Getting Started with Open-Ended Assessment

Consider the "Broken Calculator" problem: "How would you make a calculator display the number 75 if the 5 key was broken?" How many solutions can you find? What solutions would you expect from your students? **Figure 1** contains some of the correct solutions we received from students. Note the variation in students' use of operations and number sense. We also received a number of incorrect solutions (see **fig. 2**), including some that give a result of 75 but do not adhere to the condition that the 5 key is broken, and others that do not, for various reasons, give a result of 75.

We refer to problems such as the Broken Calculator problem as *open-ended* because there are multiple solution strategies and multiple correct answers. As such, open-ended assessment items encourage students to demonstrate their understanding (or lack of understanding) in creative and informative ways. In fact, as is often the case with open-ended problems, many of the incorrect solutions to the problem still demonstrate students' number sense and computational abilities. Yet others, such as 7 + 4 + 1 = 75, bring into question the students' understanding of place value and the operation of addition. Thus, the Broken Calculator problem allows students to demonstrate their sense of operations as well as their ability to choose operations that will accomplish given mathematical objectives. This problem also serves as a sort of template; it can be easily modified by changing the target number or the broken key (to an operation key or multiple keys, for example). The following are two variations of this problem:

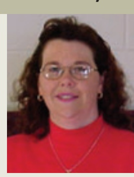
- How would you get your calculator to display 75 if all the odd-numbered keys were broken?
- How would you get your calculator to display 75 if the 5 key and the + key were broken?

Such changes can be made in order to alter the level of difficulty, to assess understanding of different mathematical concepts, and to make comparisons with students' previous responses.

Characteristics of Open-Ended Assessment Items

Using open-ended assessment items supports students in achieving the mathematical understanding that the National Council of Teachers of Mathematics (NCTM) advo-

By Keith R. Leatham, Kathy Lawrence, and Denise S. Mewborn


Keith Leatham, kleatham@mathed.byu.edu, is an assistant professor in the Department of Mathematics Education at Brigham Young University in Provo, Utah. He is interested in mathematics teacher education and in the use of technology in the teaching and learning of mathematics. Kathy Lawrence, klawrence@jackson.k12.ga.us, teaches fourth grade at West Jackson Intermediate School in Hoschton, Georgia. She is interested in differentiation of instruction to meet the needs of all learners and in teaching mathematics through connections with everyday life experiences. Denise Mewborn, dmewborn@uga.edu, is an associate professor in the Department of Mathematics Education at the University of Georgia in Athens. Her interests include elementary mathematics teacher education and professional development.

Figure 1

Correct solutions to the Broken Calculator problem

- 74 + 1 = 75
- 82 – 7 = 75
- 30 + 30 = 60; 60 + 10 = 70; 70 + 4 = 74; 74 + 1 = 75
  You could also do these:
  76 – 1 = 75
  77 – 2 = 75
  78 – 3 = 75, etc.
- (8 + 7) + (8 + 7) + (8 + 7) + (8 + 7) + (8 + 7) = 75

Figure 2

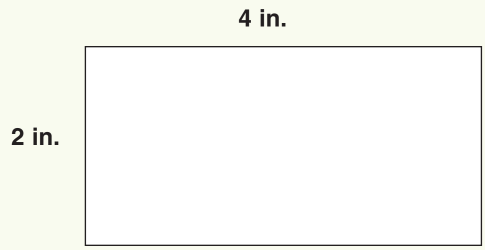
Incorrect solutions to the Broken Calculator problem (recall that the 5 key is broken)

- 25 × 3 = 75
- 100 – 25 = 75
- 60 + 5 + 5 + 5 = 75
- 7 + 4 + 1 = 75
- Put in 15 × 5 and it will put up 75.
- Press 15 times and that is how you can use five in a different way.

Figure 3

Item meeting open-ended criteria

Draw a rectangle whose area is twice the area of the rectangle below. Explain how you know that the area of your rectangle is twice the area of the initial rectangle.



cates (NCTM 1995, 2000). Open-ended problems elicit reasoning, problem solving, and communication. In addition, the types of problems that we use to assess student learning clearly communicate what we value in mathematics. If we assess only procedural skills and algorithmic thinking, we lead students to a limited view of mathematics. If, in addition to procedural competency, we assess reasoning, problem solving, and communication, however, we let students know that mathematics is a subject that can be understood conceptually, requires reasoning, and demands communication beyond equations and formulas.

High-quality, open-ended assessment items should (1) involve significant mathematics; (2) have the potential to elicit a range of responses, from incorrect to simplistic to generalized; and (3) strike the delicate balance between providing too much information, which makes the problem restrictive and closed, and too little information, which makes the problem ambiguous. **Figure 3** shows a problem that meets these three criteria. This problem involves significant mathematics because it requires that students understand the spatial quantity we call area as well as the relationship between the two linear measures (length and width) and the resulting area measure. For example, common errors on this problem include doubling each linear measure (which results in a four-fold increase in area) or computing the new area correctly (36 square units) but selecting linear measures whose sum is 36 (for example, 24 and 12). Although most responses would involve some computation, this is not the primary focus of the problem. The problem yielded a wide range of responses when used in the classroom. This problem also offers an example of the balance between giving too little and too much information. If the directions had been more explicit by asking students to first find the area of the new rectangle and then provide possible dimensions, the problem would have been less open, less challenging, and less revealing of students' thinking. If the problem had been more open by merely asking students to produce rectangles with an area greater than that of the given rectangle, it would have been less challenging and less revealing of students' thinking about the relationship between area and the linear measures that comprise the area measure. For more information about the characteristics of open-ended assessment items, see Cooney, Sanchez, and Ice (2001) and Cooney et al. (1996).

Starting to use open-ended assessment items with their students is often difficult for teachers because creating items that meet these criteria is both challenging and time-consuming. We engaged in a project to develop a pool of these items and make them accessible to teachers because we believe that these types of items have tremendous power to elicit student thinking, and that teachers are eager to use and eventually create their own such problems when given the opportunity to experience their value. Kathy Lawrence, one of the authors of this article, was one of the teachers who pilot-tested items in her fourth-grade classroom. As a result of this project, a set of approximately 450 items spanning the content Standards in *Principles and Standards for School Mathematics* (NCTM 2000) is now available at www.heinemann.com/math in a searchable, online database with accompanying examples of student work (Cooney et al. 2001). (A demo is available at no charge; full access requires a registration fee.) In the next section, Kathy describes how she got started using open-ended items, the various ways in which she has used these items in her classroom, and how she and her students benefited from using them. Our comments are added in italics.

Open-Ended Assessment in a Fourth-Grade Classroom

I have been using open-ended assessment items with my culturally and economically diverse third and fourth graders for four years. I initially began using open-ended assessment items because I was asked to pilot-test them as part of a project through the local university. I have continued to use these items in my teaching—for both instructional and assessment purposes. I gain new insight into my students' thinking every time I use these items.

Getting started

My involvement in the project gave me a large supply of items and a motivation for getting started. I was curious to see how my students would do. I thought that I had a pretty good idea of where my students stood, both individually and collectively, with respect to their understanding of specific mathematics topics. Therefore, I was eager to use the items and to read my students' responses. Getting started, however, was not as simple as just passing out the problems. Open-ended items almost always ask students to explain the reasoning behind their responses. My students had seldom been asked to explain themselves in mathematics, so at first they did not understand what explaining their thinking meant. They could do certain computations, but they did not know how to explain why they did them or why they worked. When I first started giving students these problems, they often simply mimicked what I had said in class.

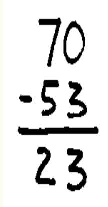
I created a "name" for these kinds of problems so that my students would begin to see them as a genre of mathematics problems (see Reardon [1990] for a discussion of helping students see assessment items as fitting within a particular genre). We referred to these as the "Terrible Tommy" problems because "Terrible Tommy" is the name I give to the fictitious student in my classroom who is always getting into trouble, goofing off, or falling behind. The task became for students to explain their answers in a way that Terrible Tommy would understand. This seemed to help the students understand that their goal was to provide enough detail about both their thinking and the mathematical processes they used so that another 9-year-old could follow their reasoning.

The first problem I gave the students to solve in the Terrible Tommy setting involved a common error or misconception with subtraction (see **fig. 4**). My instructions to the students were as follows: "Terrible Tommy has done this problem, and he did it wrong. Tell him what is wrong and explain a way to fix it." I was confident that my students would do well on this problem because we had spent the first couple of weeks of school reviewing place value and operations. When I got the responses back, I was surprised. Many of my students simply wrote, "Terrible Tommy forgot to borrow." I could not tell for certain what the students did or did not understand from this response, and I realized that I needed to better communicate my expectations to my students.

Figure 4

The first "Terrible Tommy" problem

What is wrong with Terrible Tommy's reasoning?



The first few open-ended items that I distributed were the cause of much whining and complaining because many students did not understand why they had to explain if they "just knew" the answer. In many cases, they meant that they could solve the problem in their heads and did not see the need to explain how they solved it. They were also reluctant to draw pictures to supplement their explanations because they thought they had outgrown drawing pictures. With persistence, I was able to get them to explain what they did in their heads, and over time, their reluctance disappeared.

I used several strategies to help students understand what I was looking for when I asked them to explain their answers. First, I removed the assessment component of the task to make it less threatening. I explained to students that I wanted them to do their best work on these items because it would help me determine when we were ready to move to a new topic and when we needed more work on something. I told them that I would collect and read their responses, but I would not enter a grade in the grade book. After I collected and reviewed their responses, I made overhead transparencies to show caricatures of three or four responses—some very weak, some very strong, some very typical of most of the class, some with pictures, some without. As a class, we discussed what made a particular response thorough or incomplete, mathematically correct or erroneous, elegant or convoluted. I never attached a particular student's name to any of these solutions, although many students recognized a solution similar to theirs on the overhead. Occasionally, a student who had given an incomplete solution would say, after seeing a solution on the overhead, "That's what I wrote. I meant the very same thing as in the other explanation you just showed, but I just said it with fewer words." I would then take the opportunity to point out some of the other possible interpretations of the less detailed solution.

I gain new insight into my students' thinking every time I use open-ended items

The next step that I took in getting started with these items was to give students problems to solve in class and then have a sharing session. To help students feel comfortable sharing their responses, I often had them give responses on small slates. When they held up their slates, I could see the range of responses and call on students to share a variety of solutions. I made a point of praising students' willingness to share responses, whether or not they were correct. I also made a point of highlighting students' work that showed pictures, tables, or other visual aids to the written explanation to encourage more of this type of work.

Every year when I begin with the first Terrible Tommy problem, I have to spend concentrated time on making sure that students understand what it means to explain an answer. Getting students to explain what they are thinking has become easier, however, because our textbook series requires explanations for some homework and test items. This is particularly evident this year because I have students who have been working out of our textbook series since first grade. They come to me in fourth grade with a fairly good understanding of what is expected when a problem asks for an explanation.

Another possible approach to helping students gain an understanding of what a thorough explanation looks like is to have them work in pairs or small groups to solve the problem and craft the response. In this type of setting, they actually have to explain their thinking to someone else, and saying it aloud first can make it easier to put their thoughts in writing.

Benefits to students

As a result of using open-ended assessment items, I have noted significant improvement in my students' self-confidence and their willingness to share their thinking with others. In fact, they begin to take pride in their explanations and find satisfaction in being able to explain what they are doing and why. They begin to see that there is a point to explaining their thinking. This leads to students feeling more ownership of their mathematical learning. Many of them have been able to achieve a new level of depth or generalization in their thinking, and they have recognized and appreciated this change in their thinking.

Solving these problems has helped my students develop a different set of values about sharing their thinking with others. Before starting this project, my students were not particularly good at sharing their ideas with or helping one another. It was as if they believed there was a finite amount of knowledge in the room, and if they shared what they knew with someone else, somehow it diminished their own knowledge. By explaining their thinking to the fictitious Terrible Tommy, my students develop empathy for others and become more willing to help one another. They also develop self-confidence in their ability to explain mathematics to others—Terrible Tommy, their peers, and me. At the beginning of the school year, some of my students absolutely refused to help a peer with anything. I now see that this is a self-confidence issue. Doing the Terrible Tommy problems helps them gain confidence in their ability to explain themselves, and they are much more willing, and even eager, to help others. In addition, they are able to help by explaining rather than by simply giving answers.

Benefits to the teacher

Prior to using open-ended items, I had fallen into the trap of thinking that because my students could do certain types of problems, they understood what they were doing. I became aware that I was wrongly assuming that my students' performance on tests and homework was indicative of conceptual understanding. My students' solutions to open-ended items gave me a window into their thinking and helped me gauge which material I needed to revisit and which material needed only occasional review.

Because I have experienced the benefits of these types of questions, I ask a lot more "why" and "explain" types of questions in my instruction and in my various assessments. After seeing the items that the project writers developed, I have been able to develop additional items to assess the concepts that are important to me. Because I have seen such variety in the solutions that students produce, I have become more aware of the varied ways in which they think. Therefore, when we solve problems as a class, I am no longer satisfied with having just one student share a correct solution. I now have a habit of asking, "Did anyone solve this in a different way?"

Kathy's experience provides an example of putting into action the Assessment Principle from Principles and Standards for School Mathematics *(NCTM 2000), which calls for blurring the line between instruction and assessment: "[Assessment] should be an integral part of instruction that informs and guides teachers as they make instructional decisions. Assessment should not merely be done* to *students; rather, it should also be done* for *students, to guide and enhance their learning" (p. 22). Kathy's students are learning from items that also give her valuable information that will help guide future instructional decisions.*

An unexpected benefit of using these problems is that it has enabled me to engage students who ordinarily do not like to write or do not like to do mathematics. For example, one child enjoyed mathematics but did not have strong language skills, and I could not get him to write in language arts. Because these problems gave him an opportunity to write in the context of mathematics, he was willing to write and even eager to explain his thinking. These problems afforded me a vehicle to reach a child whom I was not reaching previously. I have other students who would rather do just about anything other than mathematics. Because I was able to link mathematics to writing—something with which they were very comfortable—the students were willing to engage in mathematics.

The link between mathematics and literacy can also be a powerful hook for teachers who are less comfortable with mathematics instruction but are at ease with literacy instruction. If teachers can see how these open-ended items call on students to apply the same types of skills that are required in language arts, perhaps they will be more inclined to implement them in their classrooms.

The first open-ended items I distributed caused much whining and complaining

Examples of student work on open-ended assessment items are helpful when communicating with parents about their child's progress. Parents often question a teacher's assessment of their child's understanding in mathematics. Previously, I did not have a concrete way to respond to these parents, but now I can show them exactly how well their child understands a particular topic. Parents can see that, although their child may be able to accurately complete a worksheet, the child may have a fragile understanding of the topic.

Sharing examples of student work with teaching colleagues can also be a useful way to communicate about student learning. Teachers who teach the same group of students but in different subjects might benefit from comparing students' written work in several domains in order to better assess individual and collective strengths and needs. General education and special education teachers, as well as other support teachers (such as those in the gifted and early intervention programs) who pull students from class, might also benefit from discussing specific examples of a student's work. Often, teachers must collaborate in order to *develop an appropriate individualized education plan, to assign a grade, or to coordinate assignments. Samples of student work on open-ended items can provide a starting point for these discussions while keeping them grounded in student learning.*

Continuing struggles

One of the things with which I continually struggle is the interpretation of students' responses. I struggle to ascertain what I can infer about a student's knowledge from his or her response. As I have worked to interpret students' responses and what they tell me about learning, I have become more adept at modifying the assessment items in my textbook, and I have come to see the value of having a rubric for scoring student work.

Assessment problems in instructional materials sometimes appear to be open-ended but are actually constructed in such a way that discerning conceptual errors from careless ones is difficult. For example, an assessment item on a textbook test included a variety of figures and asked students to list the ones that were not polygons and explain why they were not polygons. I had students who listed some, but not all, of the nonpolygons and gave adequate explanations for why the shapes were not polygons. Other students included a shape that was a polygon in their list, but their explanations showed me that they knew what a polygon was. The second time I used that item, I changed it so that the students had to mark "Yes, it is a polygon" or "No, it is not a polygon" underneath each figure and then add an explanation for those that were not polygons. This format gave me a much better sense of which students understood the definition of a polygon. My experience with using open-ended assessment items has opened my eyes to the strengths and shortcomings of other kinds of assessment items and has given me the confidence to modify items to better reveal students' thinking.

As previously mentioned, our experience with writing and pilot-testing open-ended items suggests that they are very challenging to write. We have discovered, however, several useful "frames" for such problems that enable teachers to modify an item to assess particular content. These frames range from simply asking students to explain their reasoning to providing competing solution strategies and asking students to justify which strategy they think is correct. A discussion of these general principles can be accessed free of charge from the online database by Cooney et al. (2001) at www.heinemann.com/math. Frames of this nature are invaluable in getting started on creating meaningful open-ended items.

The problem in **figure 4** contains an example of student responses that I find difficult to interpret and that have led me to see the value in creating simple scoring rubrics. Many of my students gave responses similar to this one: "Terrible Tommy's answer is not right because if you have zero bears you cannot give three away. So you have to get more. So you borrow from the tens place. Then you take away three from ten because you borrowed ten. Then you take away what's left in the tens place." From this response I can infer that the child knows the steps of the procedure and even understands the reasons for some of them, but what can I confidently surmise about the child's understanding of place value or subtraction or the process of trading? This type of dilemma has prompted me to write what I would consider a thorough, top-notch response to an item before I look at any of the student responses. I then craft a rubric to help me think about the various gradients in student thinking that I am likely to see. I am still often caught off-guard by an unexpected student response, but having a rubric in hand before I begin reading responses really helps me think about the mathematics that I value and expect to see in the solution and explanation.

Conclusion

Our collective experiences with creating and using open-ended assessment items have convinced us that students will respond positively to the challenge of providing more detailed explanations of their solutions. Although getting started with using these items is not easy, the rewards quickly make the effort worthwhile. We suggest the following as keys to getting started with using open-ended assessment in a classroom setting:

- Begin by using existing open-ended items that fit the content you are currently teaching. Start to create your own items as you recognize important mathematical ideas in your curriculum for which you do not have items.
- Give students ample opportunities in a variety of contexts to understand your expectations. Provide examples of good and poor-quality responses and invite your students to articulate the characteristics of these responses.

- Start by using open-ended items for formative evaluation. Use them to inform your teaching, to acquire information about your students, and to help them learn how to successfully respond.
- Start slowly with using open-ended items. Students will need more time to respond to these items and they take longer to grade than traditional items.

Using open-ended items in classrooms can open a new window on our students' mathematical understanding. In particular, if we expect students to gain conceptual understanding of mathematics, we must give them opportunities to demonstrate that understanding. Open-ended assessment items can provide such an opportunity. These items have the potential to assist teachers and students in meeting the vision of *Principles and Standards.*

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The work described in this article was done as part of the project "Developing Open-Ended Assessment Items for Georgia's Teachers of Mathematics: Grades 3–11," directed by Thomas J. Cooney and funded by the Georgia Eisenhower Higher Education Program for Improving Mathematics and Science Instruction. The authors wish to express their appreciation to their colleagues, Thomas Cooney and Wendy Sanchez, and to all those who pilot-tested items. ▲