

Lesson Plan 3 Conic Sections

Appropriate for Grades 6-9

As presented by:

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Abstract/Personal Reflections

The unit in Algebra II that students and teachers dread the most seems to be the one on conic sections. To overcome this fear, I have developed a series of activities that have helped my students be more successful in this unit. The manipulative approach used in Lesson 1 of the study helps students realize that there is nothing to fear and that there is a real reason for studying these unique curves. Using technology helps students spend less time erasing their mistakes and makes them more eager to try to graph the different curves.

As they become more successful on the computer, students are more willing to take the task to paper and produce their own art work based on conic sections. The individual curves become more and more realistic as the students investigate the real-world areas that utilize each curve. Students are encouraged to investigate the use of conic sections in our world, looking at everything from magazines, to amusement parks, to the Internet

When each student completes his or her project, there is a tremendous sense of accomplishment and pride. Every student knows this is one Algebra II test that all can pass. Parents and students have acknowledged to me that one of the things they remember most about Algebra II is the art project. Success is its own reward!

For me, the biggest reward from this project has been the recognition by students and parents that Algebra II is a subject that students can master and that I will be able to extend my project's availability to students throughout the state.

Materials

- Play-doh™ (1 can per two students)
- string or dental floss
- large paper for posting results
- colored pencils, markers, tape
- magazines, scissors, newspapers and/or Internet resources
- graphing calculators or graphing program for computer.

A TI-92 graphing calculator or a computer graphing program would be helpful. A computer graphing program that allows the students to input the equations in standard form instead of function notation is recommended. The lessons can be completed without technology, but students are more successful if the computer software is available.

Time Line

Teachers will need three weeks to complete the entire unit.

Expected Student Outcomes

This project teaches the skills outlined in the Texas Essential Knowledge and Skills, which will become effective in Texas in the fall of 1998.

Students will:

- describe a conic section as the intersection of a plane and a cone;
- relate simple parameter changes in the equation to corresponding changes in the graph;
- identify symmetries from graphs of conic sections;
- identify the conic section from a given equation;
- use the method of completing the square.

After completing this project, students should be able to:

- identify how conic sections are formed and cite examples of conic sections in the real world;
- define both in words and in algebraic formulae, a circle and its center and radius, and an ellipse and its foci, latus rectum, and major and minor axis;
- solve and graph, using a graphing calculator and/or a computer graphing program, algebraic problems related to various circles and ellipses;
- define, both in words and in algebraic formulae, a hyperbola, its center and foci, and its latus rectum;
- solve and graph, using a graphing calculator and/or a computer graphing program, algebraic problems related to various hyperbolas and parabolas;
- compare and contrast different circles, ellipses, hyperbolas and parabolas;
- produce a work of art related to conic sections;
- work cooperatively towards the successful completion of a project; and
- keep journal entries describing and assessing their participation in each project activity.

Prerequisites

Teacher: The teacher must be familiar with the properties of conic sections and how to use either a graphing calculator or a computer graphing program. He or she should have a knowledge of cooperative learning strategies to help enhance student performance. Since these lessons cross most levels of multiple intelligence, i.e., verbal/linguistic, body/kinesthetic, interpersonal, logical/mathematical, visual/spatial, and intrapersonal, the teachers understanding of different learning styles and different levels of critical thinking is also helpful.

Students: Students should know how to solve a quadratic equation by completing the square, how to graph linear equations, and how to find the equation of a line. Prior experience with the graphing utility is helpful. They should be familiar with basic geometric shapes such as cones, rectangular solids, and cubes. Some experience with journal responses is also helpful because students will be writing explanations in journal form.

Assessments

Assessment tools are found after each lesson (see Lesson Plans and Activities).

Lesson 1: Introduction to Conic Sections

Note: The teacher should actually use the Play-doh first before trying the lesson with the students. Teachers may wish to pre-select the magazines or web sites that students will visit to find examples of conic sections.

Objectives:

Students will:

- be able to identify which conic section is formed by a specific cut of the cone;
- be able to compare and contrast the cuts of the cone, the sphere, the rectangular solid, tetrahedron, and cube;
- identify and label ten examples of conic sections used in the real world from newspapers, magazines, or from Internet research.

Materials:

- Play-doh
- string, markers, tape
- large paper
- magazines or newspapers and/or Internet access

Activities:

1. Divide students into groups of four, with each student serving in a particular role: recorder, materials manager, questioner, and reporter.

2. Each student in these groups is to form a cone, using the Play-doh. All the students then slice their individual cones by drawing the dental floss or string through the cone parallel to the base and perpendicular to an imaginary line or axis passing through the center of the cones.
3. The recorder draws the results on the large paper provided to the group and the group chooses a real-world object that looks like the circle they have just formed.
4. Students reshape their cones and proceed to form their second conic, the ellipse. This cut is done by slicing the cone at an angle to the imaginary axis, but not through the base of the cone. Again the recorder draws the result on the group paper and a real world example is listed.
5. The cones are reshaped and students cut a third slice. This slice is made parallel to an imaginary line on the side of the cone and passing through the base of the cone. The parabola is formed and recorded on the posting sheet with its real-world application listed.
6. The final cut is made by placing two cones, nose to nose. The string is pulled from the base of one cone parallel to the imaginary axis and through both cones to the base of the second cone. The result will be the two branches of the hyperbola. Once again, the result is drawn, an example provided, and the sheets are posted about the room for student feedback and teacher correction.
7. The students are then challenged to form the geometric solids of a cube, rectangular solid, tetrahedron and sphere. They are to perform the same four cuts within their groups and reach a group consensus of the outcomes.
8. Students are to record, individually, in journal form, the results of both cutting the cone and the geometric solids. They are to compare and contrast the results for each solid and the cone.
9. From magazines, newspapers, or from Internet resources, students are to find ten real-world pictures of conic sections, label them by name, and identify which cut of the cone was used to form that particular conic.

Assessment:

Students are to receive 10 points for each conic correctly identified in their real-world pictures, for a total outcome of 100 points.

The journal writings are critiqued for accuracy and corrective feedback provided. Students are allowed to resubmit incorrect journal writings to improve their performance.

Lesson 2: Development of Graphing Circles and Ellipses

Objectives:

The student will be able to:

- graph circles and ellipses, and write the equation of each when given appropriate information.
- Materials: graphing program for computer and graph paper

Activities:

1. Using computers, students work in groups of two. They are given the following information through direct teaching:
 - A circle is the set of all points in a plane equidistant from a fixed point, called the center. The fixed distance is the radius. The standard form of a circle whose center is at the point (h,k) is $(x-h)^2 + (y-k)^2 = r^2$. The radius is r units.
 - An ellipse is the set of all points P in a plane such that the sum of the distance from P to two fixed points is a constant. The fixed points are called foci (plural of focus). The standard form of an ellipse that lies in the x -direction with center (h,k) is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1.$$

The a^2 and b^2 positions are interchanged when the ellipse is stretched in the y -direction. The distance from the center to each vertex is a units and the distance from the center to the endpoints of the minor axis is b units. An ellipse's two foci are located c units from the center on the major axis and are found by applying the relationship $a^2 - b^2 = c^2$. The ellipse's width is found by locating the latus rectum, which is a set distance above and below each focus determined by the value

$$\frac{2b^2}{a}$$

- Students use the graphing program and determine the effect of changing (h,k) and the values of a and b for the ellipse. They record the location of the foci and the length of the latus rectum for each problem. Then they sketch the result on graph paper and formulate a conclusion about the effects of changing the values.

Student Work Sheet for Lesson 2
see page 36

Lesson 3: Development of Graphing of Parabolas

Objectives:

The student will be able to:

- graph the parabola using the standard form of $(x-h)^2 = 4c(y-k)$ and $(y-k)^2 = 4c(x-h)$;
- write the equation of the parabola when given the appropriate parts.

Materials:

- computer graphing program
- graph paper

Activities:

- The students are grouped in pairs and the necessary information given to them through the direct teaching model. Students then explore the effects of (h,k) and c upon the parabola in each of the forms.

Student Work Sheet for Lesson 3
see page 37

Lesson 4: Development of the Graph of the Hyperbola

Objective:

The student will be able to graph and write the equation of the hyperbola.

Materials:

- Computer graphing program,
- graph paper

Activities:

- Each student will work with a partner to discover the characteristics of the graph of the hyperbola. The following work sheet will be provided to the students prior to their exploration.

Student Work Sheet for Lesson 4
see page 38

Use the following problems:

1. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

4. $\frac{x^2}{9} + \frac{y^2}{16} = 1$

2. $\frac{(x-2)^2}{16} + \frac{(y+3)^2}{9} = 1$

5. $\frac{(x-2)^2}{9} + \frac{(y+3)^2}{16} = 1$

3. $\frac{(x+1)^2}{16} + \frac{(y-2)^2}{9} = 1$

6. $\frac{(x+1)^2}{9} + \frac{(y-2)^2}{16} = 1$

- A. Conclusions about (h,k): _____
 B. Conclusions about a and b: _____
 C. Based on your observations, write the equation for the following ellipses.

1. (h,k) = (3,-2) and a=5, b=2 and major axis horizontal
2. (h,k) = (3,-2) and a=5, b=2 and major axis vertical
3. (h,k) = (-1,-3) and a=5, c=4 and major axis horizontal
4. (h,k) = (-1,-3) and b =3 , c=4 and major axis vertical

- D. Use the computer program to graph the following circles and record your observations concerning (h,k) and the radius. Suppose you wanted to find the diameter of each circle. What would you do?

Problems

1. $x^2 + y^2 = 9$
2. $(x - 3)^2 + (y - 1)^2 = 16$
3. $(x+3)^2 + (y + 1)^2 = 16$
4. $(x + 1)^2 + (y-5)^2 = 9$

Conclusions

To find the diameter when given the radius, I _____

To find the radius when given the diameter, I _____

Assessment

1. Write an explanation of the effects of changing (h,k) on both the circle and the ellipse.
2. Explain the effect of a and b on the ellipse.
3. Create 5 problems of your own that would be ellipses and circles.
4. Give 5 sets of data that would be sufficient for a partner to write the equation of the circle or ellipse. For one of the circles, give the diameter instead of the radius.
5. List the problems and have your partner solve them, using the computer graphing program.
6. Write a summary of the difficulties, if any, that you and your partner experienced when checking your problems. If no problems were encountered, sketch the graphs of the problems and show the equations.

Use the following information to complete the activities listed:

- A. A parabola is the set of all points P in a plane that are the same distance from a given line and a fixed point not on the line.
- B. The fixed point is called the focus and the fixed line is called the directrix.
- C. The focus and directrix are each located c units from the vertex, but lie on opposite sides of the parabola.
- D. The parabola cannot intersect the directrix.
- E. The width of the parabola at its focus is 4c units and is called the latus rectum.
- F. A parabola's direction is dependent upon which term is squared in the standard form.

Graph the following parabolas on the computer graphics program and draw conclusions about the direction of the curve, the effect of (h,k) and c. Find the length of the latus rectum for each curve and verify from the graph. Sketch the graph of each on your graph paper.

Problems

1. $(x-2)^2 = 4(y-1)$ 2. $(x + 1)^2 = -4 (y + 1)$ 3. $(y + 1)^2 = -4 (x + 1)$ 4. $(y-2)^2 = 8 (x-1)$

Conclusions

A parabola in the form of $(x-h)^2 = 4c (y-k)$ turns _____ or _____. When 4c is a negative number, the parabola turns _____, but if 4c is positive, the parabola turns _____. (h,k) represents the _____ of the parabola. A parabola in the form of $(y-k)^2 = 4c(x-h)$ turns _____ or _____. If 4c is positive the parabola turns _____, but if 4c is negative the parabola turns _____. The focus is located _____ units from the vertex and the latus rectum is _____ times this distance.

Assessment

The student will use the following assessment to evaluate Lesson 3:

- A. Describe in paragraph form the following parabolas, then have your partner read and correct your description. Check your results by graphing each parabola on the computer. Sketch each graph on your graph paper.

1. $(x + 2)^2 = -8 (y - 1)$ 2. $(y-2)^2 = 4 (x + 5)$

- B. Write the equation of the following parabolas, sketch a graph, then verify on the computer:

1. (h,k) = (2,3) c=3, turns right

2. (h,k) = (-3,-1) c=1, turns left

3. (h,k) = (2,3) c=3 turns down

4. (h,k) = (-3,-1) c=1 turns up

- A. A hyperbola is the set of all points, P, in a plane such that the difference from P to two fixed points, called foci, is a constant.
- B. The standard form of the hyperbola with center (h,k) is
- $$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \quad \text{or} \quad \frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$
- C. The hyperbola is similar in some of its characteristics to the ellipse, but very different in others.
- D. As you graph the following hyperbolas, watch for similarities and differences between this curve and the ellipse listed next to it.
- E. The foci for the hyperbola are defined by the relationship $a^2 + b^2 = c^2$.
- F. The latus rectum relationship is the same for both the hyperbola and the ellipse.
- G. For each graph, list the vertices, the foci, the latus rectum distance, and the center.

Hyperbola	Ellipse
1. $\frac{(x-2)^2}{25} - \frac{(y-4)^2}{9} = 1$	$\frac{(x-2)^2}{25} + \frac{(y-4)^2}{9} = 1$
2. $\frac{(y-4)^2}{25} - \frac{(x-2)^2}{9} = 1$	$\frac{(x-2)^2}{9} + \frac{(y-4)^2}{25} = 1$
3. $\frac{(x+2)^2}{4} - \frac{(y+1)^2}{16} = 1$	$\frac{(x+2)^2}{4} + \frac{(y-4)^2}{16} = 1$

Answer the questions and do the activities that follow:

- What do you notice about the relationship between the size of a and b on the hyperbola that is different on the ellipse? _____
- What seems to determine the direction of the hyperbola? _____
- What determines the direction of the ellipse? _____
- Compare and contrast as many characteristics of the two curves as you can. Record your observations for each specific curve.
- Write the equation for each of the following hyperbolas and have your partner check. Check the results on the computer:
 - hyperbola (h,k) = (2,3) a = 4, b = 9 lies in the x-direction
 - hyperbola (h,k) = (-1, -2) a=9, b=4 lies in the y-direction
 - hyperbola (h,k) = (4,5) a = 4, c = 16 lies in the y-direction

Assessment

Once all observations are listed and noted, additional problems from all conics are assigned. Students graph on paper, then check their graphs on the computer. Next, students write their own equations, one per conic, and check them on the computer.

Lesson 5: Creating a Picture Using Conics

This lesson may take several days to complete.

Objective:

Students will be able to apply the graphing of conics to a real-world situation by creating their own picture.

Materials:

- graph paper
- colored pencils or markers
- computer graphics software or graphing calculator

Activities:

See worksheet below.

Work Sheet for Lesson 5

1. Use a minimum of six conic curves and four lines to create a picture of your choice. No abstract art is allowed. Your picture must include at least three of the four conics studied.
2. For one of the equations, write the relationship as an inequality to show the coloring that would be used. Algebra must be used to prove the intersection points for any two of the equations.
3. Your final product cannot be computer-generated; however, the computer may be used as a checking tool throughout the process.
4. All rough drafts must be included, with the final copy on top.

Assessment:

A 70% will be awarded for having the minimum number of curves, the inequality, and the algebraic solution. Points will be awarded for level of difficulty, extra equations, and overall presentation.

Scoring Rubric	Maximum points
6 conics/ 4 lines	50 points
inequality	10 points
algebra solution	10 points
11-25 equations	5 points
over 25 equations	10 points
presentation	10 points
level of difficulty	10 points

Opportunities for Extension and Cross-Curriculum Integration

This tactile/kinesthetic approach to conics allows students to more concretely visualize and understand the concept and then, as they research and develop their pictures, apply this knowledge to the real world. The art project encourages students to use another medium to demonstrate their mathematical abilities so that students with minimal algebraic skills can still prove mastery of the concept of conic sections.

These lessons address the needs of students who are concrete/sequential, abstract/random, abstract/sequential, and concrete/random. The compare/contrast questions require students to progress from the basic level of knowledge to the engaged level of thinking. By creating their own pictures, students are developing and thinking at the highest level of learning -- the dynamic.

Different levels of competency and learning styles are utilized as the students pair to discuss and discover the special qualities of each conic curve. Journal writing allows the more verbal/linguistic student to demonstrate his or her knowledge of the subject, even if the algebraic development is not as refined as the more logical/mathematical student.