# Situation 06: Cross Multiplication

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## Prompt

An Algebra 1 class is in the middle of an example, for which the teacher has written the following on the whiteboard:

$$\sqrt{\frac{12}{81}} = \sqrt{\frac{3 \cdot 4}{3 \cdot 27}} = \sqrt{\frac{4}{27}} =$$

A student asks, "When we're doing this kind of problem, will it always be possible to cross multiply?"

## Commentary

When learning new material, students often attempt to apply procedures they are comfortable with even though those procedures may not be mathematically appropriate for the new material. This appears to be the case here, where a student imposes a process used to solve proportions on this problem involving the simplification of a fractional radical. The first focus explains the student's apparent thinking, and the associate misapplication of the process known as "cross multiplication." The second focus gives the conditions that led to the apparent "cross multiplication" and explains when this condition will arise.

## Mathematical Foci

### Mathematical Focus 1

*Cross multiplication is a process often taught to students learning how to solve proportions. The mnemonic often becomes associated with solving problems involving fractions.*

"Cross multiply" is the name given to a process often used to solve for an unknown in a proportion. In the proportion $\frac{a}{b} = \frac{c}{d}$, one "multiplies across" the equals sign in a diagonal fashion $\frac{a}{b} \times \frac{c}{d}$ to obtain the equation $a \cdot d = b \cdot c$. It appears that the student,

while thinking about the problem on the board, has recognized that $3 \cdot 27 = 81$ (using the 3 from the numerator) and that $3 \cdot 4 = 12$ (using the 3 from the denominator) and has confused the process of simplifying fractions with the process of "cross multiplication."

Presumably, the student is thinking of something along the lines of $\sqrt{\dfrac{12}{81}} = \sqrt{\dfrac{3 \cdot 4}{3 \cdot 27}}$ .

*Mathematical Focus 2*
*Simplifying fractions requires finding factors (greater than 1) that are common to both the numerator and the denominator.*

The student's question might be more appropriately rephrased to the following: Will it always be possible to find a common factor in the numerator and denominator of a fraction? The answer to this is *yes* if the fraction is not in simplified form and one is only interested in factors greater than 1. This may lead to situations like the previous one, where it appears as if some form of "cross multiplication" is being used, though it is not.

$$\sqrt{\frac{12}{81}} = \sqrt{\frac{3 \cdot 4}{3 \cdot 27}}$$

$$\sqrt{\frac{100}{225}} = \sqrt{\frac{25 \cdot 4}{25 \cdot 9}}$$

The following counterexample shows that, when there is no common factor greater than 1, the misapplied "cross multiplication" does not produce the original fraction.

$$\sqrt{\frac{14}{15}} = \sqrt{\frac{2 \cdot 7}{3 \cdot 5}} \neq \sqrt{\frac{2 \cdot 7}{3 \cdot 5}} = \sqrt{\frac{3 \cdot 7}{2 \cdot 5}} = \sqrt{\frac{21}{10}}$$