NAME_______________________________________________________
Show work and write clearly. Answers without work to support them will not receive full credit. Answers without correct notation will not receive full credit.

Answer only 11 of the 12 questions. Where necessary, estimate to 4 decimal places.

1. Without using the allsums program, estimate the area under the graph of \( f(x) = x^2 + \frac{2}{x} \) from \( x = 2 \) to \( x = 4 \) using four approximating rectangles and midpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate? Explain.

2. Sketch the region enclosed by \( y = x - 1 \) and \( x = 3 - y^2 \). Sketch the area.

3. a. Find the average value of \( f(x) = \frac{3x}{\sqrt{1 - x^2}} \) from \( x = 0 \) to \( x = \frac{1}{2} \).
   
   b. Find \( c \) such that average value of \( f \) equals \( f(c) \). Explain.
   
   c. Sketch the graph of the function and a rectangle whose area is the same as the area under the graph of \( f \).

4. Find the volume of the solid formed by revolving the region bounded by \( y = x^3 + x + 1, y = 1, \) and \( x = 1 \) about the line \( x = 2 \). Sketch the area.

5. Find the length of the curve \( y = \ln(\cos x), 0 \leq x \leq \frac{\pi}{4} \).

6. Find the area of the surface of revolution obtained by rotating the curve \( y = \frac{x^5}{10} + \frac{1}{6x^3}, 1 \leq x \leq 2 \) about the \( x \)-axis.

7. Solve the initial value problem: \( e^{-x^2} (y^2 - 1) y' + xy = 0, y(0) = 1 \).

8. Derive the formula for the derivative of \( \sin^{-1}(x) \). Show all steps and be specific.

9. Find the derivatives of the following:
   
   a. \( y = \sin^{-1}(x) + x\sqrt{1-x^2} \)
   
   b. \( y = (\sin x)^{\ln x} \)

10. Find the following limits:

    a. \( \lim_{x \to -\infty} \frac{x^2}{e^{-x}} \)

    b. \( \lim_{x \to -\infty} \sqrt{x} \)

    c. \( \lim_{x \to \infty} \left( x - \sqrt{x^2 + x} \right) \)

11. Find \( \int \frac{\tan^3 x}{\cos^4 x} \, dx \)

12. Find \( \int e^x \cos(2x) \, dx \)