetings than the second cycle, which were spent orienting the teachers to the lesson study process, answering questions and explaining this research study, completing Internal Review Board documentation, and preparing the teachers for the process in general.

Second Cycle of Lesson Study

The teams each met four times during the second cycle of lesson study. After the public teaching, we had a post-lesson discussion with all observers followed by a revision meeting, all on the same day. The following day, we had our second public teaching of the revised lesson with a post-lesson discussion with observers immediately following the public teaching. Each public teaching had from one to eight additional observers in attendance.

I interviewed the teachers individually prior to beginning lesson study, I interviewed each team as a group after the first cycle, and I interviewed each team as a group after the second cycle. I also had them reflect and respond individually and in writing to a set of questions after each cycle. Every interaction between the teachers and me was digitally recorded. The public teachings were videotaped. I transcribed all digital recordings. I also made field notes of the meetings and interview notes, and I recorded email communications and all iterations of the lesson plans developed by the teams.

The field notes served me as facilitator and as researcher. Although I was aware that I was a researcher throughout the process, I tried to separate myself from that role during lesson study. I wanted to be only in the role of facilitator during our two cycles of lesson study, so that my interactions with the teachers were specifically toward the goal of professional learning. I wanted to look back on the process as a researcher, in order to try to see a typical lesson study setting for teachers. A lesson study setting being facilitated by a researcher constantly asking research questions did not fit my definition of *typical*. I will define that role below. I tried to conduct these lesson study teams exactly as I had any other team in my work with PRISM.

School and Participant Description

The middle school was in a southern state in the United States in a suburban area. The teachers ranged in experience from 2 to 20 years. The school had a reputation for consistently having high scores, traditionally outscoring the neighboring districts, on tests that are mandated by the state. Both the sixth-grade and the seventh-grade teams felt a tremendous amount of pressure with the new standards, new textbook program, and traditionally high test scores.

Sixth Grade

The new state performance standards were implemented 2 years prior to our work, but this was the first year for this grade level to implement the CMP textbook program. The teachers had been trained the summer before our work began in the fall. Even though the teachers had reached a level of comfort with their grade-level standards, they were struggling, as many teachers do in the initial stages of reform curriculum, with CMP. All the teachers said they valued the way the textbook required the students to think deeply about the mathematics, but they worried about other issues. They did not think the small group work in their classrooms was as effective as it should be. They spent a lot of time in our planning meetings discussing strategies, successes and failures, and how to better address the needs of students in small groups. They also were not convinced that the students were given enough practice with newly developed mathematical ideas and skills. But the teachers shied away from supplementing the textbook series or, for that matter, omitting topics or investigations from the series. They desperately wanted to do all investigations in all units. The reasons the teachers gave for this desire to complete all investigations were clear: They really liked all the investigations, so they did not want to let go of any of the work because they believed the work was that good. Also, they were convinced from the summer training on the program that each investigation was such a building block for future investigations that, in this first year of implementation, they would not be able to accurately determine the impact of choices they might have made on their own.

Prior to lesson study, this team did not typically plan collaboratively. The pairs of teachers – that is, the regular education teacher paired with the special education teacher – did work together to prepare for co-teaching. But even though the two pairs were always very close together in the textbook program, they did not plan actual lessons with one another. During the lesson study planning meetings, especially during the early meetings, the teachers worked together with a great deal of hesitation, neither wanting to seem overbearing to the other. The roles of collaboration were negotiated during our lesson study process. As the meetings progressed, this hesitation continued to diminish, and the group became more comfortable with each other, although still reserved.

The pseudonyms I used for the sixth-grade teachers were Bobbie, Amy, Trish, and Jane.

Seventh Grade

The seventh grade implemented both the new state performance standards and the CMP textbook program during the year prior to the lesson study. However, they implemented CMP without any training and then attended the same training as the sixth-grade teachers did, after this first year of using CMP and before the year of lesson study. This sequence gave these teachers a great deal of confidence, even though the first year was extremely difficult for them. During the training, when everyone else was being exposed to the investigations for the first time, these teachers remembered, often painfully, the investigations from their own class. They brought prior knowledge to the summer training. This knowledge made it much easier for them during the second year of implementation to make judgments about what parts of a CMP unit they would do and what parts they would not do. They were very relaxed about both the curriculum and the textbook program.

This team also collaborated daily about their classes. They planned for the following week together, and they met together daily to plan for any deviations from those earlier plans. They even made copies of materials together. They were very relaxed with one another, already having established a collaborative work relationship. They also had deep personal relationships with one another that contributed to the ease with which they worked together.

The pseudonyms I used for the seventh-grade teachers were Ruth, Pat, Annie, and Olivia.

Facilitator

I acted in the role of facilitator and should therefore be considered a participant. In this case, the facilitator was also the researcher, so the role merits even greater detail in order to understand the environment of the study. I had facilitated 4 other lesson study groups over three years. Part of the purpose of this study was to explore what the role of the lesson study facilitator should be. To consider that aspect, I felt it was important to separate the facilitator role from the researcher role as much as possible.

I interviewed the teachers individually before we began our first cycle, I interviewed them in grade level teams after the first cycle and again after the second cycle, and I gave them written questions to respond to after the first cycle and after the second cycle (see Appendixes A, B, and C for interview questions). Before we began any of these, I described the details of the study, very similar to the description I provided for them before we began. I explained that the study was about what teachers learn during lesson study and what experiences during lesson study helped them to learn. They were told that the experience from the eyes of teachers in this situation was a critical piece of how this process would be understood in mathematics education. But, outside of these interview experiences, I did not mention my study. The teachers occasionally asked questions about my research but only when they were concerned they were about to "do something" that could somehow hinder the research.

As stated above, I digitally recorded all our planning meetings, and I took notes during each meeting. These notes served two purposes. Most of what I wrote was with regard to the development of the lesson plan, questions that the team needed to address, and plans for our next meetings, very much like minutes of a meeting that any facilitator might be responsible for. In addition, I added comments about what was happening during the meeting for me as researcher to consider after both cycles were completed. At the beginning of the project, I listened to the recording of the previous meeting in preparation for an upcoming planning meeting. However, I found the activity of listening to previous meetings to be too tempting as a researcher to use that opportunity to think about how I wanted to guide their learning. Therefore, I abandoned this activity. Again, I wanted to study what lesson study afforded for teachers' learning, not what a researcher may afford that learning. I envisioned that the process of lesson study itself would perturb the teachers' system of knowledge. Then the particular pieces of the system that the teachers themselves had to restructure were their own choice. Although I (as lesson study facilitator) contributed when teachers had discussions during the process, which ultimately guided the teacher learning, I did not want to make the decision for the teachers about what should be discussed or probed, or about what pieces of their knowledge should be restructured. It was important to me to remember that the teachers did not request this learning experience because they felt lacking in some area as mathematics teachers. They did not approach me and ask for knowledge to be imparted to them. They were simply participants because the grant the district received had specified some level of engagement with mathematics teachers as support during the implementation of the new state performance standards. A facilitator pushing an agenda for an aspect of mathematics teaching or learning that did not originate from the teachers would not have been well received, nor would it have proven useful in this environment. My goal was to allow the circumstances of the process of lesson study to guide the team and to allow the members of the team to dictate their own focus, according to the teachers' experience.

Nevertheless, as with other lesson study teams I facilitated, I interacted with the team as a mathematics specialist and former teacher. In planning meetings, I asked questions for the team on two different levels. I asked questions as a guide for them to implement lesson study, and I asked them questions as a mathematics educator. To guide them through the process of lesson study, I frequently asked questions such as these: "What is our goal for the lesson? How does this part of the plan help the students attain that goal?" And I also asked questions as a mathematics educator such as these: "Why do we want the students to perform the division on the last step? Do they understand it as division or are they just thinking about multiplication?" Both of these types of questions were similar to ones I asked in other lesson study groups, even groups in different content areas. I was not a passive observer, nor was I only a guide through the professional learning experience. I did limit my interactions as a mathematics educator so that the teachers would not default to seeing my role as being their teacher. When a question was asked and they quietly turned their heads in my direction, I responded with questions like, "What do you think?" or "What do you do in those situations?"

Primary Knowledgeable Other

The primary knowledgeable other in this study was a mathematics educator with a vast amount of expertise in lesson study. His expertise in this role closely aligned with the description in the lesson study literature (Chokshi & Fernandez, 2004; Lewis, 2002a; Wang-Iverson & Yoshida, 2005). He received the last few iterations of the lesson plan for each public teaching and communicated with the teams via email about the plans, although he did not attend any planning meetings. He attended the public teachings, post-lesson discussions, and revision meetings, with the exception of the second teaching of the second cycle for sixth grade. Even though he had planned to attend that day, family circumstances prevented him from being able to do so. During the post-lesson discussions, he reserved his comments from the lesson until the end when I specifically asked him to address the team, as is often the practice in Japan. Additionally, the primary knowledgeable other in this study participated in the revision meeting. The teachers often used that time to ask him specific curriculum questions and general questions about the lesson study experience.

The pseudonym I used for the primary knowledgeable other was John.

Secondary Knowledgeable Others

The secondary knowledgeable others in this study were mathematics professors working with the school district as content specialists for the grant awarded to the district by the state. They were, therefore, not only working with the sixth- and seventh-grade teachers in lesson study, but they were also working with the high school teachers in content delivery. The first public teaching of each cycle for each grade level was attended by one of the mathematicians. They attended the post-lesson discussion for that lesson, and the comments from these secondary knowledgeable others influenced each of the second public teachings. Additionally, one of the mathematicians attended a planning meeting immediately prior to the public teaching he attended.

The pseudonyms I used for the secondary knowledgeable others were Matt and Luke.

Data Analysis

This study was a design experiment in the sense that it entailed "both 'engineering' particular forms of learning and systematically studying those forms of learning within the context defined by the means of supporting them" (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003, p. 9). The purpose of this study was to understand what mathematics teachers may learn about the practice of mathematics teaching as they examine practice while participating in lesson study, hopefully linking certain key experiences of lesson study with their learning. According to Cobb et al., a design theory explains why designs work and suggests how they may be adapted to new circumstances. My goal was to lead the teachers through lesson study and gather data about what happened to them in the process. I wanted the data to illuminate the experience in order to “leverage learning at new sites” (Lewis, Perry, & Murata, 2006, p. 10). Therefore, I used a grounded theory approach to analyze the data. Crotty (1998) described this methodology as research that builds theoretical ideas and claimed that it “seeks to ensure that the theory emerging arises from the data and not from some other source. It is a process of inductive theory building based squarely on observation of the data themselves” (p. 78). This was precisely my intent when I developed the structure of this study.

After transcribing the digital recordings, I used a printed version of the transcripts to begin coding. I started the process with the first cycle. So, I read through the transcripts of the sixth-grade planning meetings and the seventh-grade planning meetings. I coded categories that the teachers discussed at length or that they mentioned frequently. I then used the same procedure with the second cycle, looking first through the transcripts of the sixth- grade meetings, then the seventh-grade meetings. My list of codes grew as I went through the two cycles, so when I had been through the entire set and had a complete list of codes, I started the process over, in the same order, with the complete list of codes.

Then I placed related codes into categories that were chunked together because of some relationship. For instance, I initially had a code called “student models” and one called “scaffolds for student learning.” I put both of those into a chunked category called “student learning.” I went back through all the data, again in the same order – sixth-grade first cycle, seventh-grade first cycle, sixth-grade second cycle, and seventh-grade second cycle – this time coding everything with the chunked categories. After I completed coding the set of data, I went through the data to study the coding system.

Using the new categories, I found additional pieces of data that had not been previously coded. As I studied those, it became apparent to me that with the new chunked categories, my grain size for the data had become more general. Placing these initial codes into chunked categories provided me an opportunity to refine and better define the categories. Therefore, I was able to pick up certain pieces of dialogue that were not initially evident with the first set of codes. Similarly, some of the data that had been coded originally did not properly fit in the chunked categories. I further studied the context of the first coding and not the second coding and was convinced that the chunked categories contained the data that was important to my research purpose and questions and omitted the codes that did not make the second cut. Those data seemed to be centered on very specific building issues. For example, one of my first codes was "depends," which was used when the teachers responded to questions with "it just depends." They used this phrase to refer to a variety of issues: the class, individual students in the class, the day, the time of year. None of these could be tied to the purpose for my research. I also had a category labeled "CMP." With this, I coded the times that the teachers referred to CMP as the one and only source for determining what happened in class. Although both of these categories play a part in the overall context and environment for these teachers, they did not contribute to my research questions.

Table 3 (Data Codes)

First Set of Codes:	*Description*
CMP	Teacher views CMP as the final authority
C	Teaching models consistent with constructivist perspectives (Simon, 1997)
Est	Students using estimate / considering reasonableness of answer
D	Teachers using phrase "it just depends…"
T	Looking for facilitator or knowledgeable other to provide answers
F	Having faith in the teacher's way only
L	Teaching that is constrained by level of class
Mo	Student models
Co	Teacher collaboration
M	Mathematics
Sca	Scaffolding student learning / helping students make connections
KO	Role of knowledgeable other
Second Set of Codes:	*Description*
St L	Student Learning
Sc	Scaffolding
M	Mathematics
G	Lesson Goal
Third Set of Codes:	*Description*
RC	Role of Teachers in Classroom using Reform Curriculum
M	Mathematical Knowledge for Teaching
Focus	Lesson Focus

At this point, I went to the teacher interviews after the first and second cycles and to the written responses the teachers had given me after the first and second cycles (which had also been coded twice as had the rest of the data). I used these to help me think about what these teachers claimed as having learned during lesson study, and even what they simply found to be really important to them in our experience in lesson study. I studied this set of data alone, and I began to see that the teachers were describing categories of their own learning. Those categories became the three threads of growth that I name as what the teachers learned during lesson study in chapter 4: role of the teacher in a classroom using reform curriculum; mathematical knowledge for teaching; and lesson focus. I used these categories of teacher learning to go back to the data and recode. I then went through the data in a different order. I reviewed the sixth-grade data from beginning to end (first and second cycles), then I reviewed the seventh-grade data from beginning to end (first and second cycles). I looked at the data line by line again. I found that only the data that had already been coded with the chunked categories actually fit one of the three learning categories.

In addition to these codes, I also coded evidence in the data of the knowledgeable others' roles and the lesson study facilitator's role. In explicitly discussing the roles of the knowledgeable others with the teachers, the links to the teacher learning from the data-determined teacher learning categories are apparent. As I tried to make transparent the role of the lesson study facilitator, however, I had to go to the data seeking specific insight about a role, actually finding boundaries of the role, rather than exact specifications. Although I have gained insight in looking at the data this way, it revealed to me more about what I failed to do as lesson study facilitator that would have better aided the process of teacher learning. I embarked on this journey because I wanted to know what teacher learning occurred in lesson study; and what conditions specific to lesson study, for example, the role of knowledgeable others, supported that learning. I then wanted to use this experience to describe the role of lesson study facilitator.

Researcher Role and Subjectivities

Peshkin (1988) said, “I have looked for myself where, knowingly or not, I think we all are – and unavoidably belong: in the subjective underbrush of our own research experience” (p. 20). From my very first year of doctoral study, I came to know one of my own “subjective I’s” (p. 18), as he would describe it. My most pronounced subjective I has always been my “teacher-defender I.” As a result, my areas of research have catered to that part of me. This study is not about the judgment of teachers, their roles in the classroom, their understanding of those roles, or any other value-laden opinion about teachers or their classrooms. As selfish as this may be, my research interest has focused on trying to determine what will make me a better teacher because I view myself as just that, a teacher. This study was an in-depth look at what happened to teachers in lesson study, good or bad. The “teacher-defender I” promised the teachers in this study to report it as just that. Therefore, one of my subjectivities could be easily put aside since this study was not about what teachers do or do not do well.

I personally knew two of the eight teachers and had a working relationship with two others. I did not know the remaining four teachers prior to our work together, although all four knew who I was either for personal or professional reasons. I found that these relationships aided in my gaining the trust of the group. I also felt a deep sense of personal commitment to these teachers because of these relationships and because I was a former teacher in the district. I realized that my research was secondary to what I was asked to do for my job with PRISM. I was asked to facilitate the lesson study, outside of this research study. As a result of this opportunity to facilitate the lesson study, I was able to collect data about the process.

Limitations

This study was limited since I worked with only two middle school teacher teams. These teachers were also in the midst of implementing new state standards and a new textbook program. Each of these played a role in the teachers' environment. These conditions placed the teachers under a tremendous amount of added stress. This stress, undoubtedly, was significant in what things the teachers discussed, what was important to them, and what the teachers ultimately learned while participating in lesson study.

My research findings were also limited because the teachers had no previous experience in lesson study. Much of what could be reported as being learned in two cycles is a clearer understanding of the process itself, but that learning was not relevant to my research questions. Moving beyond that into a more reflective understanding of mathematics teaching and learning requires multiple levels of learning to occur simultaneously for the teachers. Although the data showed that this, in fact, did occur, I suspect the rate and depth at which it may occur would be faster and deeper with a group who already knew the process. If I had facilitated lesson study with the same teachers for the year after this study, they would potentially have been much more focused and adamant about the direction of their learning.

CHAPTER 4

ANALYSIS

The purpose of this study was to examine what mathematics teachers learn as they participate in lesson study and what experiences in lesson study, including the interactions of the knowledgeable others and lesson study facilitator, contribute to that learning. This examination of teacher learning may provide insight about the structure of lesson study to afford opportunities for teacher learning at other lesson study sites. In this chapter, I present the data about teacher learning from this study of lesson study. First, I describe the categories of teacher learning observed in the study. Next, I detail the contributions of the knowledgeable others that may have been a factor in the teacher learning. Finally, I account for the ways the lesson study facilitator may have affected the process and the teacher learning.

Teacher Learning in Lesson Study

The data from this study reveal three threads of growth in what and how the teachers thought about mathematics teaching. These threads were identified by the teachers, by me, or by both. Although other instances that suggest teacher learning exist in the data, these threads were chosen for analysis for two reasons: One, each thread is prevalent in the data for the duration of the work; and two, the prevalence of each thread can be linked, at least partly, to certain characteristics, or key experiences, of lesson study. The teachers in this study showed development in their understanding of the following three categories: the role of the teacher in a classroom using a reform curriculum, mathematical knowledge for teaching, and the importance of lesson focus. These threads of growth serve to provide answers for the first two questions of this study:

- What do mathematics teachers learn when they participate in lesson study?
- What do mathematics teachers consider to be key experiences of lesson study that help them learn?

Role of the Teacher in a Classroom using a Reform Curriculum

The teachers in this study struggled from the beginning with their role in the CMP materials, which I called a reform curriculum. By reform curriculum, I mean one designed to facilitate students developing a conceptual understanding of mathematics; one that does not have predominantly skill-driven and review exercises, especially in excess, but rather focus on problem solving with skills and mathematical reasoning; and one that guides the teacher to plan around student mathematical thinking, both developmentally and conceptually. From the CMP training the teachers received during the summer, they had a fairly clear general notion of what the students were supposed to do; the teachers had themselves played the part of the student in the training. They were less clear about what the teacher's responsibilities were. A short exchange between teachers reveals one way the problem surfaced in the fifth planning meeting:

> Bobbie: We don't want to do all the teaching [as we set up the problem at the beginning of class].
>
> Trish (with disbelief in tone): Right, because [the students] are supposed to figure all this stuff out. That'll be exciting.

The conversation above among the teachers, and others similar to it, implied that they thought they either had to tell the students everything or the students had to figure everything out themselves, with no scaffolding between these extremes. It was this comment and others like it that revealed how the teachers were struggling with what their role should be with this nontraditional curriculum. The data that follow are meant to show how the teachers worked through that struggle during our lesson study experience.

In lesson study, the planning team must spend a considerable amount of time anticipating student responses that will occur during the lesson. They also prepare questions for the observers to use throughout the lesson that will guide the observations. Answers to these questions are beyond the scope of what the average teacher can normally achieve while in the act of teaching. They require observers to watch and listen carefully to students, often individual students and for an extended amount of time. The teachers on the planning team must carefully choose these questions for the answers to reveal something about what the students are understanding and learning at that moment in the lesson. While choosing these questions, the teachers in this study spent a lot of time discussing what the students should do in the lesson and how they should be thinking about the mathematics in the lesson. These conversations, inherent to the lesson study process, provided an opportunity for the teachers to discuss their own role in a classroom using a reform curriculum, as I highlight below. These examples show how teachers changed their own actions and roles in the classroom based on how they began to consider student thinking. The subsections included are student models, number choices, teacher guidance, time issues, and student capabilities. These are in no way to be interpreted as reform curriculum issues in general. They simply represent the ideas that the teachers themselves struggled with during this school year. In the data presented in the categories, the teachers ultimately battled with their own roles in teaching the curriculum in the manner they interpreted as being fairly prescribed.

First Cycle

Student models. During the first cycle of lesson study, the lesson goal was for students to use area models to find part of a part (see Appendix D for the lesson plan). This lesson is from a CMP unit. In the problem, brownie pans were being sold at a fair, and partial pans could be purchased. One pan was 2/3 full of brownies, and someone wanted to buy 1/3 of that. In our lesson planning meeting, the teachers drew models for themselves in order to consider how the students may think about their own models so that they could anticipate student responses. Two of the four teachers drew representations similar to the one on the left, and two drew representations similar to the one on the right.

Figure 3: Teacher models

These representations resulted in the first discussion of student models, an important aspect of the reform textbook program. The teachers worried that students would believe 1/3 of 2/3 to be 2/6 if they used the model on the right without extending the segments into what the teachers called the “empty” part of the brownie pan. The team decided to choose a student whose model was in nine pieces, similar to the one on the left, to share with the class. In that way, instead of the teacher’s example, they would be using student work to share, which is often the recommendation in CMP and similar programs, and they would prevent confusion for students who may have misunderstood the model on the right. At that time, the teachers did not discuss how they would negotiate the situation if students asked questions about a model like the one on the right.

During the first teaching, the model was shown after the students worked on the problem. The student explained his reasoning, and the teaching teacher further explained the model after the student finished. The students were then working on the next part of the problem when the teacher saw a student who had not gotten the correct answer for 1/3 of 2/3 and had not extended the segments. The teacher told the student to extend the segments, but without giving an explanation. After the teacher went to another group, the student called the co-teacher to his desk and asked if he was supposed to extend his segments in his drawing. Not knowing the previous conversation, the co-teacher told the student exactly what the teacher had said, also without explanation. When the co-teacher walked away, the student stared at the model in disbelief and after many seconds reluctantly extended his segments.

During the post-lesson discussions, John and Matt, the primary and secondary knowledgeable others, addressed the issue of students clearly not understanding the purpose of the segment extensions into the pan. Matt used this example of student confusion to discuss with the teachers how the whole referred to in the problem shifts from the original pan to the amount of brownies remaining in the pan, and back to the original pan. In the revision meeting, the teachers discussed this idea at length among themselves. They were very disappointed that several students did not seem to understand what had happened when the class discussion moved from the whole being 2/3 of the pan to the whole being the entire pan. For the revised lesson, they decided not to force the students to extend segments, but to allow each of them the opportunity to draw the model that made sense to them.

Number choices. After the first public teaching, the teachers realized the observers could not determine if the students understood how to draw the model representing the amount remaining in the pan. The first problem the class worked together was finding 1/3 of 2/3, and the second problem the students worked with a partner was finding 1/2 of 2/3. The students used 2/3 of a pan in the first problem and then had to use it in the second problem also. The teachers decided to change the first problem for the revised lesson. The revised first problem had the students find a fraction of 3/4 of a pan of brownies, and the second problem had them find a fraction of 2/3 of a pan of brownies. John complemented the teachers on this choice after the second teaching. He said, "In the original plan, the second problem seemed only to allow the students more practice. In the revised plan, the second problem allowed the students further concept development." Other issues surfaced during the second teaching, but the teachers were confident that using student models in a less directive way was an important aspect of the findings of the two public teachings.

Teacher guidance. Jane was a teacher on this team whose comments during planning meetings and interviews are indicative of the evolution I witnessed in the larger group. Throughout our planning meetings in the first cycle, she continued to hold fast to her notion that these students needed to see her teacher model first and then to develop their own models. In discussing how the students would create models of this problem, she said, "I will do the problem with them. I don't think they will understand the model if we don't do the problem as a model." This comment was made at our third planning meeting on September 19. At the fifth planning meeting on October 17, she said, “I’m probably going to model it at this point because this is the launch [problem for CMP]. When we’re working in the problem, I’ll have them show me. But, right now, I’ll model this first so they’ll know what we’re talking about.” As facilitator, I suggested that the teaching teacher use construction paper to cut out a brownie pan and strips that represent thirds. Trish said she liked the idea of the students seeing the pan. Jane said, “I’d probably do another example and draw it so they see it drawn out, because that’s what we’re expecting them to do.” Trish asked Jane if the teaching teacher would solve the problem. And she responded by saying, “It doesn’t say we solve it here, but yes, I’m going to show them what I expect.” At the end of this meeting, Jane said, “I’ll read the [launch problem with them] first…. I’m probably going to do a model so they have a clue of which way to go. This is my experience so far, they don’t do it. They don’t want to try.” Later, on November 14, Jane said, “I’m thinking I want to use different color chalk and use vertical lines as my model, because I want them to see how I want them to do it… so they will see how it will look on their paper.”

The revision meeting for the first lesson was on November 29, immediately following the first teaching and the post-lesson discussion. The teachers were astounded at what they had seen the students do during class. Jane really thought deeply about CMP and how the students, in general, not just in this class, responded to it. She shared with the team an experience during the last week that seemed to go hand in hand with what she was experiencing after seeing the public teaching.

> Kids make connections to learn. It was a big deal this week when a kid realized that 3 ½ times 2 8/9 was almost 3 ½ times 3. I’ve not really seen that before when we were not in CMP. I have always gotten them to estimate first, but it was almost like they were just doing it because they were told to do so. This time, this

> student saw it without me having to tell him to think of an estimate, it just occurred to him. I think that came from drawing those models. He was thinking, "I need 3 ½ almost 3 times."

This comment was the precursor to her even more reflective thoughts in the interview after the first cycle of lesson study was complete. I asked her if she thought about mathematics teaching and mathematics learning differently because of something we did or discussed during lesson study. She replied, "Yes, I thought about the way students show fraction models in a different way. Usually, I walk them through the process and everyone's model looks just like mine." At this point, her reflections seemed to suggest to her that the students may not need the kind of guidance she had been accustomed to giving.

Second Cycle

Time issues. Nevertheless, the first meeting of the second cycle revealed the ever-present and continued struggle of the teachers in a classroom using reform materials. While discussing some of the teacher comments at the end of the first cycle, in preparation for the second cycle, Jane expressed concern about the standardized test that would be administered in April:

> I think we have to work realistically [with regard to] time. I feel really pressured to get certain things taught before April. I know it takes time for students to stop and think, but sometimes you have to take them…where you want them to be, you know, because of time constraints. … I know what is right and what I should do, but I'm always thinking of where I've got to go and about what I've got to do.

This comment set the tone for the second cycle. The teachers were under a great deal of stress: The state-mandated test in April had very high stakes associated with it. The feelings Jane expressed implicitly suggested that too much of the teachers' time, in and out of the classroom, would be used in lesson study and in teaching in this reform manner required by the textbook program. But adapting as always, the teachers were determined to complete the commitment they had made to lesson study.

The goal of the lesson for the second cycle of lesson study was to have students find the percent discount of an item that was on sale (see Appendix E for the lesson plan). At the last meeting prior to the first public teaching of this lesson, the team discussed the evaluation questions they had written. One question was as follows: Were the students able to understand at least one way of solving this problem? I asked the teachers to describe the multiple ways, implied in the evaluation question, that the students may solve the problem. Jane said, "I don't feel like we're giving them a whole lot of flexibility…. Pretty much, we're telling them where we want them to go." She was referring to the ratio that was given in CMP: amount off / regular price. The team decided to go forward with the lesson as it was written but changed the goal to reflect the fact that multiple ways of solving the problem were not expected. There was a sense from the conversation at this point that because this was the last scheduled meeting before the public teaching, there was not time to make greater changes in the structure of the lesson. Nevertheless and although it was apparent in the first cycle that Jane wanted to direct the students' models, her actions at this meeting in the second cycle made it clear that she was not content with how the lesson had unfolded, especially with regard to the lack of opportunities the students would be given to construct meaning.

During the post-lesson discussion for the first lesson, the teachers were considering what to change for the revised teaching of this lesson, and Jane's concern was even stronger. "I wonder if [working backward] would be a good approach to this problem instead of just throwing…how did they get this 'amount off / regular price'?

How could that connect somehow better?" she asked emphatically. Jane suggested that they go back to what the students had been doing in the entire unit, that is, finding a percent of a number. The teachers soon saw where she was going with her suggestions, and they excitedly began reconstructing the lesson. They focused the lesson on the algebraic equation – percent discount * regular price = amount off – which is what the students had been doing. They discussed how some students might use this equation with a blank where percent discount is and guess what the number should be. They also thought some students would use knowledge of fact families (If 2 * 3 = 6, then 6 / 2 = 3 and 6 / 3 = 2) to write the division problem. They even suggested some students might consider the sale price instead of the amount off to find the solution. The teachers were much more satisfied after the revised teaching that they had gathered evidence of student understanding rather than evidence that the students were just manipulating something that was given to them.

In the post-lesson discussion for the second teaching, Jane articulated again the tension between what she thought to be right for students and what she was constrained to do:

> I really think I need to give kids more time to discover things on their own. But, I'm so stressed about time. How do I balance that? I loved seeing what the kids did themselves, but sometimes it takes so long. I don't know what to do or how to get things done.

In the interview after the second cycle was complete, Jane's responses suggested that she was searching for that balance outside of lesson study. She admitted that she now "spends more time listening to a student's way of thinking instead of making them settle on doing things [her] way." She concluded that "we do students a disservice when we lead them too much."

Student capabilities. Bobbie wrote about her own learning at length after the first public teaching:

> I realized … that what I wanted to talk about yesterday after school was teaching math. Not about discipline problems, not a funny story about a kid, not frustration with what these kids don't know, but really about what the students really understand, how they think, and how we can help them construct meaning from it. … Our public teach[ing] has given me a new way to think about what our students can do, and what we do to our students when we just lead them where we want them to go without figuring out what they're thinking first, and letting them have some time to think and struggle on their own.

In a discussion after the second cycle of lesson study was complete, the teachers communicated how the public teaching itself widened the lens through which they saw the teaching practice. Each teacher, continuing from where the one before left off, seemed to be reshaping how he or she saw the teacher interacting with students from the position of observer during the class.

> Jane: Watching other teachers with their students makes me more aware of how students actually learn things. When I'm teaching, I'm so worried about "did I get the concept, did this happen, did that happen," that sitting back and watching while you were doing it, I was able to think more about the whole process, the numbers, the time for students to think things through.
>
> Bobbie: Are we giving them too much time? Too much help? Where do we give them help? How that can sometimes hinder and enable them. But when you're watching someone else, and you're thinking from the point of view of this kid that is sitting over here….When I'm in the classroom, even with two of us in the room, I don't think that way, I don't see that way.
>
> Amy: Or what would have happened if you hadn't given them so much, and would they eventually have gotten it?

The teachers showed obvious disequilibrium in the organization of their structure of the role of the teacher. These comments reveal how they were reflecting on the source of the disequilibrium and trying to reorganize that structure.

Summary

Each of these examples – student models, number choices, teacher guidance, time issues, and student capabilities – became part of the teachers' discussions as a result of the process inherent in lesson study: teachers anticipating student responses in the lesson, therefore deeply considering student thinking; teachers determining evaluation questions for the observers, therefore conceptualizing how observers will know about student thinking; teachers observing the lesson they are so intimately a part of, therefore critically deciding about flexibility of student thinking. These instances were vital to the post-lesson discussions after the public teachings and were central to many of John's comments.

Mathematical Knowledge for Teaching

The intimate, detailed planning of a lesson in such a visible and audible manner provided an environment that required the teachers to arrive at common understandings of mathematics teaching, evidence of the teachers' growth in their mathematical knowledge for teaching. As the teachers immersed themselves in the lesson, new and developing notions of common content knowledge, specialized content knowledge, and pedagogical content knowledge began to develop for the group and for the individual teachers.

Common Content Knowledge

Understanding the whole in fraction problems. In the brownie pan investigation from CMP that is discussed above, the teachers initially struggled to make sense of their own models. The idea that students would not understand whether 1/3 of 2/3 was 2/6 or

2/9 emerged because the teachers could not explain why both models the teachers drew could represent 2/9. Figure 4 is the same as Figure 3 shown earlier in the chapter.

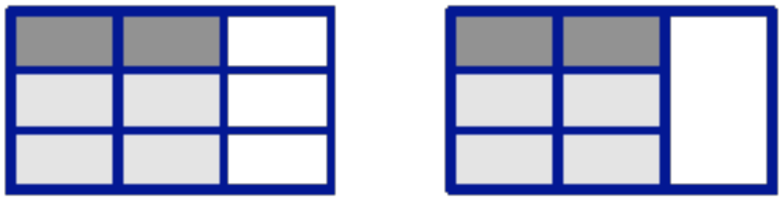

Figure 4: Teacher models

During one discussion, Trish looked at her model and thought the model was incorrect, because she knew the answer was 2/9. She said, pointing to the picture on the right, "But, here, this is 2/6 of the pan but not 2/6 of what's left…. It asks what part of the whole pan you bought. Am I doing that wrong?" Bobbie said, "Your answer is going to be what part of the pan you buy…. No, wait, I don't know, I'm not sure." Jane said, "I think you are talking about two different things." After a bit more work Trish asked, "Aren't these two different questions?" The teachers then reworked the problem from the beginning, talking through the steps as they went. The content that was at issue was not finding 1/3 of 2/3, but properly representing and understanding multiple models. I interpreted this discussion to be about common content knowledge, since the challenge was in understanding the whole at different stages of the problem. The teacher who created the model was uncertain about the model representing 2/9. That challenge was not about specialized mathematics used in teaching, because if a student had drawn the same model and interpreted it as 2/6, that student would have a misunderstanding of content knowledge. Therefore, I have labeled this episode as common content knowledge. The teachers were discussing their own models, not the students. However, in the upcoming section on specialized content knowledge, I describe how it became a challenge in specialized content knowledge when the focus of the conversation turned to how the use of these models would unfold in teaching.

In the post-lesson discussion after the first teaching, Matt, who was a mathematician from a university, commented on the challenge.

> [The concept of] fractions, in general, is a very hard concept to teach and to think about ourselves. It's important to say a fraction of what. We could draw different pictures to show the different steps instead of superimposing [pictures] on top of each other. So, show the pan in thirds first (see figure 5). Then show just 2/3 (see figure 6). Then show 1/3 of that (see figure 7).

Matt drew the figures 5, 6, and 7 as he discussed this with the teachers.

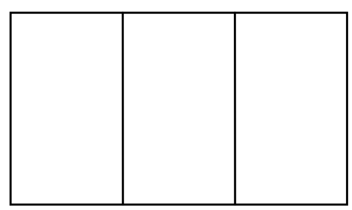

Figure 5: Matt's brownie pan showing thirds

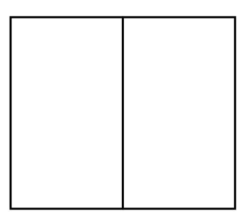

Figure 6: Matt's brownie pan showing 2/3 of the pan

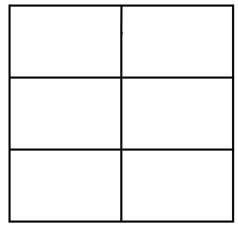

Figure 7: Matt's brownie pan showing thirds of 2/3

These pictures show the different wholes during the different stages of the problem. Using different pictures, instead of shading on top of the original, was not how the teachers worked through the mathematics during the planning meetings; it was from the collaboration with a mathematician that the teachers were exposed to this simple and powerful suggestion. The root of the common content problem was in understanding what the whole was in different stages of the problem. This discussion provided further evidence for the teachers to understand what had previously been discussed.

Understanding mathematical definitions. The goal of a different lesson was to have students explore the use of the order of operations by thinking about rational numbers and how they work (see Appendix F for the lesson plan). At the fourth of seven planning meetings for this lesson, I asked the teachers to write on a piece of paper what a rational number was, either formally or informally. The reported answers were as follows:

- A number that can be written as a / b, where a and b are integers.
- Positive and negative integers.
- Any number that can be written as a fraction.

After the team agreed on the formal definition, the teachers were then asked for examples of rational numbers. Two of the four teachers gave these examples: -2/9, 0.75, 50, and 3, and the other two teachers did not provide examples. The teachers were then asked for examples of numbers that were not rational. One teacher answered pi, and there were no other answers. I then asked about specific numbers.

Facilitator: What about square root of 4?

Teachers: Rational.

Facilitator: Why?

Annie: Because it can be written as a fraction.

Facilitator: What about square root of 8?

Pat: Rational.

Facilitator: Why?

Pat: It can be written as a fraction, also.

Facilitator: What is square root of 8 as a fraction?

Pat: 2 square roots of 2 over…1? How do you get the square root of 2 out?

Later, at the sixth planning meeting for the same lesson, problems with the definition were still evident. The activity in this lesson provided students with four integers and asked them to use the numbers once and create the largest possible value using operations only once. I asked if the teachers could foresee any problems with answers the students might give. One teacher replied, "Division. They could get fractions, so they may not get a rational number." The team again worked through the definition as a group, having an opportunity to develop a deeper understanding about sets of numbers in general.

Understanding the use of parenthesis. In the same lesson, the teachers were discussing the use of parentheses in these expressions with four integers. They were writing anticipated student responses and discussing what students would be thinking about to use the negative integers to create a large positive number. The teachers were trying to make a decision about calculators since the integers could be used as exponents. I suggested a problem that might occur with the use of calculators. If a student entered "-3 squared" instead of "(-3) squared," the answer would not be what the student expected. All the teachers confessed that they did not know the two expressions were different. As they thought about it and discussed what had happened in previous questions, Ruth said, "It really makes sense, if you think about it. None of us would have questioned the difference between '4 – 3 squared' and '(4 – 3) squared.'"

Specialized Content Knowledge

Understanding the mathematics of student models. The team that taught the brownie pan problem worked through the mathematics of that problem, as illustrated above in the Common Content Knowledge section. At the last meeting before the first public teaching, the discussion about that mathematics had developed into an opportunity to articulate and mold specialized content knowledge. I asked the group how the teaching teacher should respond when a student determines that a customer buying 1/3 of what remains in a pan that is 2/3 full is buying 2/6 of the pan. One teacher replied, "You say, 'Yes, it is 1/3 of the brownies that are in the pan, but what fraction of the whole pan did he buy?'" At this point, the discussion recorded earlier as common content knowledge became specialized content knowledge because the teachers were using their content knowledge to determine how to unpack or decompress the knowledge to make the content "visible to and learnable by students" (Ball, Thames, & Phelps, 2007, p. 35).

Use of mathematical notation with students. The goal for a lesson in the second cycle was for students to write and solve equations using the properties of equality to solve and check solutions. On the day before the public teaching, the students were given pictures of coins, pouches with an unknown number of coins, an equal sign, and a total for the coins and pouches. The students used the pictures to determine the unknown number of coins in the pouches. On the day of the public teaching, the students were asked to use similar pictures to write and solve equations.

During the last planning meeting before the public teaching, Luke was present. The teachers were going through the lesson, making final comments and small changes to prepare for the teaching.

> Pat: Are we using *x* for pouches? [The lesson] says assign *x* [as the variable], these kids will be writing *b*'s,…
>
> Olivia: These kids will want to write *p* for pouch, *c* for coin, …
>
> Annie: I think the kids understand that a variable can be a different letter and represent the same quantity. Some people use *c* for cost, *m* for money, *d* for dollars, and they all can get the correct answer.
>
> Luke: If they use *p* for pouches, 5*p* might be 5 pouches. But, they really want to think of *p*, not as the pouch, but as the unknown number of coins in a pouch. Somehow, you need to speak to that: *p* is not the number of pouches. [In this problem,] 5 is the number of pouches.

This clearly shows a discussion of mathematics, influenced by possible student misconceptions, which influenced the decisions the teachers made during the lesson. After Luke's statement, the teachers actually fell silent momentarily, taking in what had just been said and possibly thinking through similar episodes from other classes. The notion of letters simply representing quantities and letter choice being just that, a choice, was so ingrained in the teachers' thinking that Luke's comment provided an opportunity for the teachers to decompress that knowledge. One teacher wrote this in her interview questions: "Before lesson study, I did what I thought was best, not necessarily what was best for the students. I just concentrated on the mathematics I was teaching at the moment instead of looking more deeply."

Mathematically accurate. In their written interview responses, five of the eight teachers reflected on how they thought about their own actions in class after the experience of lesson study.

- When students have new methods, I always think, can we do that? Why does that work?
- I have thought more about how it is more important for students to have math make sense to them rather than follow our "normal" algorithms.
- It is very important to choose the numbers used in a problem. They can be manipulated to scaffold student learning in many ways.
- I learned that I should not use a standard equal sign while checking work.
- I really feel the importance of vocabulary now.

Pedagogical Content Knowledge

Productive Student Groups for Mathematics. Trish commented after the first public teaching of the first cycle: "After seeing that class, I did the desks in my class like Jane did." Trish was convinced that this small adjustment of group size had provided many more opportunities for the students. On a different team, Pat questioned the size of the groups before the first teaching. She said she had tried it numerous ways, but she wanted to hear different perspectives. The collaborative nature of lesson study afforded these teachers an opportunity to contemplate possibilities.

Student groups dominating teacher's time. During the post-lesson discussions for both grade levels, a recurring theme was the conversations of the teaching teacher with individuals or pairs. The teachers claimed that having students think through the problems in CMP alone or in small groups was exhausting to the teacher since the teacher spent all her time running from one group to the next answering questions and guiding progress. The teachers were convinced that the students were not independent thinkers.

After seeing the public teachings, the teachers witnessed from a new perspective what it looks like for a teacher to allow herself or himself to be forced to constantly move from group to group to group. John presented options for the teachers to consider about how to involve all students when one small group has discussions that should be shared. Teacher interviews after the first cycle produced these responses from 4 teachers:

- I am more aware of the number of students waiting on a teacher for help and therefore not working.
- It is more important to make more of our group conversation public, even if it means stopping students from working earlier and having a class discussion.
- This really brought to my attention how much time I spend guiding individual groups.
- I'm listening more to students as they work together. I'm requiring students to work with a partner and hold questions for me for a given number of minutes. I want the kids to explain and question each other first – or at least try to come up with a strategy and/or answer.

At our first meeting of the second cycle, Pat said, "I was thinking about [how much the students need me during group time] from the lesson study public teachings…. I am thinking, 'Do they truly need help? Or is it more of just wanting to check work, or is it a habit?'" She told the students that she was going to discuss constructions with them and then let them do some constructions with a partner, and she would not be available to help them. Pat said she had done that for 3 days and that, even after such a short period of time, the students were taking ownership of their learning, based on how often she was being called to their desks for help. She said the students' work, completed with partners with very little intervention from Pat, revealed that they were learning how to correctly perform the constructions.

Teacher moves to structure embedded skill practice. The teachers also frequently mentioned that CMP did not provide enough practice for students. Knowing there was insufficient practice did not solve the problem for the teachers, since they were already overwhelmed with the new standards from the state and the new reform textbook. Frustrated by the post-lesson discussion of the second cycle and the state-mandated standardized test looming on the horizon, Jane appeared beaten when she said that her sixth graders were so weak in simple division that it made many CMP lessons futile. Rhetorically, she asked, "Do we just stop teaching the concepts and practice [instead]?" This led to powerful input from John about how to think about practice, when to have the practice, and how to structure the practice so that it would be focused enough on one chunk of content. He told the teachers to determine in advance where the students were lacking in knowledge in division. He told them to construct only a few problems targeting that one part of the skill of division, spending very few minutes of a day's lesson on it. He explained that students would then have the opportunity to make incremental and correct changes in their procedural fluency. "[There is] danger of practicing too soon, then [students] will only perfect their imperfections."

Summary

These examples show growth in common content knowledge, specialized content knowledge, and pedagogical content knowledge. The data showed that in the thorough and detailed planning and the discussions about the planning, the teachers were provided a forum for discussion about mathematical knowledge for teaching.

Lesson Focus

During the first cycle for the sixth-grade team, the lesson goal was for students to use area models to find part of a part. As mentioned before, the teachers used a lesson from CMP. These teachers were implementing CMP for the first time, having been trained to use it during the summer. The implementation of this reform curriculum was difficult, as the teachers knew it would be, and they were trying very hard to remain true to the program. For instance, as the teachers were truly beginning to plan the lesson in the third planning meeting, I asked if the teachers were going to use all the parts of the investigation. Trish replied, "Yes, we need to because [the program] builds so much." The teachers were further developing the lesson in the fifth meeting, describing to each other the specific steps of the investigation. Amy read the problem to the group, saying 2/3 of the brownies were left in the pan and a customer wanted to buy 1/3 of that. I asked if they wanted to use 1/3, when the students had already divided the pan into thirds for the first part of the problem. Trish said, "Well, that's what it says." Acknowledging that CMP did use 1/3, I asked if the teachers really wanted to go with the number used in CMP. Jane and Trish emphatically and without hesitation said, "Yes!" This example shows how important it was to the teachers that they remain loyal to the program in the sense that they not make choices about anything that could be otherwise changed.

In the CMP lesson, after the students found part of a part (1/3 of 2/3), they had to calculate what the fraction was of an integer (2/9 of 12). The customer wanted to buy 1/3 of the 2/3 of the brownie pan that remained. The cost of a full pan was $12. In the last meeting before the public teaching, the team was reviewing the plan to determine if it had the right flow and if it was a reflection of their intentions. I asked the group, "Do you want to include this part of the problem about the cost? Does it achieve what you want it to achieve? Or, do you think it's a good break to stop and think about a fraction of a whole number?" The question was motivated by John's suggestion that this portion of the lesson could be taken out, since it did not contribute to the lesson goal.

Trish: That doesn't have anything to do with the goal.

Jane: Where do you think [the students] will get confused?

Bobbie: 2/9 of 12, because to break 12 into 9 parts is not easy.

Jane: But, we've been doing that a bunch, and it doesn't always work out evenly.

Bobbie: That's true.

The teachers agreed to leave this portion dealing with a fraction of an integer in the plan.

The first lesson was taught, and the teaching teacher did not have enough time to complete the plan. A considerable amount of the revision meeting was about how to shorten the lesson in order to complete it during the class period. John suggested that part of the issue may have been that the students had two major tasks: draw part of a part and figure out how much to pay. He claimed these were two very different ideas and suggested that the teachers reconsider the focus of the lesson. He made the point that in mathematics classes, teachers often want students to solve the whole problem, because the teachers find the problem interesting and challenging. However, trying to do too much in a problem may just slow down how, or if, the teacher achieves the lesson goal. Nevertheless, finding the cost of the pan was not taken out of the revised lesson planned for the next day. The teachers did not find John's claims compelling enough to make the change.

A similar example of narrowing the lesson to align with the goal became an issue for this team in the second cycle. The goal for the lesson was for students to find the percent discount given the regular price and the discounted price of an item. One of the last problems had the students estimate, according to the directions, the percent discount of an item that was discounted to $8.00 and was regularly $8.50. The team discussed what was intended by the word *estimate* for this problem. The students would begin with the ratio .50/8.50 and simplify this to 1/17. According to the CMP teacher edition, the students would then divide, and convert the quotient to a percent. The teachers thought about reasons for this choice and decided to leave it as it was written for the students, again alluding to the teachers' desire to follow the program faithfully. However, they based this decision on the fact that their students were notoriously lacking in computation skills, and this activity would be a source of practice within another problem to work on the skill of division.

The first public teaching became almost painful to watch as the students struggled through the long division of 50 ÷ 850 or 1 ÷ 17. The teachers found this struggle to be evidence affirming their earlier statement about the students' weaknesses and also confirming the need for practice.

Jane: The thing that came out of this for me is the problem with division.

Amy: Yes, 1/17 was so hard.

Trish: They don't get the place value.

Bobbie: I feel like they understand place value in isolation, but I don't know how that is in a problem where they have to use it.
...

John: In the way CMP presented the lesson, it is not quiet clear what the main goal is.

Jane: You think, since division is weak for us, that it is an opportunity…

John: Even in that case, you probably don't want to get into 1 divided by 17.

The discussion continued as the teachers thought about how they should handle this seeming dichotomy. On the one hand, students needed practice with division. And, on the other, when students are given practice in the context of another problem, progress toward a new concept grinds to halt in order to address, yet again, problems of computation skills, specifically division. The discussion in the meeting continued in the same circular fashion that had caused problems for the students during the lesson.

The point seemed lost until the very end of the second cycle. During the final group interviews, the teachers discussed how important the lesson focus had become. They referenced the first cycle, when the fraction of 12 proved not to be related to the goal, and the second cycle, when the 1/17 division problem proved not to be related to the goal. The teachers questioned other lessons that might have not been keenly focused. These comments seemed to pull together the experiences in the two lesson study cycles of John's suggestions about the cost of the part of a brownie pan in the first cycle and the long division of 1/17 in the second cycle.

> Jane: One of the things that I consider when planning a lesson now (after lesson study) is the type of questions I ask. I also try to make sure the numbers I use in a model or example actually address the standard I am addressing.

> Trish: When planning by myself, I find now I am asking myself if these are the best numbers and situations for the students to solve. I wonder if the students will be able to take the "launch" and then solve the problems. I learned to focus more on the actual goal of the lesson. I now think about almost everything I say during class. Did I say that the correct way for the student to be able to understand what I expect? I concentrate more on making my lessons better, presented in a way to help reach the lesson goal.

These types of comments spilled into the interviews of the other team after the first cycle for one teacher.

> Ruth: I have been thinking about the point of the lesson a lot more since lesson study, the point we're trying to make in both the teaching and the assessing of information. Outside of lesson study, I focus on exactly what we're trying to teach, regardless of the way we do it. I try to really know what the point of the lesson is and to think about how students will react to it.

And in the reflections after the second cycle, more teachers discussed the importance of lesson focus.

> Ruth: I definitely spend more time planning for what I really want the students to know now. I have brought it up in every planning period I've had with other teachers. What are we really assessing? What are our goals for the information? Why is this important?

> Annie: Although I reflect a bit more after the fact since lesson study, I find myself planning more thoroughly before the lesson. What do I really want kids to know at the end of the class is the question that seems to be taking center stage.

> Trish: It was really important to me to see how we tried it one way, and we changed it for the second teaching to try it a different way. It makes me think about the numbers more. I'm thinking about 1/17. We did that fraction from CMP and then we changed it. So, it makes you know that everything written in a program is not always what you want to use.

> Bobbie: It made me look at some of the problems differently, the number choices of the program, and how that affects student learning.

Teachers learned to be critical of details of class. They frequently asked these and similar questions during planning meetings: What do I want them to leave with? What will this question tell me about their understanding? They also reported they asked those questions of themselves when planning alone after completing lesson study.

Summary

The teachers spent so much time in planning meetings, consciously thinking about and explicitly stating reasons for the choices they were making, that when they changed one of those choices after seeing the first public teaching in preparation for the second, it was powerful to see the effect of that choice. The data above that shows the teachers' growth were gathered as the team reflected about the intent of the lesson and what the teachers observed in the lesson.

The Role of the Knowledgeable Others

Showing what the teachers articulated about the role of the knowledgeable others and also what the data reveals about those roles addresses the next two questions:

- How do mathematics teachers describe the role of the knowledgeable other in lesson study?
- How does the role of the knowledgeable other influence the learning of the mathematics teachers?

I use the data in this study to supply evidence that the knowledgeable others provided the teachers with (1) a different perspective when reacting to the lesson study work of the group and (2) information about mathematical knowledge for teaching. These align with some of the duties of the knowledgeable other as reported by Fernandez, Yoshida, Chokshi, and Cannon (2001).

Different Perspective

During the sixth-grade post-lesson discussion of the first teaching of the brownie problem, John gave a summary of the student models he saw. When the teachers reflected on the models the students had used, they expressed excitement about what the students had done. Amy exclaimed, “I can’t believe what they came up with on their own!”

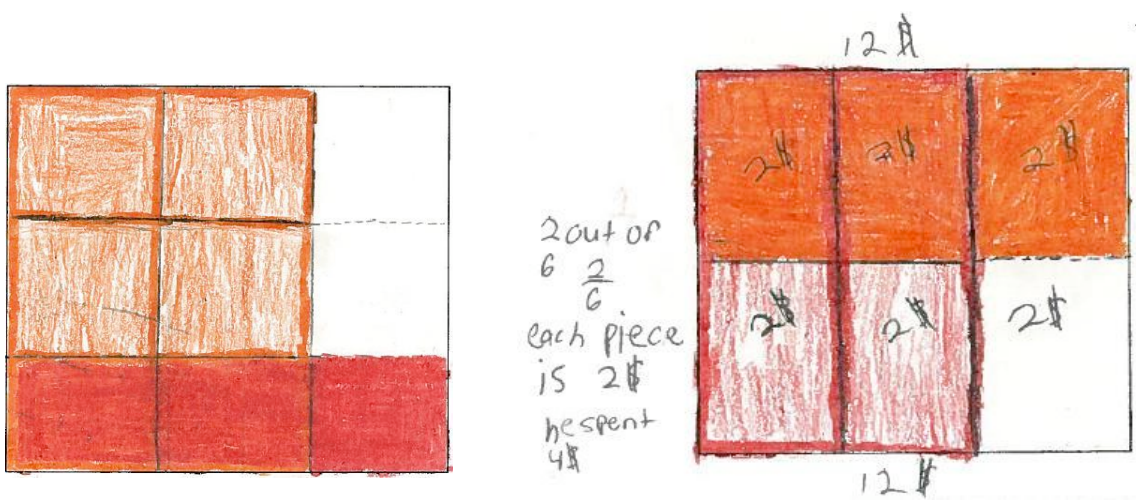

Figure 8: Brownie problem student 1 model, problem 1 (1/3 of 2/3)
and problem 2 (1/2 of 2/3) – first teaching

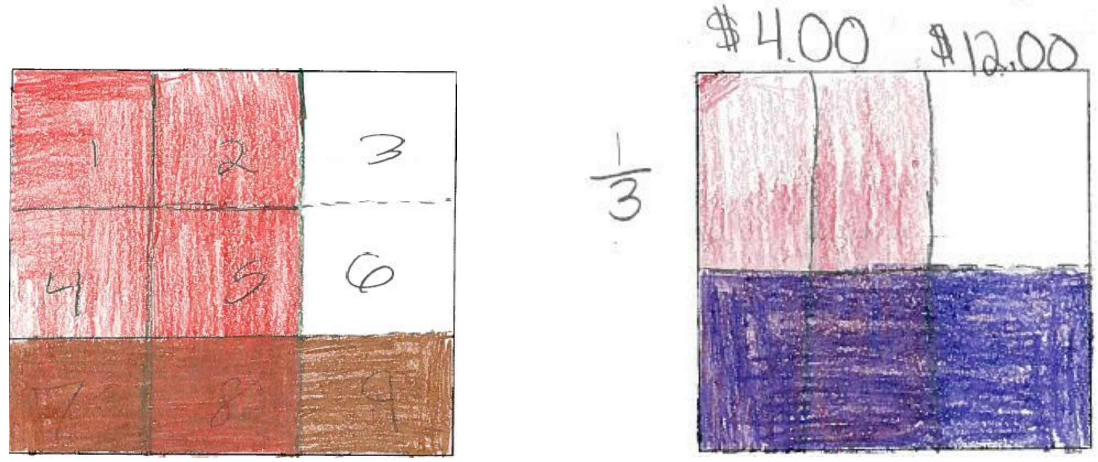

Figure 9: Brownie problem student 2 model, problem 1 (1/3 of 2/3)
and problem 2 (1/2 of 2/3) – first teaching

During the post-lesson discussion of the revised teaching, John again began with a summary of the student models he saw. He elaborated on how the models from the two lessons were very different, largely because the teachers had revised one of the problems, so 2/3 of a pan of brownies was used in the first teaching, and 3/4 of a pan was used in the revised teaching.

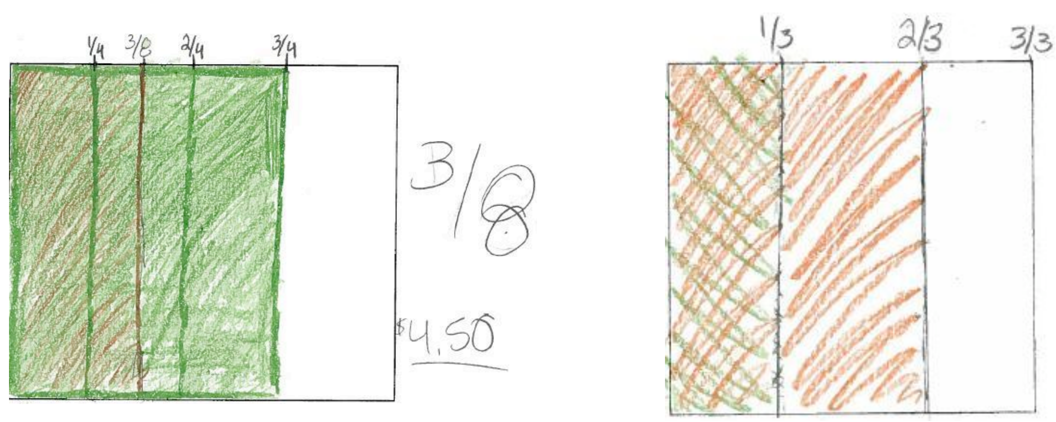

Figure 10: Brownie problem student 3 model, problem 1 (1/2 of 3/4)
and problem 2 (1/2 of 2/3) – second teaching

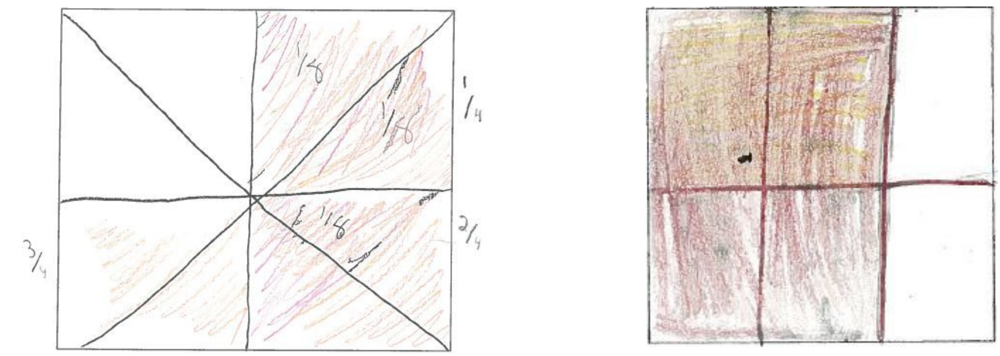

Figure 11: Brownie problem student 4 model, problem 1 (1/2 of 3/4)
and problem 2 (1/2 of 2/3) – second teaching

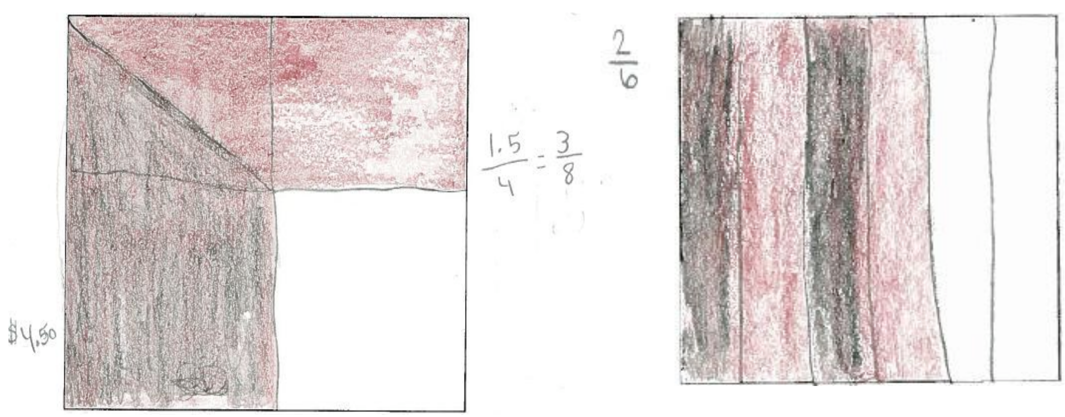

Figure 12: Brownie problem student 5 model, problem 1 (1/2 of 3/4)
and problem 2 (1/2 of 2/3) – second teaching

However, the teachers were again shocked by the vastly different ways the students tackled the problem in fourths. In addition, they were disappointed because when they had anticipated what the student models would look like, they did not produce one of the models that many of the students used. This made it difficult for the teaching teacher, because she was not prepared for the unique representation (1 ½ fourths as shown in figure 9).

John talked with the teachers about these different student ideas. He proposed that the student ideas could be used to guide the students as they learn mathematics. He urged the teachers to consider in the future posting the models around the room during a lesson, providing an opportunity during class to put otherwise private conversations in the public domain. He described several "rich conversations" that the students had with the teacher, but these conversations were confined to pairs of students working together. "We must find a way to make these rich, private conversations public," he said. The experience and conversation provided an opportunity for the teachers to reflect further on the importance of the models in the students' construction of knowledge, and not just their own knowledge, and they began to think about how one student's model might facilitate another student's understanding. As evidenced by the comments from Jane in the teacher learning section above, prior to these discussions, the teachers seemed to view the student models more as an activity to engage the students than as a means for students to use to develop understanding. The progression of comments from Jane showed how her thoughts, in particular, evolved during the study.

For the seventh-grade team, John's comments during the post-lesson discussion for the first teaching were ones the teachers reflected on. As stated earlier, this lesson had students use a particular group of 4 numbers to create the largest integer, using the numbers and operations only once. John began his commentary with two questions: "What is it the students needed to learn in this lesson? What kinds of ways of looking at numbers did you mean in the goal?" This comment opened the floor for the teachers to, again, narrow the lesson and determine what outcomes they wanted from the students in the revised lesson. In the revised lesson, the teachers added questions for the students to consider in order to guide them to think about the effects of changing numbers or operations on the mathematical expressions. They added questions to the plan like these: “What did Wendy do that made her answer so much larger?” “How did we end up with a positive number when we have negatives in our expression?”

As a result, the students in the revised lesson seemed to move much further in their thinking. Several students did not want to leave the class when it ended because they were in a discussion about which would produce the bigger number: increasing the magnitude of the base and leaving the exponent fixed or increasing the magnitude of the exponent and leaving the base fixed. During the post-lesson discussion of the revised lesson, an outside observer said, “Today’s lesson really did have a conclusion! You felt the kids left with new knowledge!” This may seem like a natural consequence of teaching, but it is not necessarily an easily obtained goal. The teachers on this team were ecstatic because the lesson was contained within the class period, since they had run out of time in the first teaching. And they were ecstatic because they could actually place a finger on what the students learned. They could not do that in the first teaching. This was directly linked to the revision and the comments from John in his original commentary.

John constantly reminded the teachers on both teams that a focused lesson was important when developing new mathematical content. He commented on it in his emails about initial iterations of the plans, in post-lesson discussions, and in revision meetings. He pushed the teachers on this point to force them to make choices about the lesson that teachers often leave to the textbook. As apparent in comments from each of the teachers, in the above sections and in the comments that follow, this idea was very powerful for them.

The teachers reflected on their interactions with the knowledgeable others in this project, and five of the eight shared the following written responses (three of the teachers did not respond directly about their interactions with the knowledgeable others):

- I think the knowledgeable other brings a "fresh look" at the lesson and an objective thought process to the teaching / learning aspect of the lesson and the discussion afterward. As part of the lesson study planning team, it was a little more difficult to be objective about the lesson. The knowledgeable other was able to help focus our conversation and bring out many interesting points.

- I think the role of the advisor was to offer advice on how we could improve the lesson and our teaching strategies. However, I think some ideas may work for some students but not for all.

- The knowledgeable other guided us and provided insight as to how our kids are thinking. He offered valuable comments as to where the kids need to go with respect to next week, later in the year, and beyond. He stressed the importance of how students should begin to think that will allow for greater achievement later in their education. He was supportive yet made us think critically about how we can improve our instruction. The end result will be our kids achieving a greater level of success in mathematics. He was invaluable, in my opinion.

- I saw his role as helping us focus on what it was we wanted our students to be able to do. He was also very good at providing situations that would help us lead students in the direction we wanted them to go, but also leave us with situations we could continue with, if we wanted. He was excellent in helping us see the big picture in terms of how what we are doing will be used later on in higher mathematics.

- The knowledgeable other has experience in what we're trying to accomplish with lesson study. He is another set of eyes to see and hear what the kids are saying and doing, and he is an experienced teacher who may be able to give us ideas and activities that we had never thought of.

Mathematical Knowledge for Teaching

In the brownie problem, the teachers had a discussion about the mathematics during the planning meetings. They worked and reworked the problem together, focusing on one model that appeared to show 2/6 and another model that showed 2/9. They spent quite a bit of time discussing these two models in the planning meetings. In the post-lesson discussion, Matt wanted to discuss what happened with the students' models. He was unaware of the amount of time the teachers had spent working on the problem in the planning meetings, sorting out their own adult mathematics, or common content knowledge. However, even though he was unaware of the previous conversations, it was his contributions at this time that morphed the common content knowledge into specialized content knowledge. Although his commentary on the problem was still beneficial for the teachers' own mathematics, he weaved into the conversation choices that the teachers could make to better guide how the students needed to think about that mathematical content. Three distinct pictures, instead of one picture with three different phases drawn on it, was one suggestion he provided for the teachers. These comments led the teachers to make the revisions in the lesson for the second teaching that have already been discussed.

In the percent discount problem, John asked the teachers in the post-lesson discussion, "Would the students have been able to find the percentage based on the sale price and subtract that from 100 to find the percent discount? ... We didn't mention how those numbers relate. … Especially if that is what you did previously. … It might be helpful for them to make that connection." This comment seemed to be the impetus for Jane to express her concern and begin to change the revised lesson to be focused on how they had already been working with percents. Again, this seems like an obvious choice for a teacher to make, that is, to connect the lesson to the previous ones, but the CMP materials did not make that connection. As a result, making that connection did not occur naturally to the teachers, they were trying to "stick to the program." It was only after John's comments on the first lesson that Jane began to conceptualize what had been bothering her throughout the planning of the lesson.

Luke posed two situations to the seventh-grade teachers that the teachers revealed to be pivotal in their work. During the planning meeting immediately prior to the first teaching, he discussed with them the possible problems that may result in allowing students to choose, as he said it, "*p* for pouch or *b* for bag." This situation is described above. Additionally, in the post-lesson discussion after the first public teaching, the observers were discussing how the teachers intended the students to solve the equations mathematically. He asked the teachers, "Do [the students] understand that $4x/4$ is the same as 1/4 of $4x$? … That seems to me to be a critical conceptual thing [in this discussion]." One of the teachers commented at the end of this discussion that she had never thought about $4x/4$ as 1/4 of $4x$. But she said she thought it would make so much more sense for students if we tried to help them see it that way. She commented again about this same idea in her final interview. "I know that sounds so simple, and I'm almost embarrassed to admit it, but that simple phrase struck a chord with me. I never thought of it that way – I never made it that simple."

These are written reflections from five of the eight teachers that highlight the conversations that involved content (the other three teachers did not comment):

- I enjoyed hearing the discussion from the professors. It really gave a fresh perspective on the math content in relation to the daily grind of teaching and reaching the students.

- They made me think about the numbers which were used in the lesson. I now ask myself what numbers would make the launch more transferable into the lesson. I think that was very evident in the brownie pan problem.

- At times their contributions helped me see another way of looking at a problem or how a student might look at a problem. Other times, I felt as if they were giving advice without knowing the full story, without seeing the complete picture.

- They gave us a lot to think about down the road as well as for current lessons. All contributors stressed the importance of vocabulary.

- They made me think about what I say and how I say things. I try to be more specific in things that I say. I think about if what I am to say is really true mathematically before I say it. For example, if we say this will always work or this will never work, we make sure we try it before we say that. We also think of different ways to approach problems so that we present as many ways to solve a problem as we can.

The Role of the Lesson Study Facilitator

In this section, I use the data to address the last question of this study:

- What is the role of the lesson study facilitator?

The seventh-grade teachers chose the lesson they wanted to teach in CMP at the beginning of the first cycle for many reasons. That particular lesson was scheduled to occur at the time we targeted for the public teaching, and they enjoyed the lesson themselves when they did it at the CMP training during the summer. At the third meeting, I began discussion about the goal for the CMP lesson versus the state standards the teachers intended to address. The CMP lesson was focused on teaching students about order of operations, which was a standard for fourth grade. In an effort to correct the problem of a curriculum that had too many standards, the state had not only reduced the actual quantity of standards, but also deliberately coached teachers to focus on their own grade-level standards so they would address them at an appropriate depth. Discussing this with the teachers, I asked them what things they could change about the lesson to refocus it on a standard from this grade level. The teachers talked to each other for several minutes, going back and forth in conversation about what should be taught, what the lesson was teaching, what was important, what was not important. Finally, Olivia, who had been silent, spoke with frustration in her voice. “I don’t understand why this is so hard. Am I missing something? We’re trying to figure out what to teach, right?”

In the interview at the end of the second cycle, Olivia said, “I feel like you [the lesson study facilitator] would ask a question, and we wouldn’t say anything, like we didn’t know where we were supposed to be going, what we were supposed to be thinking.” Annie said at that time, “We presented a lot of questions, and we didn’t have answers.” Pat agreed saying, “Sometimes I felt like I would leave our meetings confused.”

These comments give evidence of my own struggle that mirrored how the teachers struggled to understand their role in a classroom with a reform textbook program. I struggled to understand where I should scaffold the teachers in this reform professional development model, while still giving them space to make their own decisions, without “giving them the answer that I wanted.” There were many times I wanted to weigh in on the discussions with my own ideas, and many times that I did. But I viewed that as a dangerous act because it could have moved me into the role of the superior knowledge source, as having the right answer to any question, and that the questions themselves were just exercises. I believed that if the teachers saw me as having the answers they sought, they would spend our time trying to guess what I wanted them to say. I worked through the process trying to maintain that these were decisions the teachers needed to make without my input.

John served as my knowledgeable other for my role as facilitator, especially since he had a vast and varied amount of expertise in lesson study. I communicated to him in an email 2 months after we began planning meetings that I needed help understanding how I could get the teachers to focus on the overarching lesson study goal that was to make students be more responsible for their own learning. He replied as follows:

> Perhaps you may want to ask them if they can describe what those students who take responsibility of their own learning would look like/act like, and how those who don't take that responsibility would look/act like. Perhaps you may need to prompt them a bit - in your mind what would those students look/act like? Some things that might come to my mind are: taking good notes, asking metacognitive questions to themselves, being persistent, etc. Once we have some specific descriptions of the students who meet the goal they set for themselves, maybe they can think about how and what they will do in class (through the problem they assign to how they interact with the students) may contribute to the development of desired outcomes.

In the same email, I told him that I felt the teachers viewed the planning meetings as just that, meetings, not as opportunities to learn. His advice to me was similar to his advice above:

> I guess I might ask you what those teachers who see these meetings as learning opportunities might look/act like and how that is different from what you are observing. Perhaps then you can focus on one or two specific traits you would like to see these groups develop and funnel your discussion in that direction.

In another email communication on November 19, 2007, I told John that I had been trying to only ask questions of the teachers and not to give advice. But, the teachers often interpreted my questions as intentions to lead them to a particular answer. So, when I asked the question, "Why aren't you going to use calculators in this problem?" they thought the question was my way of trying to get them to use calculators. And, instead of critically thinking through and articulating reasons for their choices, they asked, “Do you want us to use calculators?” John, again, gave me a different lens through which to think about my own role as a facilitator of the teachers’ learning and how I could consider scaffolding that learning:

> As much as we math educators like "why" and "how" questions, maybe what we need is a bit more specific question. So, if they are not going to let students use calculators, we can ask, "so, you are not letting students use calculators because that will make it more likely to raise the question about the order of operation (due to errors some students may make), or is it because the results of the computation should be small enough for them to mentally calculate them? [or whatever the reasons we could come up with] Or is it for some other reasons?" Perhaps the idea of "scaffold" may mean that we make some suggestions on what kinds of things they can/should be thinking about. We may have to be much more directive at first, by asking questions we want them to ask - in a very specific way. We can perhaps think about a series of questions in case if they pick one of the reasons we give to follow up.

A specific example from this study of how the teachers’ responses to my questions led them to deeper reflection again points to the lesson focus. Lesson study is not a time for teachers to create new lessons. Although this is an option, it is not the norm. Therefore, the teachers are using pre-existing lessons, lessons that may have proven effective in the past or even that have proven deficient and in need of adjustments. Since the lessons used in this study were from the textbook, I constantly asked the team to state the goal of the lesson and then state how any particular part of the lesson contributed to the attainment of that goal. It was a reminder to them to always think of their own purpose, and to realize that their purpose may force the plan into new directions since their purpose may not align necessarily with the textbook author’s purpose. According to the teachers, the constant focus on the student learning goal during planning and observation redefined how they approached lesson planning individually.

These teams were new to the process of lesson study, which affected the entry points, those points specifying understanding of not just mathematical content or mathematical teaching, but also the lesson study process itself. Participating in lesson study is not easy or natural. That participation requires scaffolding for teachers by the facilitator. As teams grow in comfort with the lesson study process, the facilitator's role grows also. The teachers in both groups often asked me if what they were doing was right. "Can we do it this way?" or "Is it all right for us to co-teach, since this is a class that is co-taught normally?" Many of our conversations revolved around the process of lesson study. In this group, it was also important for me to provide the teachers with trust in the knowledgeable others because they did not have a prior relationship with the professors. I used time in our planning meetings to prepare the teachers for a non-evaluative observation and also to prompt the teachers about how to ready their classes for the observations. When the practice of lesson study becomes more automatic, the role of the facilitator should evolve.

Separating the role of lesson study facilitator and researcher was a difficult task. I tried to carefully balance these roles so that from the teachers' perspectives, I was the facilitator. Furthermore, in defining that role for myself, I did not want to be their teacher. I wanted to help them understand the process of lesson study in order that they would use that process as a tool for their own professional learning. It was of utmost importance to me for the teachers not to think I held knowledge that I was going to impart to them as an instructor that would make them a better teacher. Becoming a better teacher was a notion I wanted them to struggle with shaping and manipulating as individuals and as a group of mathematics educators.

But the teachers' comments that expressed their confusion after many planning meetings show that they needed more scaffolding from the facilitator. And the comments showed that I should have made my own struggle more explicit so that when I contributed to a discussion, everyone understood it to be my opinion. It was dangerous for me not to scaffold enough what the teachers were learning. Looking back on the comments from the teachers, they do not indicate to me the teachers were searching for my answers, but rather, they needed more support. John's comments show one way to provide that extra guidance, to give them support as they explored and developed their own knowledge, rather than handing them my knowledge. John echoed Mewborn's (2003) statement, "Just as one cannot expect students to learn something simply by being told that it is so, one cannot expect teachers to change their teaching practice simply because they have been told to do so." He simply said, "In a way, lesson study requires teachers to think very differently about teaching mathematics and planning lessons. However, how can we more effectively support teachers learning to think differently than they are used to? We can't just tell them to 'think differently,' can we?"

Concluding Remarks

The data from this study show that no single experience of lesson study is the defining characteristic but that many experiences may afford different learning opportunities, different entry and exit points for the learners that are the participants. The length of time spent planning was difficult for the teachers, but important in the reflections at the end of a cycle. They poured so much of themselves into the planning process, as they have often poured themselves, mostly as individuals, into a lesson. But, even after the sum of the pouring out, the evidence of student learning was powerful in what it showed to them, signifying that a teacher pouring himself or herself into a lesson does not always connect to the students learning what the teacher intended. The link between the teacher's plan and the student outcome is not obvious or natural. After the first public teaching, the teachers walked into the post-lesson discussion exhausted, and Olivia looked at me and said, "Now I understand what lesson study is about." Bobbie said, "Up until [the public teaching], … I would have said that I thought lesson study was the biggest waste of time that I'd ever been put through. … I feel like the student who [is standing at the board] and knows something new and feels like he's really learned it, but can't articulate it in words. … The experience was meaningful."

These teachers participated in lesson study in a less than voluntary manner. And still, they used the process and benefited from it, even in these circumstances, according to the data. They responded out of professionalism to a mandate that may or may not have made sense to them. This gives new breath to a demanding process like lesson study to show that even when teachers are not fervently seeking the benefits of lesson study, many benefits are still apparent to the teachers in the end.

The experiences of the process that enabled these benefits include the sum of planning collaboratively, anticipating student responses, creating evaluation questions for observers, observing the public teaching, and discussing and reflecting on the observations. Furthermore, the data from the teachers in this study suggest the somewhat linear progression of these experiences, one naturally following the other, to be as important as the experiences themselves. It was in the collaborative planning that the teachers had to constantly refocus on the student learning goal. In focusing on that goal, the teachers had to determine ways the students would respond. Those responses led to the teaching teacher's responses. In order to prepare observers for what the team expected, the teachers had to insert answerable questions throughout the lesson in order for the observers to respond with evidence from the class. These questions focused the team, acting as observers in the public teaching, on connecting the teachers' plan with the outcome of student learning.

The data in this study suggest the teachers learned many things, some of which have not been explicitly stated here. But the experiences in this process caused perceptible disequilibrium at many different stages during the year. The teachers worked as a team and as individuals to reorganize their constructs of mathematics teaching and mathematics learning. This pivotal experience of knowledge construction was apparent in the teachers' reflections and my observations and interviews.

CHAPTER 5

CONCLUSIONS

Summary

The purpose of this study was to examine what mathematics teachers learned as they participated in lesson study and what experiences in lesson study may be linked to that learning. The research questions that guided the study are as follows:

- What do mathematics teachers learn when they participate in lesson study?
- What do mathematics teachers consider to be key experiences of lesson study that help them learn?
- How do mathematics teachers describe the role of the knowledgeable other in lesson study?
- How does the role of the knowledgeable other influence the learning of the mathematics teachers?
- What is the role of the lesson study facilitator?

To design this study, I reviewed literature on professional development, mathematical knowledge for teaching, and lesson study. I considered environments for professional development that provided opportunities to learn. Then I used the mathematical knowledge for teaching literature to reflect on what knowledge the teachers needed to develop. Finally, I used a lesson study model to create a design with the purpose of allowing teachers the opportunity to develop professional knowledge.

I focused on effective characteristics of professional development, as reported by Loucks-Horsley, Love, Stiles, Mundry, and Hewson (2003), Garet, Porter, Desimone, Birman, and Yoon (2001), and Guskey (1995). These effective characteristics describe professional development that provides teachers the opportunity to collaborate, to examine practice, and to learn to reflect. In addition, teachers should be able to focus on their own teaching practice within a community of learners (Featherstone, Pfeiffer, & Smith, 1993; Franke & Kazemi, 2000; Putnam & Borko, 2000; Stein & Brown, 1997). Ball and Cohen (1999) claim that teachers must learn in and from practice; that the best way to improve teaching and teacher learning is to give teachers the ability to learn about teaching as a part of teaching.

I reviewed the literature on mathematics learning in order to consider what the students would be learning from the teachers and what the teachers would be learning from the professional development experience. I used Steffe (in press), van Oers (1996), and Simon (1997) to frame my own understanding of a constructivist theory of learning and knowledge. A person organizes knowledge from experience into an operating system. If an experience is encountered that does not "fit" into the system, meaning the system is in disequilibrium, the system must be reorganized, which is generally where learning occurs, because the new experience now works within the operating system, it is "viable" (van Oers, 1996). In considering teachers as learners, I wanted to design an environment grounded in their practice that would allow them an opportunity to experience perturbations in their operating structure of mathematics teaching.

I have used Ball and colleagues' definition of mathematical knowledge for teaching in order to better understand and explain the teacher knowledge described in this study.

Ball, Thames, and Phelps (2007) divide mathematical knowledge for teaching into two categories: subject matter knowledge and pedagogical content knowledge. Here is an example of how they pictorially represent mathematical knowledge for teaching.

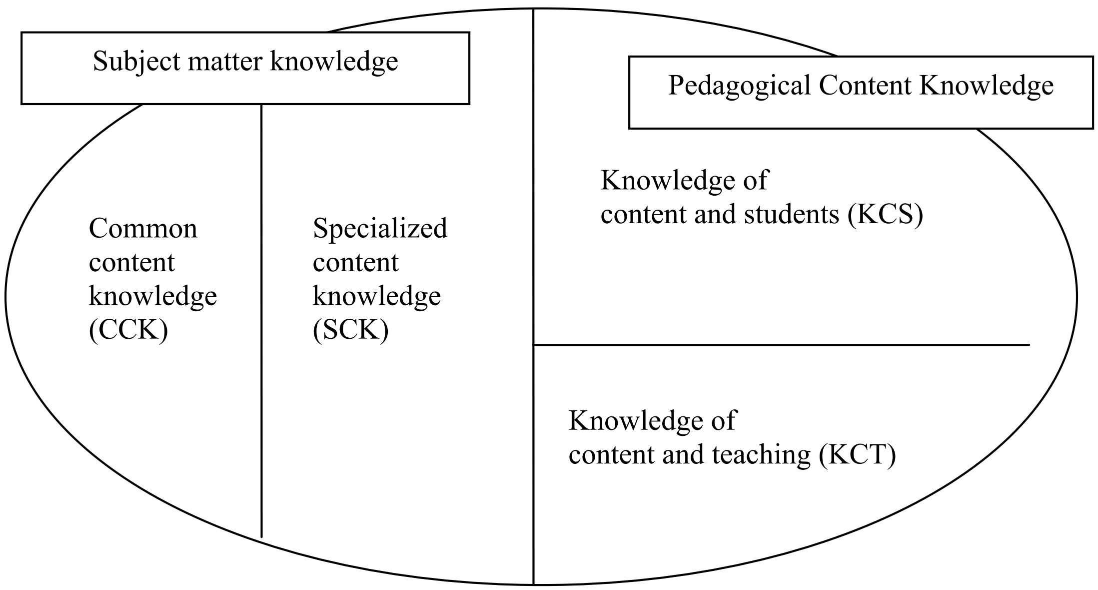

Figure 13: Mathematical knowledge for teaching (Ball & Bass, 2005)

Ball et al. (2007) divide subject matter knowledge on the left side into common content knowledge and specialized content knowledge. Common content knowledge is the "mathematical knowledge and skills used in settings other than teaching" (p. 32), which is what I describe as the adult's mathematics. Specialized content knowledge is described as the ability to "make features of particular content visible to and learnable by students" (p. 35). On the right side, they claim to elaborate Shulman's (1986) definition of pedagogical content knowledge by subdividing it into knowledge of content and students and knowledge of content and teaching. They describe this facet of mathematical knowledge for teaching as being knowledge of mathematics that is bundled with either knowledge of students or knowledge of teaching. However, for the purposes of this study, I did not subdivide pedagogical content knowledge because the instances in the data that I used to illustrate this knowledge were sometimes a combination of knowledge of content and students with knowledge of content and teaching. The combination of these two types of knowledge under Ball et al.'s (2007) description returned me to Shulman's (1986) definition of pedagogical content knowledge: the many ways the subject can be represented or modeled to others in order that others may comprehend the content, linking the content with the pedagogy, bringing a focus of student learning to the content matter.

For this study, lesson study is defined in the work of Lewis (2002a), Fernandez (2002), and Yoshida (1999). Teachers collaboratively plan, implement, revise, and re-implement a lesson. Stigler and Hiebert (1999) stress that lesson study is a process that provides incremental changes over a period of time. Fernandez (2002) describes it as providing an opportunity for systematic inquiry into teaching. During the lesson study process, teachers spend the bulk of time planning a detailed lesson, in which they include anticipated student responses, intended teacher moves, and evaluation questions for observers. The lesson is taught by one member of the team, observed by the entire team and invited observers who gather evidence about student learning during class. The evidence sought is guided by questions written by the teachers in the planning stage. After the lesson, the observers and team discuss the plan and the evidence. The team revises the lesson based on this discussion, and the revised lesson is taught and observed.

I facilitated two lesson study teams of mathematics teachers at a middle school, a sixth-grade team and a seventh-grade team. I met with each team about once every 2 weeks and completed two cycles of lesson study. The public lessons were observed by members of the planning team and other invited guests. The planning teams each met seven times before the public teaching in the first cycle and four times before the public teaching in the second cycle. The planning meetings, post-lesson discussions, and revision meetings were digitally recorded, and the public teachings were videotaped. The teachers were interviewed individually before lesson study began and as a group after the first and second cycles. They also responded to reflection questions after each cycle.

I used a grounded theory approach to analyze the data. After transcribing all the digital recordings, I coded the transcriptions according to ideas that were discussed at length or that were discussed often. I developed the list of codes as I went through the data, so I used a complete list to go back through the data. I then considered any relationships between codes and chunked the codes into categories, which better defined for me the strands of discussion among the teachers. I went through the data by cycle; that is, first cycle sixth grade and seventh grade, then second cycle sixth grade and seventh grade.

Next, I focused on the interviews and written responses of the teachers. I gleaned from these transcribed notes what the teachers claimed and showed to have learned. These defined the three threads of growth documented in chapter 4. I went back through the data, by grade level this time (sixth-grade first cycle, sixth-grade second cycle, seventh-grade first cycle, seventh-grade second cycle), to recode according to these three threads. In doing so, I found that the previous codings from the chunked categories fit nicely into these three strands, and no additional data needed coding. At that time, I was convinced I had strong arguments for what the data showed with regard to teacher learning.

Last, I coded the data for any interactions from the knowledgeable others or the facilitator that appeared to contribute to the threads of teacher growth. Most of these coded data were in the transcripts of the planning meetings and post-lesson discussions. I also coded the data that appeared to contribute to the roles of the knowledgeable others and the facilitator. Most of these coded data were in the teacher interviews and written responses.

Major Conclusions

Both the teachers' reports and their actions revealed three threads of growth for the group of teachers. Specific experiences in lesson study worked together to stimulate and shape the teachers' growth. Detailed collaborative planning, anticipating student responses, determining questions for the observers to use in evaluating student learning, observing the live lesson, and interacting with the knowledgeable others, each worked with the others to guide the teachers as they (1) began to explicitly define their teaching role in a classroom using a reform curriculum, (2) gained new mathematical knowledge for teaching, and (3) focused on the lesson goal to streamline the lesson.

These teachers had not participated in lesson study before this study. They had been introduced briefly to the overall process in post-planning; they had a short meeting during pre-planning to answer questions; then they began the process in grade-level teams. These teachers did not begin this study as experts in lesson study. They learned the process as we participated in it, which is how Stigler and Hiebert (1999) assert that it should be done:

> In our view, lesson study is not the kind of process in which teachers must first develop a list of capabilities and then begin to design improved lessons. Lesson study is, in fact, the ideal context in which teachers *develop* deeper and broader capabilities. This is what we mean when we say that lesson study is a form of teacher development as well as a program for improving teaching. (p. 152)

As these teachers were in a teacher development program that they needed to learn about, the data provide evidence that the teachers were simultaneously learning about teaching and ways to improve it. Clearly, as they develop a level of automaticity with the process of lesson study, they will have more opportunity to develop their own teaching, especially at a deeper level. The adaptation of the critical lenses (Fernandez, Cannon, Chokshi, 2003) is probably not possible until the process of lesson study is understood deeply and flexibly by the participants. But this study provides evidence that even as the participants are learning the process, powerful growth can occur.

The teachers started to develop a place for themselves as teachers in a classroom using a reform curriculum when they began considering intentional interventions that would engage students to construct mathematical meaning (Simon, 1997). As the data showed in chapter 4, the teachers initially expected the Connected Mathematics Program [CMP] task itself to carry this burden. In the percent discount task, the teachers originally used the lesson exactly as written in the CMP materials. After the first public teaching, the teachers began to question how the students were supposed to make connections. They decided to redesign the lesson to build upon what the students were learning in the unit prior to that lesson. In that way, they took the burden of helping students construct mathematics away from CMP and placed it on themselves.

Each teacher team had to develop two lesson plans, one for each cycle of lesson study. These lessons were developments of existing lessons, as is often the case in lesson study, and specifically these were from CMP units. As the teachers developed these plans, they engaged in the mathematics of the lesson themselves. They solved each problem at each stage of the lesson. Tackling the mathematics, especially the mathematics used in this reform curriculum, was a task in itself for the teachers. Their conversations supplied the data to reveal how they were developing their mathematical knowledge for teaching as they developed the lessons.

Ball, Hill, and Bass (2005) claim that "representation involves substantial skill in making these connections. It also entails subtle mathematical considerations" (p. 20). As reported in chapter 4, the seventh-grade team made a decision to allow the students to use any letter to represent the unknown number of coins in the pouch in the CMP problem. It was at that point that the secondary knowledgeable other, Luke, spoke to the group about how the students could misunderstand the term $5p$ to represent 5 pouches, when the term actually represented the number of coins in 5 pouches. This was a subtle mathematical consideration that had eluded the teachers prior to this discussion. Even in their own conversations, the teachers realized they were saying phrases that promoted misunderstanding: "Students will use $5p$ for the 5 pouches." This is not a case of common content knowledge: all the teachers knew how to get the correct answer for the task. It was also not a case of pedagogical content knowledge, even though there was a discussion about student misconceptions and about a better model to use for the term (that is, using x instead of p). But, it was a discussion of specialized content knowledge for these teachers because of the subtle mathematical meaning that was being presented in the imprecise vocabulary they were using. This important discovery was not about the teachers' knowledge of content and students or the knowledge of content and teaching (both terms from Ball et al., 2007, used to describe pedagogical content knowledge), it was about specialized knowledge of mathematics "uniquely needed by teachers in the conduct of their work" (Ball et al., 2007, p. 34).

Two questions in this study required clarification about the defining roles of the knowledgeable others and the lesson study facilitator. These questions were meant to make transparent the work of these support personnel in lesson study. The data in chapter 4 that showed teachers experiencing disequilibrium and then attempting to restructure their understanding in order to restore equilibrium are linked to sustained dialogue with the facilitator or a knowledgeable other. By sustained dialogue, I am specifically stating that the learning did not occur because of any one statement or even one discussion. Some of the incidents were discussed, and seemingly dismissed, in the first cycle; then discussed, and seemingly dismissed, in the second cycle; then elaborated in the reflection questions at the end of the second cycle. For example, both the facilitator and the primary knowledgeable other attempted to steer the sixth-grade team to leave out a portion of the lesson in the first cycle that seemed to be diverging from the lesson goal. The teachers were adamant that it stay. In the second cycle with the same team, a similar problem occurred when a major portion of the students' work time was spent on one part of a problem, where that specific time-consuming part had nothing to do with the lesson goal. Nevertheless, even in the post-lesson discussion, the teachers wavered a bit in deciding whether or not to remove that part for the revised lesson. But, in the final reflection questions, the teachers responded with conviction that this issue of focus was an extremely important realization for them. The conversation that the teachers claimed to change them was one that could be traced through each lesson study cycle to the end of the year-long process. The teachers would not have appeared to find warrant to this notion in a snapshot view of them during the process. Rather, the understanding of this idea of a narrowly focused lesson seemed to begin to be realized at the end only after many conversations and actual examples in the classroom.

As another example, in a certain planning meeting, one lesson study team repeatedly said they wanted students to think about rational numbers in the activity in the lesson, not just haphazardly choose numbers and operations. The teachers were asked how they would know the students were thinking about rational numbers in the way the teachers wanted them to. Several seconds of silence passed until one teacher spoke, “They will have conversations like we’re having.” The teachers were realizing the value of listening to students. They had worked through the activity together during a planning meeting and were thinking about the important aspects of the mathematics and analysis. In doing so, they came to see that listening to how the students were talking about the problem would be more important, in this case, than seeing a finished problem. In order for the teachers to arrive at this conclusion, they had to make a departure from the textbook program and deeply consider what they wanted the students to gain during this class period. That departure would have been harder to make if the teachers were not guided by the outside perspective of the facilitator since it was the questions from the facilitator that led the teachers to this departure.

These data show that when conclusions can be made about teacher growth in lesson study, that growth is often linked to ideas pushed to the foreground by either the facilitator or a knowledgeable other or combination of those. Because these teams were grade-level specific, the lesson study teams were the existing grade-level content teams.

The roles of the members were already established. If the team had been from different buildings, where working relationships were not already in place, the members themselves may have been responsible for pushing ideas. However, in this case, the established roles would have been difficult, at best, to change. For example, the role of leader was established for the teachers since they began work together two years prior, and to question an idea, or push a different idea other than the leader's probably would have been met with resistance and soon dismissed. The working relationships among teachers in the United States can often be shaken or damaged by personal feelings. Therefore, when hard questions need to be asked, or when ideas need to be challenged, a person outside of the teaching role and the established teacher group would probably be beneficial. In addition, as indicated by many comments about the knowledgeable others in the teacher interviews, two levels of outside perspectives seem to be necessary. The outsider at one level of involvement was the facilitator, who could guide and ask questions at each planning meeting. But, also of great importance to the teachers was the knowledgeable others, who were not part of the planning team. They offered a fresh perspective, as described by many of the teachers, to see the plan and the lesson more objectively. A guiding view outside of the teachers from the facilitator and outside the lesson from the knowledgeable others is an important facet of the roles of the facilitator and knowledgeable others.

Implications

Lewis, Perry, and Murata (2006) call for research "to explicate the mechanism by which lesson study results in instructional improvement" (p. 5). They express concern about a focus on surface features of lesson study that may cause the process to fail.

Throughout this study, I continued to think about that petition. I found myself always going back to a quote that Lewis (1998) documented from a Japanese teacher experienced in lesson study, “A lesson is like a swiftly flowing river; when you’re teaching you must make judgments instantly.” Lesson study is the process that provides teachers with a slow-motion button. The process of lesson study slows down teaching in order to consider and re-consider the judgments teachers make and to be able to critically think about and discuss what effects different teacher actions may cause. Of great importance is that the teaching being considered by a lesson study team is specific to that team. The teachers have claim on the lesson and the students. The mechanism of lesson study that results in instructional improvement and in teacher growth seems to be the slow-motion button, the experience of lesson study that provides for the swiftly flowing river and all the instant judgments to be suspended, critically discussed, intentionally chosen, and tested. This forced articulation of teacher actions and reactions contributes to learning and therefore knowledge. This study suggests lesson study, as specifically defined by Lewis (2002a) and Fernandez and Yoshida (2004), may be the mechanism that results in instructional improvement. It is an experience in meta-cognition: to hold the act of teaching out at a distance to look at it from all different perspectives, and then to place the teacher back into the act of teaching and to monitor what happens. Sherin (2002) makes the claim this way: “It is not sufficient that teachers learn new ideas about the domain and new teaching practices to implement reform. Instead, the process of reform-based teaching is itself a learning process and should be recognized as such.” It is this experience in meta-cognition, put in motion by what Lewis et al. (2006) call surface features, that leads to teacher growth. Teachers often lack the time to attend to their own intentional learning about mathematics teaching. This mechanism, called lesson study, provides the means, the environment, and the space for teachers to attend to their own learning, allowing the teachers' time to be spent efficiently attending to their own learning while also planning a lesson and accomplishing other tasks in the work of teaching.

In order for other school districts to be able to use a lesson study model like this one, several considerations must be addressed. Time was structured during the school day for these teachers to meet. They were under a most demanding and crushing amount of stress. To have asked them to meet after school as one more duty to add to all the others would have meant certain failure for the process. Even the requirement of once every 2 weeks during the school day was paralyzing to them during certain time periods of the year. Schools that want to utilize professional development as demanding as lesson study should be prepared to provide the teachers with the space and the time to carry it out successfully.

Additionally, the data from this study and others (Fernandez, 2005) suggest the role of the knowledgeable other and facilitator are critical in creating disequilibrium for the teachers. First, the teachers, as evidenced in this study, need support to learn the process of lesson study. Also, they need organizational support in carrying it out, not as "one more duty," but support in making the meetings run smoothly. The teachers also need to be challenged in what they believe and in what they do. They need a person with an outside perspective who has a robust knowledge of mathematics education research and a profound respect for teachers. It is the responsibility of the knowledgeable other to help teachers as they grow in understanding lesson study and to begin to deeply examine their own practice in a manner that allows them to move from thinking about knowledge *in* practice to the bigger view of knowledge *of* practice (Cochran-Smith & Lytle, 1999). The knowledgeable other and the facilitator must help the teachers to focus on the learner and the quality of the mathematics being learned (Sfard, 2003, p. 386).

But teachers have a responsibility in this process, as all learners do. Teachers must begin to see practice as a "site for learning and of themselves as actively in charge of their ongoing learning process" (Fernandez et al., 2003, p. 182). Teachers should, as Ball and Cohen (1999) state, learn how to learn in and from practice. This learning-how-to-learn acknowledges the need for the adaptation of lesson study in the United States to require the work of outside experts; that is, the process cannot be completely and solely teacher-led (Fernandez, Cannon, & Chokshi, 2003). Teachers often admit to the inevitable struggle they experience when they are asked to reflect on instruction without guidance (Stein, Smith, Henningsen, & Silver, 2000). Teachers then need guidance from outside experts, or knowledgeable others, in order that they may learn how to reflect and critically examine practice as they are reflecting and critically examining practice. The data from this study suggest that the input of knowledgeable others is important and can be beneficial at all levels of lesson study implementation, to guide these learning-how-to-learn stages. The knowledgeable others in this project often disturbed the equilibrium in the teachers' logic, while the lesson study facilitator needed to nurture the process of the restructuring experiences so that equilibrium could be restored in the teachers' operating system of mathematic instruction. Simon (1994) claims that the phrase "let students construct the ideas" is not a useful way to guide teachers to think differently about the nature of mathematics and how it is learned. Neither is the phrase "let teachers construct different ideas about mathematics instruction" a useful way to guide teacher educators to think differently about the nature of mathematics instruction and how it is reformed. Careful, deliberate decisions and goals must be made by teacher educators to interact with teachers' construction of mathematics instruction in a productive manner that causes the teacher to reorganize these concepts.

Suggestions for Future Research

Lesson study is a very new form of professional development in the United States. Having specified experiences that seem to be critical to teacher growth in this study and in others (Fernandez & Yoshida, 2004; Lewis, 2002b) that formed a foundation for this study, a longitudinal study to follow teacher learning over several years of lesson study implementation, with teachers who are not new to the process, might provide evidence of student growth and of further change in individual instruction for participating teachers. A study of this nature and magnitude has been reported by Lewis, Perry, Hurd, and O'Connell (2006) and would benefit from further contributions from different sites with different researchers. These contributions would provide additional researcher perspectives as well as determining if geographic locations or populations influence the findings.

Further research should also be conducted to understand how the process of lesson study continues to unfold to teachers as they understand lesson study and use it as a vehicle to learn more about teaching. Researching teacher learning with teachers experienced in lesson study should provide additional insight about the mechanism of lesson study. Are teachers experienced in lesson study simply further along the learning curve than ones who are learning lesson study for the first (or second, or third) time? Or, do teachers experienced in lesson study learn in different ways? At what stage do teachers develop the critical lenses referred to by Fernandez et al. (2003)? Also, does sustained lesson study over a period of years influence teachers' dispositions about mathematics, mathematics teaching, or mathematics learning? What do university faculty, fulfilling the role of knowledgeable others, learn from lesson study?

Using teachers in different grade levels would further round the understanding of lesson study in this country. Breaking into the area of high school teachers, in mathematics as well as other content areas, provides rich territory for the brave researcher. Would a particular key feature of lesson study, like collaboration, become more important than other features, if the participating teachers had not previously experienced it? Do teachers of different grade levels, with overall different content needs, learn the content in different ways?

Concluding Remarks

As I was learning about lesson study, I always tried to think about what I was learning from my perspective when I was a classroom teacher. I forced this perspective on myself for a number of reasons: I was certain there were other teachers like me who shared that perspective; I felt most comfortable from that perspective; and I knew that perspective would keep me grounded in my conclusions. However, even with my grand attempts, I do not believe I could feel the depth of what the teachers in these lesson study teams felt. I found myself breathless after most meetings as I tried to consider the pressure cooker atmosphere in which they lived. Life occurred every day at such a fast pace with so much at stake that the emotional strain almost seemed unbearable. In the last 3 years, these middle school mathematics teachers had endured new state standards, a new model of teaching, a new reform textbook program, new state mandated tests with higher stakes attached, scores printed for public scrutiny, editorials to heap blame and distrust upon teachers, and yet they returned to the classroom. They returned and tried even more new things, like a year-long two-cycle implementation of lesson study. With all of these complications, I examined what happened to them in this process.

When the action of teaching is suspended, as I have said earlier, and decisions are carefully regarded, the teachers have an opportunity to meaningfully discuss teaching. Fernandez (2005) suggests that these conversations the teachers have allow them experience outside of their classrooms to handle mathematical challenges occurring in the act of teaching. Stigler and Hiebert (1999) claim "lesson study can help teachers establish meaningful goals for their students' learning and their teaching that reach well beyond the few study lessons that they work on together" (p. 224). Gill (2005) shared a teacher response: "As I sit and plan, I can almost hear my team members' responses. I think, 'Now Martha would say I'm staying too abstract'" (p. 60). That comment is similar to one shared by a teacher in this study: Annie said, "Since we began this process, I have stressed points that I would not have last year since I knew what we were working toward in our public lesson." These comments taken as a whole suggest that learning in lesson study may produce generative growth (Franke & Kazemi, 2001) since the teachers are describing opportunities of continued learning in their practice outside of the professional development experience.

I have reported that the teachers gained new knowledge, that they learned about their own teaching. In general, I saw the overall nature of teacher conversations evolve from being about what the students' reactions should be to the teaching into what the teaching reactions should be to the students. These conversations, which first surfaced in the lesson study process of writing observer questions, created perturbations in the teachers' organized system that represented the classroom environment. Those perturbations could have easily been dismissed. But, as they became more pronounced and more often apparent (first in planning sessions, then observed in public teachings, then in post-lesson discussions), accommodations had to be considered and made. That is the process of learning. However, I do not intend to suggest lesson study is the only way teachers can learn certain things about mathematics teaching. I only present this as one experience that offers an opportunity for teachers to learn *about* the act of teaching while *in* the act of teaching.

And I have to be realistic. My guess is that these teachers did not necessarily look vastly different in the classroom the year after this lesson study implementation. However, they had experienced a process that teased their mind with certain new ideas. These ideas, based on the teachers' comments, may prove to continue to create disequilibrium for them when they have similar experiences in the classroom. Therefore, I willingly concede that the teachers' operating system of knowledge may only be in a perturbed state, more than in an accommodated one. That state of disequilibrium may even lie dormant for a period of time. But if the teachers can continue in an examination of their practice, the possibility remains that the disequilibrium would create learning and even produce deeper change.

We do have an imperative to act. If a student can gain more from a year of instruction with a teacher who has more mathematical knowledge for teaching than one who has less (Hill, Rowan, & Ball, 2005), and if a teacher can professionally develop that knowledge (Hill & Ball, 2004), then mathematics teachers must seek to constantly improve this mathematical knowledge for teaching. If studies like this one can show lesson study as a viable option to develop teachers professionally, and especially if experiences like lesson study develop this kind of mathematical knowledge for teaching, then we have a responsibility to make this type of long-term, committed experience a possibility for teachers.

REFERENCES

Ball, D. L. (1993a). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *Elementary School Journal, 93*(4), 373-397.

Ball, D. L. (1993b). Halves, pieces, and twoths: Constructing representational contexts in teaching fractions. In T. Carpenter, E. Fennema, & T. Romberg (Eds.), *Rational numbers: An integration of research* (pp. 157-196). Hillsdale, NJ: Erlbaum.

Ball, D. L. (1996). Teacher learning and the mathematics reforms: What we think we know and what we need to learn. *Phi Delta Kappan, 77*(7), 500-508.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83-104). Westport, CT: Ablex.

Ball, D. L., & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, W. Martin, D. Schifter (Ed.), *A research companion to principles and standards for school mathematics* (pp. 27-43). Reston, VA: National Council of Teachers of Mathematics.

Ball, D. L., & Bass, H. (2005). *Learning the mathematical work of teaching.* Retrieved June 25, 2008, from University of Michigan Web site: http://www-personal.umich.edu/~dball/presentations/092305_aamte.pdf

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In G. Sykes & L. Darling-Hammond (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3-32). San Francisco: Jossey Bass.

Ball, D. L., Hill, H. C., & Bass, H. (2005, Fall). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, 29*(3), 14-22, 43-46.

Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed.). New York: Macmillan.

Ball, D. L., Thames, M. H., & Phelps, G. (2007). *Content knowledge for teaching: What makes it so special?* Manuscript submitted for publication.

Bransford, J. D., Brown, A. L., & Cockling, R. R. (Eds.) (2000). *How people learn: Brain,*

mind, experience, and school. Washington, DC: National Academy Press.

Chokshi, S., & Fernandez, C. (2004). Challenges to importing Japanese lesson study: Concerns, misconceptions and nuances. *Phi Delta Kappan, 85*(7), 520-525.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9-13.

Cobb, P., Wood, T., & Yackel, E. (1990). Classrooms as learning environments for teachers and researchers. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on teaching and learning mathematics* (Journal for Research in Mathematics Education Monograph No. 4, pp. 125-146). Reston, VA: National Council of Teachers of Mathematics.

Cochran-Smith, M., & Lytle, S. L. (1999). Relationships of knowledge and practice: Teacher learning in communities. *Review of Research in Education, 24*, 249-305.

Crotty, M. (1998). *The foundations of social research: Meaning and perspective in the research process*. Thousand Oaks, CA: Sage Publications.

Darling-Hammond, L. (1998). Teacher learning that supports student learning. *Educational Leadership, 55*(5), 6-11.

Educational Testing Service (2005). *The Praxis Series: Mathematics Content Knowledge*. Retrieved June 9, 2008, from http://www.ets.org/Media/Tests/PRAXIS/pdf/0061.pdf

Featherstone, H., Pfeiffer, L., & Smith, S. P. (1993). *Learning in good company: Report on a pilot study* (Research Report 93-2). East Lansing: National Center for Research on Teacher Learning, Michigan State University.

Fernandez, C. (2002). Learning from Japanese approaches to professional development: The case of lesson study. *Journal of Teacher Education, 53*(5), 393-405.

Fernandez, C. (2005). Lesson study: A means for elementary teachers to develop the knowledge of mathematics needed for reform-minded teaching? *Mathematical Thinking and Learning, 7*(4), 265-289.

Fernandez, C., Cannon, J., & Chokshi, S. (2003). A US-Japan lesson study collaboration reveals critical lenses for examining practice. *Teaching and Teacher Education, 19*(2), 171-185.

Fernandez, C., & Yoshida, M. (2004). *Lesson study: A Japanese approach to improving mathematics teaching and learning*. Mahwah, NJ: Erlbaum.

Fernandez, C., Yoshida, M., Chokshi, S., & Cannon, J. (2001). *An overview of lesson study.* Retrieved June 9, 2008, from http://www.teacherscollege.edu/lessonstudy/presentations_slides.html.

Franke, M. L., & Kazemi, E. (2001). Teaching as learning within a community of practice. In T. Wood, B. S. Nelson, & J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary school mathematics* (pp. 47-74). Mahwah, NJ: Erlbaum.

Garet, M. S., Porter, A. C., Desimone, L., Birman, B. F., & Yoon, K. S. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research Journal, 38*(4), 915-945.

Gill, A. (2005). For knowledgeable others: AFT's experience in Volusia County. In P. Wang-Iverson & M. Yoshida (Eds.), *Building our understanding of lesson study* (pp. 53-62). Philadelphia: Research for Better Schools.

Grouws, D. A., & Schultz, K. A. (1996). Mathematics teacher education. In J. Sikula (Ed.), *Handbook of research on teacher education* (2nd ed., pp. 442-458).

Guskey, T. R. (1995). Professional development in education: In search of the optimal mix. In T. R. Guskey & M. Huberman (Eds.), *Professional development in education* (pp. 114-131). New York: Teachers College Press.

Hill, H., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Education Research Journal, 42*(2), 371-406.

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education 35*(5), 330-351.

Kazemi, E., & Franke, M. L. (2003). *Using student work to support professional development in elementary mathematics* (Document W-03-01). Seattle, WA: University of Washington, Center for the Study of Teaching and Policy.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Lave, J. (1996). Teaching, as learning, in practice. *Mind, Culture, and Activity, 3*(3), 149-164.

Lewis, C. (2002a). *Lesson study: A handbook of teacher-led instructional change*. Philadelphia: Research for Better Schools.

Lewis, C. (2002b). Does lesson study have a future in the United States? *Nagoya Journal of Education and Human Development, 1*(1), 1-23.

Lewis, C., Perry, R., Hurd, J. (2004). A deeper look at lesson study. *Educational Leadership, 61*(5), 18-22.

Lewis, C., Perry, R., Hurd, J., & O'Connell M. P. (2006). Lesson study comes of age in North America. *Phi Delta Kappan, 88*(4), 273-281.

Lewis, C., Perry R., Murata, A. (2006). How should research contribute to instructional improvement? The case of lesson study. *Educational Researcher 35*(3), 3-14.

Lewis, C., & Tsuchida, I. (1998). A lesson is like a swiftly flowing river: How research lessons improve Japanese education. *American Educator, 22*(4), 12-17, 50-52.

Loucks-Horsley, S., Love, N., Stiles, K. E., Mundry, S., & Hewson, P. W. (2003). *Designing professional development for teachers of science and mathematics* (2nd Ed.). Thousand Oaks, CA: Corwin Press.

Marks, R. (1990). Pedagogical content knowledge: From a mathematical case to a modified conception. *Journal of Teacher Education 41*(3), 3-11.

Mewborn, D. (2003). Teaching, teachers' knowledge, and their professional development. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 45-52). Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Perry, R., Lewis, C., & Akiba, M. (2002, April). *Lesson study in the San Mateo-Foster City School District.* Paper presented at the meeting of the American Educational Research Association Meeting, New Orleans.

Peshkin, A. (1988). In search of subjectivity: One's own. *Educational Researcher, 17*(7), 17-22.

Putnam, R. T., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher 29*(1), 4-15.

Rowan, B., Chiang, F., & Miller, R. (1997). Using research on employees' performance to study the effects of teachers on students' achievement. *Sociology of Education, 70*(4), 256-284.

Sfard, A. (2003). Balancing the unbalanceable: The NCTM standards in light of theories of learning mathematics. In J. Kilpatrick, W.G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 45-52). Reston, VA: National Council of Teachers of Mathematics.

Sherin, M. G. (2002). When teaching becomes learning. *Cognition and Instruction, 20*(2), 119 – 150.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Simon, M. A. (1994). Learning mathematics and learning to teach: Learning cycles in mathematics teacher education. *Educational Studies in Mathematics, 26*, 71 – 94.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114–145.

Simon, M. A. (1997). Developing new models of mathematics teaching: An imperative for research on mathematics teacher development. In E. Fennema & B. S. Nelson (Eds.), *Mathematics teachers in transition* (pp. 55-86). Mahwah, NJ: Erlbaum.

Sowder, J. T. (2007). The mathematical education and development of teachers. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157 – 223). Charlotte, NC: Information Age Publishing.

Steffe, L. P. (2004). On the construction of learning trajectories of children: The case of commensurate fractions. *Mathematical Thinking and Learning, 6*(2), 129 – 162.

Steffe, L. P. (in press). Perspective of children's fractional knowledge [Monograph]. *Journal of Mathematical Behavior*.

Stein, M. K., & Brown, C. A. (1997). Teacher learning in a social context: Integrating collaborative and institutional processes with the study of teacher change. In E. Fennema & B. S. Nelson (Eds.), *Mathematics teachers in transition* (pp. 155-191). Mahwah, NJ: Erlbaum.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). *Implementing standards-based instruction: A casebook for professional development*. New York: Teachers College Press.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom.* New York: Free Press.

Van Oers, B. (1996). Learning mathematics as a meaningful activity. In L. Steffe, P. Nesher, P. Cobb, G. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 91-115). Hillsdale, NJ: Erlbaum.

Wang-Iverson, P., & Yoshida, M. (2005). *Building our understanding of lesson study*. Philadelphia: Research for Better Schools.

Watanabe, T. (2002). Learning from Japanese lesson study. *Educational Leadership, 59*(6), 36-39.

Watanabe, T., & Wang-Iverson, P. (2005). The role of knowledgeable others. In P. Wang-Iverson & M. Yoshida (Eds.), *Building our understanding of lesson study* (pp. 85-91). Philadelphia: Research for Better Schools.

Wilson, S. M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: An examination of research on contemporary professional development. In A. Iran-Nedjad & P. D. Pearson (Eds.), *Review of research in education* (Vol. 24, pp. 173-209). Washington, DC: American Educational Research Association.

Yoshida, M. (1999). Lesson study: A case study of a Japanese approach to improving instruction through school-based teacher development. *Dissertation Abstracts International, 60*(11), 3895-4399. (UMI No. 9951855)

Yoshida, M. (2005a). An overview of lesson study. In P. Wang-Iverson & M. Yoshida (Eds.), *Building our understanding of lesson study* (pp. 3-22). Philadelphia: Research for Better Schools.

Yoshida, M. (2005b). Using lesson study to develop effective blackboard practices. In P. Wang-Iverson & M. Yoshida (Eds.), *Building our understanding of lesson study* (pp. 93-100). Philadelphia, PA: Research for Better Schools.

APPENDIX A: Interview Questions before Lesson Study

- Describe what happens when you and your colleagues collaborate.

    How often?

    Which colleagues?

- Describe how you plan a lesson.

- When you are teaching, describe what you observe that you consider evidence of student learning. (What do you look for so you know students have learned?)

    How do you use that evidence? (How do you know when you can move on to next topic?)

    How often do you make use of it?

- How and when do mathematics teachers learn the mathematics they need to teach?

APPENDIX B: Interview Questions after 1st Cycle

1. Reflecting on the first cycle of lesson study, what have you learned or what have you thought about differently as a result of our conversations and work? (collaboration , mathematics, pedagogy, expectations for students, evidence of student learning, CMP, etc.)

2. Did you at any time, during the first cycle of lesson study, plan or instruct or even think about mathematics teaching and mathematics learning differently because of something we did or discussed during lesson study?

3. What things have we discussed during lesson study that you had not thought of before or that were surprising to you or that you disagreed with?

4. What have you tried because of our work in lesson study in your classroom, outside of the lesson we planned together?

5. What do you see as the role of the knowledgeable other in lesson study?

Interview Questions after 2nd Cycle

1. What opportunities to learn do teachers have that participate in lesson study?

2. With regard to amount of time spent, what can happen that makes lesson study easier or more difficult?

APPENDIX C: Written Interview Questions after Lesson Study

1. Describe how you and your colleagues collaborate (outside of lesson study). This may include planning to teach, discussing a lesson that has already been taught, or discussing a unit that is going to be taught.

2. Tell me how you plan a lesson when you are preparing without your colleagues. Are there things that you consider now as you make your plans that you did not consider before this experience in lesson study?

3. When you are teaching, describe what you observe that you consider evidence of student learning. What do you look for so you know students have learned?

4. What did you learn while participating in lesson study? This could be many things: mathematics content, how students think mathematically, how your colleagues view some aspect of teaching different from your own view, how you interpret standards or curriculum, etc.

5. Describe any contributions of the knowledgeable others. This may be about the enactment of a lesson, the mathematics of the lesson, etc. How did their contributions influence what you learned?

6. As you think about the things we focused on in lesson study, how might this experience influence the way you examine practice, either your own, a student teacher's, or another colleague's? You may want to describe how you may have "looked" at the practice of teaching before lesson study and how you may "look" at it now.

Appendix D: Sixth-Grade Lesson Plan, First Cycle

I. Background Information

Goal of the Lesson Study Group
Students will be motivated enough to take ownership in their learning.

Narrative Overview of Background Information
This is the second year of a math-science partnership grant for the mathematics teachers in this district from the state department of education. During the first year of the grant, the teachers worked in depth on mathematics content training. This year, the teachers are taking this work into their own classrooms to reflect on the mathematics for students in the context of actual lessons.

The sixth grade team decided to focus on the Connected Mathematics unit “Bits and Pieces II.” The teachers chose this unit because it will be taught during November and December. This coincides with the time they want to teach the public lesson. They also wanted to look in depth at this content.

In this unit, students will explore to what integers certain fractions & decimals are closest. They will estimate sums and differences and develop algorithms for adding & subtracting fractions with unlike denominators, making use of fact families. Students will then discover that finding a fraction of a number uses the operation of multiplication and then also determine a part of a part, eventually working with mixed numbers and writing an algorithm. The unit concludes with division of fractions, having students explore models and then writing algorithms.

Students will know from a previous unit how to find equivalent fractions and how to use benchmarks to compare fractions and how to convert between fractions/decimals/percents. In this lesson, students will work with an area model for fractions in the context of brownie pans. They will make sense of the problems using models. They are not expected to develop a deep understanding of the algorithm yet. They are expected to think about what it means to find “part of a part.” Understanding that “part of a part” means multiplication is introduced. It is intended to help students decode whether multiplication will help solve a problem in other situations.

II. Unit Information

Title of the Unit: Using Fraction Operations (Connected Mathematics – Bits and Pieces II)

Goals of the Unit: Students will have an understanding of the meaning for computations with fractions.

III. Lesson Information

Title of this Lesson: Finding a Fraction of a Fraction

Goals of this Lesson: Students will think about what it means to find a part of a part and use area models to apply it.

Study Lesson Process:

Learning Activities Teacher's Questions and Expected Students' Reactions	Teacher Support and Things to Remember	Method of Evaluation
Warm up – fact practice.		
T: We've been talking about addition and subtraction in real problems. I want you to think about this real situation. We are going to take inventory at a sporting goods store. There are 5 ½ boxes of footballs in the stock room, and there are 12 footballs in a full box. How can you find the total number of footballs without opening all the boxes?	Students discuss whole group (students are not reading along, teacher presents problem as story).	Are students interested in problem? Can they comprehend the situation?
S: If I know there are 12 in each box, then it is 12 + 12 + 12 + 12 + 12 + 1/2 S: no, 12 +…+ 12 + 6 S: 12 * (5 + ½) S: 12 * 5.5 S: 5 ½ + 5 ½ + … + 5 ½ (twelve times) S: 5 ½ + 12		
T: So we know we have 12 in a box and 5 ½ full boxes so, the most efficient way would be 12 * 5 ½. Sometimes we are going to multiply, like this. Can you think of other times when you would multiply?	Encourage students to share similar multiplication situations. Give students lab sheet and colored pencils.	

T: We are going to the school fair and work in the brownie booth. Look at your first brownie pan on your lab sheet. There is a pan with ¾ of a pan of brownies left. That means you've already sold how much? S: ¼ of a pan has been sold. T: Draw a model of what is left in the pan. (Give students time to draw.) A customer wants to buy ½ of what is in the pan. So, what is he buying ½ of? S: ½ of the pan. S: ½ of what is left in the pan. S: ½ of ¾. S: ½ of the brownies left. T: What does it mean to find of ½ of ¾ ? Will you get something greater than or less than ¾ ? S: Less than. T: So, let's say that Mr. Jones wants to buy ½ of what is left in the pan, so he wants to buy ½ of ¾. On your lab sheet, show how much he is going to buy.	Identify misunderstandings in drawings (check equal parts on pictures, are they using vertical and horizontal lines for sections) Draw pan of brownies on board. Show ¾ of a pan of brownies in it. Use visual.	Can the students draw ¾ of the pan? Do students have a sense of how big or small products are? Are they able to visualize with model drawing?

(Allow students time to draw.)	Walk around to identify students who can and cannot show model.	Can the students show ½ of ¾ on the lab sheets?
What should the picture look like?	Encourage different ways to look at the model.	
Now I see how much Mr. Jones bought. Use the picture to tell me what fraction of the pan Mr. Jones bought. S: 1/8 S: 3/6 S: 3/7 S: 3/8	Students share explanations with class.	Are students actively thinking? Are they using the pictures to help them think?
T: Remember, we want to know what fraction of the whole pan he bought.	Teacher (or student) shows different representations, with the fraction 3/6, 1/8, 3/7. Let each person explain why.	Do students present ideas clearly? Are students learning after misunderstandings are discussed?
T: Now which one of these is correct? How did you decide what fraction of the whole pan is being bought?		
T: Let's say that all brownie pans are square and a pan costs \$12. You can buy any fractional part of a pan of brownies and pay the fraction of \$12. For example, ½ of a pan costs ½ of \$12. If this whole brownie pan costs \$12, how much would Mr. Jones have paid?		Can students articulate / show model example? How do other students respond to student explanations?

		Are models accurate?
T: Now I want you to work on Problem A on page 33. Think about the size of the answer when you are finding a part of a part. You need to do two things: 1. Draw models to show how the brownie pan might look before a customer buys part of what is left. 2. Then you need to mark the part in the brownie pan to show how much the customer buys.		
(Allow students time to work.) (Have them share, with partner, strategies used to solve the problem.)	For students who just want to use multiplication to get answer, teacher should ask: "How does your drawing help someone else see the part of the whole pan involved? What can you do to make your drawing clearer?	
T: What fraction of the whole pan does Mr. Williams buy? S: 2/6 S: 1/3	Have students show representations to whole class and describe strategies. Ask students to explain how they arrived at their answers. Pick different ways of representing to share.	
T: What does he pay? S: 1/3 of $12.00 S: $4.00	Share at tables or possibly in whole group, if there is time.	
T: In pairs at your table, work on B. (Allow students time to work.)		

T: Share your work in your table group. T: I want group ___ to come to show their work. SUMMARY: T: How can I draw a model of finding a fraction of a fraction? What does that mean? T: Now work on C (numbers 1, 2, 3, 4) individually, draw the models and find the answer.		

Evaluation:

Can students accurately draw models for finding a fraction of a fraction?

Do students explain models meaningfully?

Appendix E: Sixth-Grade Lesson Plan, Second Cycle

I. Background Information

Goal of the Lesson Study Group
Students will be motivated enough to take ownership in their learning.

Narrative Overview of Background Information
This is the second year of a math-science partnership grant for the mathematics teachers in the district from the state department of education. During the first year of the grant, the teachers worked in depth on content training that was theoretical in nature. This year, the teachers are taking this work into their own classrooms to look at the mathematics of actual lessons.

Students have already looked at typical situations in which discounts, taxes, and tips help to think about taking a percent of a number. The discount and tax situations helped students to consider the amount left when a reduction is made and the total when taxes are added.

In Investigation 5, students will develop a strategy for finding the percent of discount an amount taken off a price represents. They will use percents in estimating taxes, tips, and discounts.

Summary of Problem 5.1 Students find what percent one number is of another in the context of buying an item when they have a coupon for a percent off, or a dollar amount off, the original cost.

II. Unit Information

Title of the Unit: Bits and Pieces III

Goals of the Unit: Students will understand what decimals and percents mean, the relationship between them, and be able to compute with them.

III. Lesson Information

Title of this Lesson: Problems with Percent Discounts

Goals of this Lesson:
Students will develop a strategy for finding the percent of discount an amount taken off a price represents.

Study Lesson Process:

Learning Activities Teacher's Questions and Expected Students' Reactions	Teacher Support and Things to Remember	Method of Evaluation
	Warm up: Fact practice with percents as a review of what students already know about percents.	
T: Look at this coupon. The sale price is ____ and the regular price is _____ . I wonder what percent discount (percent off) it is.	Teachers show coupons from newspaper and discuss what information is on the coupon about the original and sale price.	
T: Let's look at one here on the elmo. This shampoo that costs $5.00 is now $1.50 off. How would we figure out what percent discount this is? S: percents need it to be 100.	Teacher shows coupon on page 62 on ELMO, students are not looking at book.	Does students' prior knowledge help them to think about this lesson?
T: We've been given a percent discount and we've been finding how much we save and what the sale price is. Now we're going to find the percent discount when we know what the sale price and the original price are.		
So, work with your partner to find the percent discount if you get $1.50 off the regular price of $5.00. Start by writing a fraction that will represent the percent discount.	Teacher monitors partners to find strategies to share.	
T: What fraction do you have that will give the percent discount?		
S: 150/100 divided by 500/100 S: 15/50 multiply by 2/2 S: 1.50/5 multiply by 20/20 S: 1.50/5.00 * 100/100 then divide by 5/5 S: 5.00-1.50 = 3.50		Do students understand why thinking about pennies helps?

T: So, we have 1.50/5.00. What will that be in pennies?	Teacher writes on board: $\frac{\text{Amount off}}{\text{Regular price}} = \frac{?}{100}$	
S: 150/500		
T: The question asks us to find the percent discount. So, this is the amount off over the regular price. How do I find a percent from that?		
S: Reduce 150 / 500 to be over 100.		
T: Work with your partner to find what percent 150/500 is.		
T: What is the percent discount?	Teacher writes as student gives reasons.	
S: 30%		
T: What did you do?	Page 63 in student book.	Can students comprehend the situation of the problem?
S: 15/50 *2 or (150/500)/5=30/100		Are students carefully reading the problem?
T: Now let's look at problem A. Let's read directions together, . If it is $0.75 off the regular price of $3.00, how much will we pay for it?		Do they relate it to pennies?
S: $2.25	Students will work in pairs	
T: With your partner, decide what fraction will help you find the percent discount then use that fraction to find the percent discount.	Walk around room to see how students will react to problem.	Are students actively thinking?
(5 - 8 minutes without help)	Share selected examples.	Are they showing their work?

T: What fraction will help you find the percent discount? S: 0.75/3.00 S: 2.25/3.00 S: 0.75/100 S: .75/2.25 S: 75/225	Teacher may write both: 0.75/3.00 or 75/300 AND 2.25/3.00 or 225/300 then write the percent represented by each to discuss reasonableness of answers. Refer to fraction with words "amount off" over "regular price".	Do they present their ideas clearly so their peers can understand their solution?
T: What is the percent discount? S: \$0.75/\$3.00 = \$0.25/\$1.00 = 25% S: 75/300 = 25/100 = 25% S: 75/300 = 0.25 = 25% S: 2.25/3.00 = 75% T: Which answer is reasonable for this problem? If we are paying \$2.25 instead of \$3.00, is it more reasonable to think we saved 25% or 75%? T: 25%	Check for examples of ways to solve: • Figuring out how much you get off for each dollar (using pennies to think of percent). • Using the 75/300 fraction to find an equivalent fraction of 25/100 or 25%. • Dividing 75 by 300 to find the decimal 0.25 and then making it into a percent.	
T: So, once we figure out the fraction of amount off over regular price, we can use that information to find the percent of the discount. But if we use sale price / regular price, what do you find? S: % of regular price you will pay.	Teacher may write "sale price" over "regular price" to contrast the different percentages.	

T: Now let's work on B1, but on this one you do not need to estimate- we will do that in B2. How much do you save? How much off?	Discuss whole group.	Can students transfer what they did in the previous problem and apply it to a different problem?
S: 50 cents saved.		
S: 50 cents off.		
T: How can I find what percent off? Remember what you did earlier with the notebooks in problem A? How did we use this fraction (amount off over regular price)?	Remind students this can be thought of in terms of pennies, 50/850, to eliminate decimals.	
S: 50/850	Also, remind students to simplify fractions to lowest terms.	Can students round .058 to 6%?
T: With your partner, use that fraction and find the percent discount.	Remind students of the fraction with words on the board.	Are they showing their work?
(Students work with partners.)		
T: What is the percent and how do I find it?		
S: Divide 17 into 1 and get 0.058, which is 6%.		
S: Divide both by 10 and get 5/85.		
S: Divide both by 5 and get 10/170.		
T: Now B2. We will estimate the numbers. What information does the coupon give us?		
S: Regular and sale price.		
T: What do we need to know before we can find the percent discount?		
S: How much we save.		
S: Regular price.		
T: How can we find our percent discount (like we have been doing) using estimates of the numbers given? What fraction should we use?		

S: (Possible answers) 18/30, 20/30, 11.80/29.50, 10/30, 17.70/29.50, 12/30 T: These are estimates. When we estimate, we want to use numbers that make our work easier. Which of these will make it easier to find the percent discount? S: 10/30 T: So, once we figure out the fraction of amount off over regular price, we can use that information to find the percent of the discount. Work with your partner to find the percent discount with 10/30. T: What is the percent discount? S: 30%	If student used 20/30 then discuss finding the percent discount from this information.	Can students explain the answers they chose? Can students transfer what they did in the previous problem and apply it to a different problem? Are they showing their work?

Evaluation:

Were students able to understand at least one way of solving this problem?

Appendix F: Seventh-Grade Lesson Plan, First Cycle

I. Background Information

Goal of the Lesson Study Group

Students will be motivated enough to take ownership in their learning.

Narrative Overview of Background Information

This is the second year of a math-science partnership grant for the mathematics teachers in the district from the state department of education. During the first year of the grant, the teachers worked in depth on content training focused in large part on the teachers' adult mathematics. This year, the teachers are taking this work into their own classrooms to reflect on the students' mathematics in actual lessons.

The seventh grade team decided to focus on the Connected Mathematics unit "Accentuate the Negative." We chose this unit because it will be taught mid-November through December break. This coincides with the time we want to teach the public lesson.

Even though order of operations is now addressed in the in fourth grade, these particular 7th grade students were not using the state curriculum during 4th grade. Given the students' background in the old standards, we felt this lesson would be beneficial and useful to contribute to students' better performance on computation problems. A competent understanding of order of operations will further benefit the students as they proceed into higher level mathematics.

Students are working alone at first because we want them to be able to think individually and then compare answers with a partner to decide who has the best strategy for finding the greatest and least values and to be able to just put strategies on the table to discuss. Students find larger numbers first because we wanted the students to be able to think about how numbers can be manipulated to create larger numbers. Typically, students seem to work better with multiplication and addition which are used by students to create larger numbers. We then have students find smaller numbers because we wanted students to be able to switch their thinking and consider how numbers can be manipulated to create smaller numbers. For the first round of our game, students work with 2 negatives and 2 positives. In the second round, we have students use 3 negatives and 1 positive, requiring them to think deeper about what needs to happen with negative numbers. The numbers can also be paired to easily divide.

We wanted students to focus on how rational numbers "work" to yield the greatest and least values. On the day before this lesson, the students reviewed GEMA (grouping, exponents, multiplication & division, addition & subtraction). Students worked problems explicitly using whole numbers, decimals, and fractions. We used the CMP launch included in Investigation 4 on page 61. They did problems B & D. Today we are focusing on A & C.

II. Unit Information

Title of the Unit: Integers and Rational Numbers

Goals of the Unit: Students will understand how to add, subtract, multiply, and divide rational numbers.

III. Lesson Information

Title of this Lesson: Properties of Operations

Goals of this Lesson: Students will explore the use of the order of operations by thinking about rational numbers and how they work.

Study Lesson Process:

Learning Activities Teacher's Questions and Expected Students' Reactions	Teacher Support and Things to Remember	Method of Evaluation
T: What are the answers for the warm up? S: 130 ----- 106 S: 27 ----- - 9 T: In a game, the goal is to write a number sentence that gives the greatest possible result using the numbers on four cards. Jeremy draws the following four cards: 4, -3, -6, 5. Joshua writes 5 – (-6) * 4 + (-3) = 41. Sarah says the result should be 26. Who is correct and why? S: Joshua is correct. S: Wendy is correct. T: Wendy starts by writing -3 – (-6) + 5^4 = What is her result?	Warm up: 6 + 20 * 5 4 – 2*10 + 7 Teacher writes the expression on board. approximately 3 minutes	Does students' prior knowledge help them think about the warm up? Are they using GEMA correctly? Are students comprehending the problem? Do they understand that one is right and one is wrong?

S: 628 S: 23		
T: How did you get the answer?		Can students explain how they got the result?
T: Now we are going to compete in a game using a similar situation to Joshua and Wendy. Using these same numbers (-6, -3, 4, 5) and using operations only once, make an expression that gives the largest answer. I want you to work individually on this.	Teacher writes the four numbers on the board. approximately 5 minutes	Are they listening to instructions for being able to solve the problem? Are students experimenting with different orders and combinations in order to find larger numbers?
T: Now, look at the responses of the person beside you. Take just a couple of minutes to check each other's answer and see if the two of you can discover an expression that gives an even bigger answer.	approximately 5 minutes If any students get rational answers that are not integers, teacher needs to specify integer answers.	Are students checking each other's answers? Are students asking for clarification about partner's explanations?
Now, check answers of the people in a 4 member team. Write an expression for the group that will give the largest answer. Write the expression for your group on white board.		How do other students respond to student explanations? Can the students explain mathematically what they are doing and why?
T: Let's decide which is going to give the largest answers. Which need to be taken down because we know they are not the highest and why do you know that?		Are students using estimation skills to decide?

T: Now using these same numbers (4, -3, -6, 5) again with any operations, make an expression that gives the smallest answer. Work on this alone.	Teacher needs to stress to the kids that they should check each other's work to be certain no mistakes were made.	How are students thinking about arranging numbers (are they just using guess and check, are they considering the bigger numbers first, are they thinking about the negatives, etc)?
Now check answers of the people in your group and decide who has the smallest answer. Remember to check the answers! Decide, as a group, if there are ways to get an even smaller answer.	Teacher writes the four numbers on the board.	
T: Now we will have another round. Find the expression that gives the largest answer with these numbers: -2, -3, -6, 4. Work on this alone.		How are students thinking about getting large numbers using three negatives and one positive?
Now check answers of the people in your group and decide who has the largest answer. Remember to check the answers! Decide, as a group, if there are ways to get an even larger answer.		Are students considering how they can use negative numbers to create positive numbers?
T: What expression gives the least answer with those numbers? Work on this alone.	If we have time for a third round: Round 3: -4, -6, -8, -12	
Now check answers of the people in your group and decide who has the smallest answer. Remember to check the answers!		Same evaluation questions apply if we have a third round.
T: For a ticket out the door, I want you to do this expression: 25 * (-3.12) + 21.3 / 3.		

Evaluation:

Are students of different states of learning being supported adequately?

Are students thinking about which operations to use with which numbers to achieve the desired outcome (especially without using some sort of guess and check)?

Are students generalizing their ideas about order of operations with positive and negative numbers so that when they look at a group of expressions they do not always have to compute the answer to know when one is larger or smaller?

Were students able to learn from their mistakes (ideas for making larger/smaller numbers) and apply it to a new problem (or a new round)?

Appendix G: Seventh-Grade Lesson Plan, Second Cycle

I. Background Information

Goal of the Lesson Study Group
Students will be motivated enough to take ownership in their learning.

Narrative Overview of Background Information
This is the second year of a math-science partnership grant for the mathematics teachers in the district from the state department of education. During the first year of the grant, the teachers worked in depth on content training that was theoretical in nature. This year, the teachers are taking this work into their own classrooms to look at the mathematics of actual lessons.

Students in this class generally require a significant amount of assistance with mathematics. Computation and reasoning skills need developing. Applying learned knowledge to a new, but similar situation is often challenging. Typically, students do not work well in groups. Off task behavior will result. Individual work is sometimes an option if the requested task was just modeled by the teacher.

Most recently, we have stressed linear relationships and a general equation of a line: $y = mx + b$. Students' prior knowledge includes identifying y-intercepts (and their meaning), recognizing coefficents, and substituting an ordered pair into a linear equation to determine if it is a solution. Most of the aforementioned work has been studied using real world examples (Walk-a-thon - money, distance).

II. Unit Information

Title of the Unit: Moving Straight Ahead

Goals of the Unit: Students will recognize linear relationships from equations, graphs, and tables; translate between these different representations; and make connections between the different representations.

III. Lesson Information

Title of this Lesson: From Pouches to Variables

Goals of this Lesson: Students will write and solve equations using the properties of equality to solve and check solutions.

Study Lesson Process:

Learning Activities Teacher's Questions and Expected Students' Reactions	Teacher Support and Things to Remember	Method of Evaluation
T: Remember the problems we did yesterday? Look at this problem on the board. We want to know how many coins are in each bag. S: 3 coins	"Getting Ready" p. 51 Teacher will show problem on the screen. Teacher gives students worksheet for warm up and problems.	Are they marking off or circling coins?
T: How can we represent this mathematically using numbers & symbols? If we do not know how many coins are in a bag, what can we say about those bags, if it is something we don't know? S: That's a variable.		What words are the students thinking about? (variable, unknown, ?)
T: If we let x represent the unknown number of coins in each pouch, how can we write symbolically that we have 5 of them? Write on your paper that x represents the number of coins in each pouch. S: 5x	Teacher needs to write that x is the number of coins in each pouch.	
T: And with those pouches, we have some other things. S: Coins. T: How can we symbolically write everything that is on the left side of that equals sign?		Do the students recognize the number of coins in each pouch is the unknown, and that this is different from the number of coins given?
S: 5x + 8 coins S: 5 pouches + 8 coins S: 5x and 8 S: x5 + 8 S: x + 8 T: So, 5 things we do not know the value of, we will call 5x. With that, we have 8 coins, so we will write 5x + 8. Now, what is all that supposed to be?		Do the students understand that the operation required is addition?
S: 23 coins. S: 23 T: How can we symbolically write this as an equation? S: 5x+8=23 S: 5 bags + 8 coins = 23 coins S: 5b + 8c = 23c S: x + 8 = 23		Do the students assign variables appropriately (not trying to write things like 5b + 8c = 23c)?

S: 5 * 1 pouch + 8 coins = 23 coins S: 5b 8c 23c		
T: Now that we have an equation, let's think about how you figured out there are 3 coins in each pouch. When you looked at the picture, what did you do first?		Are students participating?
S: I took 8 coins off this side and 8 coins off that side. S: I took off 8 coins. S: I tried numbers until I figured out I needed 3 in each pouch.		Are students thinking about mathematical operations?
T: So, taking off 8 coins means what, when we are thinking about mathematical symbols? What operation are we using?	Teacher uses vertical representation on board – maybe even write in words: take off 8 coins.	
S: Subtracting 8.		
T: So, we have 5x + 8 = 23. In order to maintain the equality, we take off 8 coins from both sides of the equals sign. Now we have 5x = 15, what is x?		
S: 3 S: 3 coins		
T: If you have 15 coins here and 5 pouches, how can you figure out how many coins are in each pouch?		
S: We multiplied. S: We divided the number of coins into 5 pouches.	Teacher may use picture first to check answer then use equation to check.	Can students use the equation to check the answer?
T: Alright, you are taking your bag to the king. Do you want to be sure you have the right answer? How would you be sure? We know what x is, so if we have x=3, and we have 5 pouches and 8 more coins, how many do you have?		
S: 23 gold coins	Teachers gives students the worksheet from CMP (with pictures already on it).	Are they writing what x represents?
T: Now, look at problem A. Just like in the warm up, represent each picture mathematically as an equation, solve the equation, show what you did on your equation. Do 1 and 2.	Teacher monitors student work, trying not to answer any questions yet. Teacher goes over problems 1 and 2 with the whole group.	Are they having x represent the unknown number of coins? Are students participating?

T: What should the equation be for number 1? S: 4x + 2 = 18 T: What does the x represent? S: Number of coins in a pouch S: Pouches	Teacher careful to see if students make distinction here between x being the number of coins in pouch and x being the pouch.	Do the students identify the variable correctly? Do they understand what the variable is?
S: Number of pouches T: We know the answer by looking at the picture. But, sometimes the equations may be a bit harder to solve, so we need additional strategies. What would we do to this equation if we only had the equation and not the picture? What did you do first to the picture? S: Take off 2 coins. S: Subtract 2 coins. S: Subtract 2 coins on the left and right sides. T: Okay, then what? S: Then you have 4 bags is 16, so each bag has 4 coins in it.	Have student picture on the elmo and have students. Student is translating as teacher writes.	
S: 4x = 16, so x is 4. S: x = 16. T: How do I know if this is the correct number of coins? S: Open the pouch and see. :) S: Count 4 coins for every pouch. S: Put 4 in the equation for x. T: What should the equation be for number 2? S: 3x + 6 = 12 S: x + 2 + x + 2 + x + 2 = 12 S: 3 (x + 2) = 12 T: What does x represent in this equation? S: Number of coins in the pouch T: And, what should we do first in order to solve this equation? S: Take off 2 coins.	Teacher needs to explain that they are to do one question per half sheet or quarter sheet??	

S: Take off 6 coins. T: And then what? S: Then 3 bags and a 6, so 1 bag has 2 coins in it. S: 3x = 6, so x = 2. S: x = 6	Students set up number 3.	Can students set up equation correctly?
T: How do I know if this answer is correct? S: Check it. S: Solve the equation with 2 in it. T: Now set up number 3. What is the equation?		Did they understand a variable is on each side and coins are give on each side?
S: 3x + 3 = 2x + 9 T: What does x represent? S: Number of gold coins in a pouch.	Teacher leads class in solving it. Teacher should probe to see if students understand subtracting 3 is subtracting 3 coins, not 3 bags (on the left side, there are 3 coins and 3 bags).	Are the students participating?
T: What should we do first to find out how many coins are in each pouch? S: Take off 3 coins. S: Subtract 3 coins. S: Subtract 9 coins. T: Now what happens? What is the equation? S: 3x = 2x + 6 S: 3x + 3 – 9 = 2x T: Now what should we do? S: Take 2 pouches off. S: Take 3 pouches off. S: There are 6 coins in each pouch.		Are the students checking with only the picture or are they using the equation?
T: What is the equation? S: x = 6 T: How do we know if it is right? S: Count 6 coins for each pouch.	Students do number 4 alone. Teacher goes over number 4 whole group.	

T: Do 4 by yourself. T: Now you are going to do B. There are 4 equations for you to solve, so you get to skip the first step you did on A – you do not have to write the equation (it's already there). You can draw a picture if you want, but you do not need the picture to solve the equations.	Teacher needs to give students sheets divided in half back & front or in fourths on one side. Teacher takes these up as students leave.	Are the students drawing pictures to solve the equations?

Evaluation:
Are students writing and solving equations without pictures?

Are the students combining like terms in solving equations?

Do the students understand what the solution is and can they check to see if it is correct?

Do the students understand that equality must be maintained throughout the process?