Situation: Constructing a Tangent
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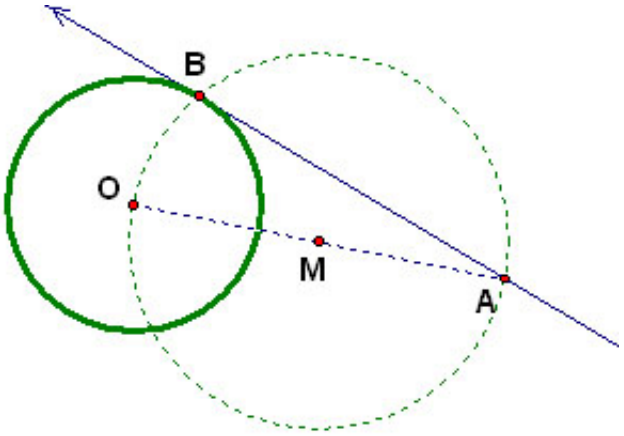
Center for Proficiency in Teaching Mathematics
7/29/2006 - Pawel Nazarewicz

Prompt

A student in a Geometry class was shown how to construct a tangent to a circle from a point A outside the circle:



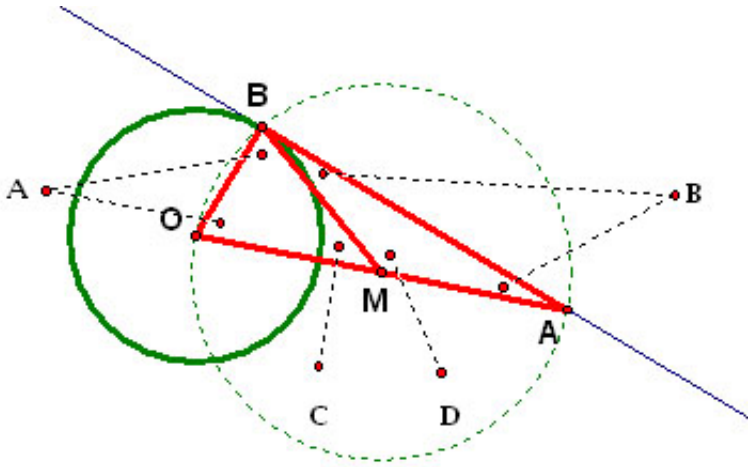
1. First, construct the segment OA and find its midpoint M.
2. Next, using M as the center of a new circle, construct a circle of radius MA.
3. The point of intersection between the two circles, B, is where we construct the tangent to. The new tangent line is now AB.

After seeing this, a student asked how we know that this is in fact the tangent line. Or, how do we know that $\angle BOM$ is a right angle?

Commentary

Mathematical Focus 1: Isosceles Triangles

Triangles with a pair of congruent sides, or isosceles triangles, also have a pair of congruent angles.



Here, the two red triangles are both isosceles because all the radia of a circle are congruent. This means that $\angle OBA \cong \angle MOB$ and $\angle MBA \cong \angle MAB$.

Let α be the length of $\angle OBA$ and β be the length of $\angle MBA$. All triangles have 180^o. Thus, if we look at $\triangle OBA$,

$$2 \cdot \alpha + 2 \cdot \beta = 180^o$$

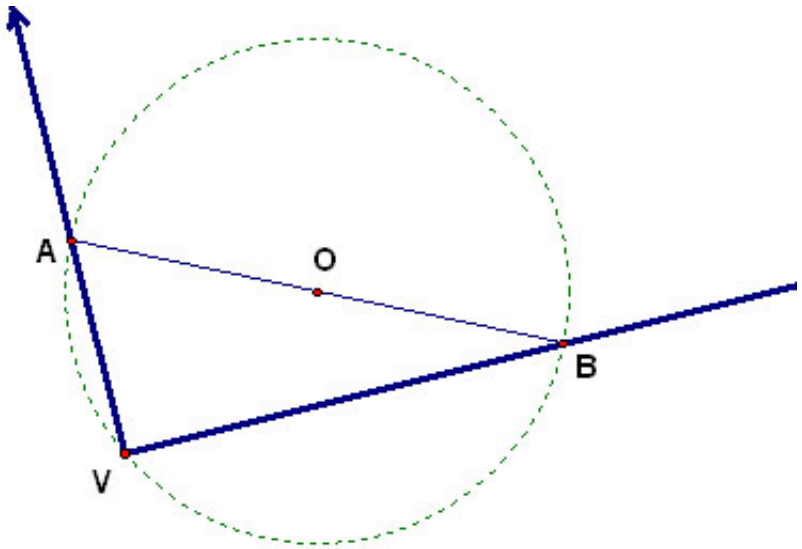
$$\Rightarrow$$

$$\alpha + \beta = 90^o$$

This is exactly the reason why an inscribed angle which spans a circle's diameter is a right angle.

Mathematical Focus 2: Inscribed Angle in of a Circle

If an inscribed angle in a circle spans the diameter of that circle, that angle will be a right angle. Conversely, given a right angle, any two points, A and B, on the two different rays combined with the vertex, V will all be on a circle with a diameter AB. See the illustration below:


A
O
B
V