Situation 12: Quadratic Equations
PRIME at UGa
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Prompt

Mr. Sing presents problems like the following to his students.

$$(x+1)^2 = 9$$

He demonstrates to them that they need only take the square root of each side to get

$$x+1 = 3 \text{ or } x = -3.$$

Then we can solve for x = 2 or x = -4. He then turns his students loose to solve some problems like the ones he has presented and is surprised to find out that many of his students are multiplying the terms out to get

$$x^2 + 2x + 1 = 9$$

and then transforming the equation so that

$$x^2 + 2x - 8 = 0$$

and factoring this equation. He notes, however, that many students still were not able to factor correctly.

He stops the class and reminds the students that they need only take the square root of both sides to solve these types of equations and then let's them continue working on the problems. A few days later, Mr. Sing grades the test covering this material and finds that many of his students are still not doing as he has suggested. At first he thinks that his students just didn't listen to him but then he reminds himself that during the class period the students seemed to be quite attentive.

What hypotheses do you have for why his students are acting in this way? What concepts are present in the material he is trying to communicate to his students? In what ways might Mr. Sing work with his students to develop the concepts he is trying to communicate? What knowledge might Mr. Sing need in order to develop these concepts with his students?

Commentary

Mathematical Foci

Mathematical Focus 1:

Introduce a quantitative situation like the following: I have a square whose side is length $x + 1$. Can you make a representation of this square and figure out the length of x when the area of the square is 9 m^2? In using this approach, a dynamic representation is helpful because it allows students to move the values of x (as seen in the GSP attachment). It leads to a number of possible levels of solution to the problem. At a basic level, it allows students to move the quantity $(x+1)^2$ in GSP to find out when the area will be 9 m^2. In constructing the representation, the student may also be able to coordinate some of the factoring issues that they seem to be experiencing in the vignette. For instance in creating the square whose sides are $x + 1$, the student can name the square algebraically either as $(x + 1)^2$ or through the sum of its parts. This may allow the student to have a quantitative experience of why $(x + 1)^2 = x^2 + 2x + 1$ and further to coordinate what the area of each region of the function is when the total area is 9 m^2. This approach can also lead to creating a linear representation of the area function and graphing it in the usual way in the plane which can lead to discussions about why this function is a translation of *x^2*. Note to get the second solution using this method the teacher would need to address the notion of negatively oriented area.

Mathematical Focus 2:

This problem helps to demonstrate a common theme in mathematics—finding special cases whose solution strategy is an abbreviated form of a more general algorithm. Some of the mathematical content involved for teachers is deciding whether to develop the general solution strategy first and then look at special cases or to look at special cases that might build to the general solution strategy. Given the way I treated the problem in foci 1, it makes more sense to move from special cases to the more general solution strategy although this technique is usually reversed in the textbooks I have read on this topic. That is usually students learn to factor quadratic equations to find their zeros and then are presented with special cases like how to solve the equation $(x + 1)^2 = 9$. In this case, I would probably start with specific cases and build to a generalized solution of quadratics, using the notion of the difference of two squares and completing the square to build up to the quadratic formula.

Mathematical Focus 3:

There is an issue in this problem also as to what x represents mathematically. In the statement of the problem x is an unknown. However, there may be significant confusion for what x stands for if a function approach is used to solve this problem. That is in a functional context x is a variable where x is any but no particular value whereas when we are solving for a particular area of square x becomes an unknown because we are looking for one particular value of x. Algebraically, however, there is no differentiation between the notation of these two ideas rather it is context dependent. Therefore, it may be important for teachers to be aware of the possible differences between thinking of x as a variable and x as an unknown and how and when they are switching between these two conceptions in given problem situations.

Mathematical Focus 4:

Important knowledge for addressing the issue in the vignette may be to have strategies for helping students see that general use of a formula may not always lead to the most insightful solution of a problem. Such knowledge might be in part at the heart of helping students to build a habit of mind in which they search for efficient problem solutions. Note foci 2 and 4 are mostly related to classroom culture or teacher pedagogy.