

MAC-MTL/CPTM/CSMC Curriculum Conference

Kathy's notes

Here are the notes from my small group (Group included Brad Findell, Iris Weiss, Glenda Lappan, Christy Graybeal, Mary Ann Huntley, Kathryn Chval, Kathy Heid) from the MAC-MTL/CPTM/CSMC Curriculum Conference:

Mathematics related to the inverse trig functions prompt:

What a function is

What composition is

Distinction between composition and multiplication

Inverse function vs. inverse

Restricted domain

One-to-one-ness

Distinctions in symbolism

$\left[\begin{array}{l} \sin^2(x) \text{ is multiplicative} \\ \sin^{-1}(x) \text{ is not multiplicative} \end{array} \right]$

f^{-1} vs 2^{-1}

$\sin^{-1}(x)$ vs $(\sin(x))^{-1}$

$f^2(x)$ vs $(f(x))^2$ vs $f^{(2)}(x)$

Identifying inverses requires identifying identities

Different ways that inverses are used in mathematics

Unit circle

Trig identities

Reflection over $y = x$ and its connection to transformation and inverses

Right triangle trig vs. trig functions

Issues that arose centering on the general idea of generating situations:

Does the commentary weave across the foci?

Is the focus an explanation a teacher could give another teacher?

Issue for foci: maintaining the pedagogical connection – tie to the pedagogical purpose

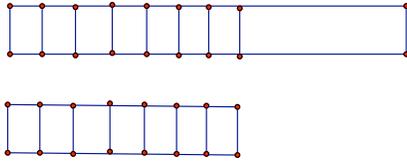
Need to distinguish pivotal foci from surrounding foci

Is it that we want to cover the domain or generate enough foci to generate the dimensions?

Small group generation of prompts

1. Glenda:

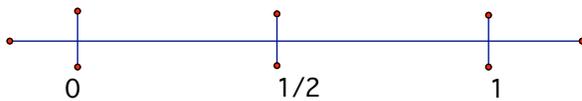
Student is asked to fold a piece of paper in 7ths. He folds the paper with creases as shown in the top of the following figure, then rips the rest of the paper off. Is he correct?



2. Glenda

Example from Connected Math:

This one focuses on the many different meanings of fractions (here the issue is position vs length)



Students are asked to mark $1/3$ and $2/3$ and to find the distance between $1/3$ and $1/2$. What are the different interpretations needed for the fractions involved in this problem?

3. Origin:?

Teacher is talking to class about how to subtract integers, noting that $5 - 7 = 5 + (-7)$.

Student asks: Does this always work?

Teacher: "Since it works for integers, it also works for rationals and irrationals."

4. Christy Graybeal shared a list that she, Sara Sword and Tim Fukawa Connolly generated:

Perplexing Algebra/Number Math Questions Asked by 5th to 8th Math Students

- Why is $(-1)(-1) = +1$?
- Why can't we divide by zero?
- Why isn't $1/0 =$ to infinity?
- Why switch direction of inequality when multiplying or dividing by a negative number?
- How do you know if a given fraction will terminate or repeat?
- When am I going to use this?
- How/why do the exponent rules work?
- Why isn't $(x+1)/x = 1$? Why can't you cancel the x's?
- Why isn't $3x+5=8x$? Why can't you add unlike terms?
- Why is subtracting a negative the same as adding a positive?
- Why isn't there a "one" product property? (like a zero product property)
- Why do we distribute when multiplying polynomials?

- Why when solving equations do the order of operations go backwards? (multiplication and division first in order of operations, addition and subtraction first in 2-step equations)
- What is a function? What does it have to do with equations?
- How can x be a negative number and $-x$ be a positive number?
- How can $|x| = -x$?
- Solutions, variables, and parameters – oh my! (These are confusing!)
- How do we know if a set of ordered pairs makes a line, parabola, etc.?
- How do you know if a “U” is a quadratic? Is every “U” a parabola?
- What’s a regression? How does the calculator “magically” get an equation?
- Fractions – Why don’t you add across? Why don’t you need a common denominator to multiply? Why do you flip and multiply to divide?
- Why do we need to factor?
- Why does $.9$ repeating $= 1$? How do you convert repeating decimals to fractions?
- How do you know irrational numbers “don’t end”?
- How do I know that the calculator answer isn’t good enough?
- Rules for radicals – who needs them?
- One more that I just thought of: Teachers often insist that the “ x ” goes on the left. So they tell students to write their answer as “ $x < 5$ ” rather than “ $5 > x$ ”. When students graph this on a numberline, the left of the five is shaded. The teachers I worked with a few years ago thought that the “arrow” tells you which direction to shade.

5. Christy

Student is solving a 2-step equation and asks why subtract first when order of operations goes the other way.

6. Glenda

NOW NEXT notations in Core-Plus may cause problems (going from iterative to explicit form)

Why does function notation help?

7. Glenda

What does the remainder mean in context?

8. Mary Ann Huntley

What does interpolated answer mean in a discrete data problem?

9. Mary Ann Huntley

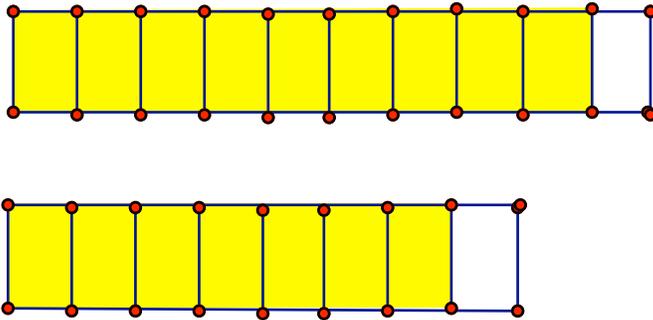
Distinguishing real world answers from math world answers (-2 as a number of cookies)

10. Kathryn Chval (from a course for prospective K-12 Special Ed teachers)

Task: compare $9/10$ and $7/8$

Heid notes from 061118 meeting in MD.doc

Student says $7/8$ is smaller illustrating like this:



Mathematical foci: Unit, partitioning

11. Mary Ann Huntley

Why is rate of change non-constant for exponential functions of the form $a \cdot b^x$

12. Kathryn Chval

Non-linear functions that look linear on a calculator (is it true that all functions are locally linear?)

13. Aisling may have probability examples

14. Kathryn Chval has sample space videos

15. Iris: Researchers' misconceptions: issues of biased samples, wrong unit of analysis, and conclusions drawn past the limitations of the data

16 Christy

using manipulatives to get the mean makes inclusion of 0 problematic

Sources for additional prompts:

Classroom videos/PD videos

Student responses to open-ended items (New Standards, AP Calc, AP Stat, NAEP, state assessments)

Field test teachers (Core Plus)

National Board certified teacher videos; presidential awardees

Chazan and Herbst animation project

NSF-MSP electronic network (e-mail Iris for info)

Jim Lewis, Bill Haver

Sharon Senk TEDS

Heid notes from 061118 meeting in MD.doc

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KAT

Gail Burrill

PCMI as venue

Nanette Seago's videos

Kids say something that is true in some cases, but not always true

Joan Ferrini-Mundy described what teachers say to kids as "The truth and nothing but the truth, but not the whole truth."

Glenda knows a source form materials by Judy Sowder, Pat and Alba Thompson, Randy (kids coming up with the right answer in the wrong way)

Student responses to open-ended assessment tasks – teachers need to make sense of them
Professional development clips

Judy Mumme, Kathy Carroll, Mark Driscoll [Fostering Algebraic Thinking Toolkit – maybe they would lend the video]

Kathryn Chval has some video clips

Glenda said she would look through the units for examples – need to send her an e-mail

Iris has videos of "Teachers as Learners"

Diane Briars

Rafaella Borasi

Linda Forman (has forums for Professional development providers)

Iris mentioned TeMat.org database – we could look in the database for high school math

Anna Graeber's collection of misconceptions

Feedback on lenses

Iris: Organization of framework depends on who we see using it – how to represent the frame depends on who you are working with

We might use different frameworks for different purposes. For PD or curriculum developers, the framework could help generate ideas but not decide on priorities. A value-free matrix would be a resource but designing a program from the matrix requires establishing priorities and values

Construct framework as a tool for conversations

Kathy Heid: What about a dynamic (n-dimensional with pop-ups) framework

Kathryn Chval: Focus on mathematical objects would appeal more to mathematicians than "big ideas"

Iris: "Mathematical activities" would turn off mathematicians.

Mark/Terry/Iris

Tie the framework to pedagogical purpose; stay with image of this as applied math

Stray from pedagogy not from pedagogical uses

What else do we need to be doing?

Write an alternative to the Usiskin/Stanley book

Try out each lens with different audiences – to get at possible misinterpretations

Articulate more fully how we are working methodologically – how are we coordinating the mathematical perspective and the pedagogical perspective [How methodologically will we keep these two perspectives in mind?]

Ideas for uses:

NSF's Learning Progressions

Encourage people to incorporate our work into theirs – so make them accessible (however, copyright them)—get reference to work placed in NSF solicitation

Mathematicians are interested in difficulties students have – this could be stealth PD for mathematicians

1-hour shadow courses to content courses