Situation: Dividing an Inequality by a Negative Number
Prepared at UGA
Center for Proficiency in Teaching Mathematics

7/8/2006 - Pawel Nazarewicz

1 Prompt

A rising ninth-grader was presented with a problem that asked her to solve for the variable x:

$$6 - 3x \leq 24$$

After subtracting 6 from both sides, the student got

$$-3x \leq 18$$

By correctly dividing both sides by -3, arrived at the solution

$$x \geq -6$$

When asked "Why did you switch the sign?", she said "I don't know, but I know I'm supposed to do that."

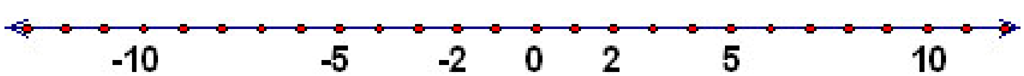
Question

What are some proofs, examples, or explorations that a teacher could do with the student to show her the reasons why $-x > 5$ implies $x < -5$?

2 Mathematical Foci

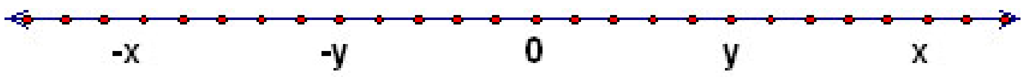
Mathematical Focus 1: An Example on the Number Line

Consider the relative value of the numbers 2 and 5 on the number line:



On the number line, a number which is *greater than* another number is to the right of that number. So, $5 > 2$. Dividing both sides by (-1) gives us -5 and -2. But -5 is to the left of -2, so $-5 < -2$.

To offer a loose generalization[1], pick any two numbers on the number line, so that x is to the right of y, or $x > y$. Their negatives will be *reflections* across the origin, like this :



So if $x > y$, $-x < -y$. We can see that by dividing both sides by a negative, the inequality sign flips.

Mathematical Focus 2: An Inductive Approach

A student could be asked to find a list (and then a set) of numbers which satisfy the inequality:

$$-x > 5$$

A key to understanding this is knowing that is seeing $-x = (-1) \cdot (x)$. So the question becomes *"-1 times what number is greater than 5?"* A student also has to understand that in order to get a positive product, a negative must be multiplied by other negative.

Students can then fill in this table and examine it inductively:

x	$(-1)(x)$	Greater than 5?
-4	$(-1)(-4) = 4$	No
-8	$(-1)(-8) = 8$	Yes
-6	$(-1)(-6) = 6$	Yes

The solution set would then be something like this:

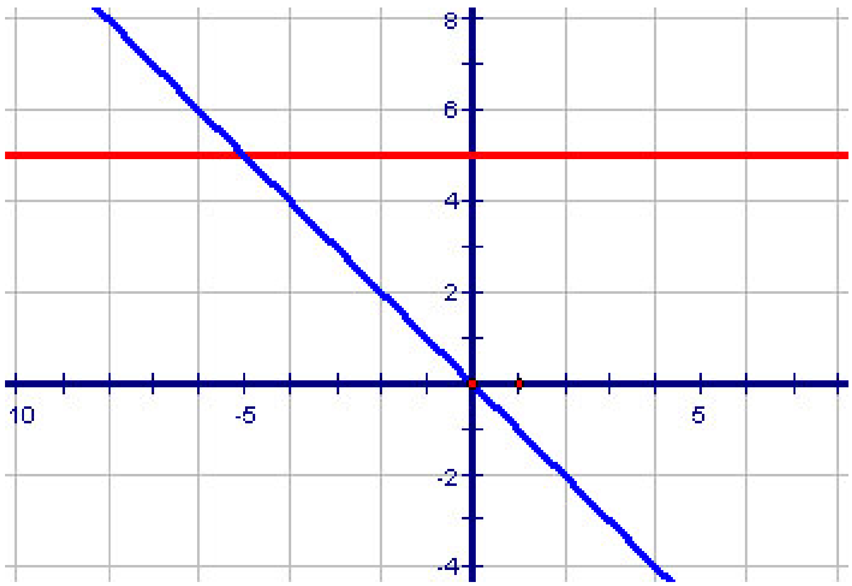
$$\{-5.01, -5.5, -6, -7, -8, -9, -10...\}$$

So basically, $x < -5$.

[1]This is a great area for discussing and developing a more *rigerous proof.* Namely, if $x > y$, prove that $-x < -y$. Students could look at a three cases: Both x and y are to the right of 0, one is to the right, one is to the left, and finally, x and y are both to the left of 0. Examples of each would follow

Mathematical Focus 3: Two Equations on the Cartesian Plane

$-x > 5$ can be seen as two equations: $y = -x$ and $y = 5$. Graphing both of those yields the following:



So the graph $y = -x$ is greater than $y = 5$ for all values of x less than 5.