## Situations Group Work Session UMD 11.18.06 Evan's notes

## Small group work with inverse trig functions prompt

Unit circle: How do we show an inverse on a unit circle?
Key idea: switching x and y values
Trig identities and the notion of shifting - thinking of phases, if you manipulate the sine function and others to get the inverse...
Constructing graphs
Understanding functions in general: What is the inverse of a function? It might be useful to think more about inverse of function.
Reflection over $\mathrm{y}=\mathrm{x}$; idea of transformations
What's the relationship between different trig functions?

## Small group generated these possible foci

Different ways of describing inverse
Unit circle, graphing, trig ideas
Notational issues
$\operatorname{Sin}^{\wedge} 2 x$ is multiplicative, $\sin ^{\wedge}-1 x$ is not
$\operatorname{Sin}^{\wedge}-1 x$ versus $(\sin x)^{\wedge}-1$
$\mathrm{F}^{\wedge} 2(\mathrm{x})$ compared to $(\mathrm{f}(\mathrm{x}))^{\wedge} 2$
One-to-one idea
Role of identity
Reflections/transformations of graphs
Right triangle trig ideas could be a source for people to focus on ideas of ratio when
thinking about trig functions

## Small group generated these possible prompts

Calculator output for sqrt8; take result and multiply by itself - new result not 8 (from Glenda)

Nesrin: During a CMP lesson that involved collecting and graphing data
Students did jumping jacks
Counted the number of jumping jacks at intervals of 10 seconds
Created a table to record
Took numbers and created a graph
The students wondered if they should connect the points
Hala: Professional development
Scale factor, unit rate and fraction distinction
One teacher seemed to be using ideas of fractions when making sense of ratio situations when she asked, "Can I say something about the scale factor? When you have a ratio you can multiply the numerator and the denominator by the same number, say 20 , you don't change the size of the fraction because you are really just multiplying by a factor of 1 . You're not changing the amount that you have, just the size of the pieces."
Are we just multiplying by one, or are we

Possible foci: unit rate vs. ratio vs. fraction
Context: What are you comparing in ratio situations?
Teacher said ratio, but talked about it in terms of fractions - meaning of the parts of the fractions
Ideas of within and between ratio comparisons, multiplicative concepts
Revised Potential Prompt from Hala:
T: Can I say something about the scale factor? This just popped to the front of my brain. Conceptually $[\ldots]$ the reason we can multiply by 20 in the numerator and 20 in the denominator is because you are really multiplying by 1 and we are not changing the size of the fraction, we are not changing the piece, we are changing the size of the pieces.
[...] You are multiplying by a factor of 1 , the whole is the same but the parts are smaller. R: So you are now trying to think about it like a fraction when you say the whole is the same. See, the ratio idea is really about comparing two quantities related to each other. T: So how does that change? Because we are now multiplying by a factor of one? We've changed the context of the problem?
From Silver et al. (under review). Where is the mathematics? Examining teachers' mathematical learning opportunities in practice-based professional learning tasks.

Angi: High school algebra class
Student divides $\mathrm{x} /(\mathrm{x}+2)$ and gets $1 / 2$.
Is this a good one to pursue?
[General area to mine: reflections from prospective teachers during field experiences.]
Possible content areas to explore for prompts: confusing fractions versus ratio; division of fractions (Why does dividing the numerator and the denominator of the dividend by the numerator and the denominator of the divisor always work?); multiplying binomials and skipping the middle terms $(a+b)^{2}=a^{2}+b^{2} ? ; a-(b+c)=a-b-c$ ?; continuous versus discrete quantities

## Analyzing Situations

Iris Weiss: We need to develop different vocabulary for different things.
Framework ideas
Can the big ideas (by themselves) at the secondary level afford a framework for mathematics teachers' knowledge? What else would we be leaving out?
Concept map: functions - what leads into functions, where do functions lead?
Connections, process standards - how do these relate to content?
How is our knowledge organized?
Can one develop concept maps that identify big ideas and link teachers' mathematical activities? Is this reasonable?
Maybe we need to begin using what we have and synthesize this into a less complicated visual representation.
Are these worth connecting?

## Alternatives to a framework consisting of three proposed lenses

Use the existing content/process standards.
Could we create a visual representation of framework? What about a matrix?
What about a database - searchable - not filling it out so much, but thinking from
different audiences and having them access the database based on what their particular interests or needs are?
The dimensions of the matrix could be the big lenses.
The framework could serve as a heuristic for analyzing preservice courses, not so much as a way to generate preservice courses. It could also help to generate needed conversation between interested parties.
Mark Thames: metaphor of applied mathematics. Identify major mathematical problems of teaching and using this as a lens. Consider the tasks of teaching as mathematical tasks of teaching or mathematical problem solving.
We need a language of mathematical activities in which teachers engage.
Keeping this work close to practice and pedagogy keeps it from becoming unrelated to what's important.

## Contacts who or resources that might help us generate new prompts

Professional developers
Professional development videos
Student responses on open-ended assessment items - AP courses, TIMSS
Field test teachers of reform curricula - Core Plus
Dan Chazan and Pat Herbst's work with animations
MSP Net
Jim Lewis, Bill Haver, other mathematicians who might be interested PCMI, Sharon Senk

What else do we need to be doing as we move forward?
Create an alternative to the Usiskin book. It seems like this project is producing information that would be valuable that is not currently out there.
Articulating more fully our methodological process
Think about audience and develop lenses for particular audiences

## How could we use these situations?

Get our work into an NSF solicitation so others know that it is there and can be used.
Develop one-hour courses that develop the mathematical ideas that are coordinated with required math content courses for prospective secondary mathematics teachers.

