1785

The University of Georgia

The Babylonian Square Root Method

If the Babylonians need to know the approximation of the $\sqrt{A}$. They first would chose a number x_0. If x_0 is too large then $\frac{A}{x_0} < \frac{A}{\sqrt{A}} = \sqrt{A}$, so x_0 is too small an estimate of $\sqrt{A}$. Similarly, if x_0 is too small then A/x_0 is too large and estimate of $\sqrt{A}$. Either x_0 or A/x_0 is an underestimate and the other is an overestimate. The Babylonians decided to take an average:

$$x_1 = \frac{1}{2}(x_0 + \frac{A}{x_0})$$

This would give them a better approximation to $\sqrt{A}$. Using an iterative method, that is substituting x_1 back into the equation over and over would give a better approximation each time.

Lets' try when A = 3, that is what is the $\sqrt{3}$? Chose x_0 = 1.

x_1 = ½(1+3/1) = 2
x_2 = ½(2+3/2) = 7/4 = 1.75
x_3 = ½(7/4+3/(7/4)) = 97/56 = 1.73214
x_4= ½(97/56+3/(97/56)) = 18817/10864 = 1.7320508

Which is correct to seven decimal places. Do you recognize the similarity with Newton's iterative formula? (Edwards)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$