

**Situation 12: Quadratic Equations**  
**PRIME at UGa**  
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## Prompt

This situation occurred in the classroom of a student teacher during his student teaching. He worked very hard to create meaningful lessons for his students and often asked his mentor teacher for advice by asking questions similar to the one's found at the end of this vignette.

Mr. Sing presents equations of the following to his students.

$$(x+1)^2 = 9$$

He demonstrates to them that they need only take the square root of each side to get

$$x+1 = 3 \text{ or } x+1 = -3.$$

Then we can solve for  $x = 2$  or  $x = -4$ . He then turns his students loose to solve some equations like the ones he has presented and is surprised to find out that many of his students are multiplying the terms out to get

$$x^2 + 2x + 1 = 9$$

and then transforming the equation so that

$$x^2 + 2x - 8 = 0$$

and factoring this equation. Mr. Sing notes, however, that many students were making mistakes in carrying out his procedure.

He stops the class and reminds the students that they need only take the square root of both sides to solve these types of equations and then let's them continue working on the problems. A few days later, Mr. Sing grades the test covering this material and finds that many of his students are still not doing as he has suggested. At first he thinks that his students just didn't listen to him but then he reminds himself that during the class period the students seemed to be quite attentive.

What hypotheses do you have for why his students are acting in this way? What concepts might be necessary for students to understand the concept of solving a quadratic equation? In what ways might Mr. Sing work with his students to develop these concepts?

## Commentary

### Mathematical Foci

#### *Mathematical Focus 1:*

##### **Using binomial reasoning as a basis for solving quadratic equations.**

In many cultures (Chinese, Babylonian, Egyptian, and Greek) solving quadratic equations arose out of solving problems dealing with lengths or areas. Perhaps the most prominent of these types of problems were one's that involved rectilinear areas and transformations performed on these areas. To this end, it is possible to build up meaning for the equations that Mr. Sing presented to his students through building up students binomial reasoning in quantitative contexts. See the attached [GSP document](#) to see a sequence of possible tasks that a teacher might use to develop binomial reasoning in whole number multiplication contexts and use this reasoning as a basis for binomial reasoning in the context of solving quadratic equations.

#### *Mathematical Focus 2:*

##### **Two methods that use a functional approach for solving a quadratic equation.**

These types of equations often come up in the context of working on quadratic functions. To see how to represent  $(x+1)^2$  as an area of a square where the length  $x$  varies over the domain of all possible lengths, see the [following GSP sketch](#). A quantitative situation appropriate for the solution of this particular quadratic equation would be: Suppose that you know the length of the side of a square is  $x+1$  where  $x$  represents a variable quantity, what length would the side of the square have to be for the area of the square to be 9? In creating the dynamic function relationship shown in GSP we have first used  $x$  as a variable quantity. However, when we are looking for a specific area (the solution of a quadratic equation), we need to switch to viewing  $x$  as an unknown. In solving an equation using functional methods, we usually switch between thinking of  $x$  first as the variable of a function and then of  $x$  as an unknown that represents a particular value of the function. Note that it is possible to represent *all* quadratics using areas of squares including making a model for quadratics that have complex roots. In order to do so involves introducing directed areas—area that has a positive or negative orientation depending on the direction one travels around the area.

Another possible functional method is to imagine each side of the original equation as being a function. Then the intersection of any two functions can be thought of as the solution of an equation. In this case, we can think of  $f(x) = (x+1)^2$  as representing an area

function where the graph of  $(x+1)^2$  represents all possible areas of squares as  $x$  ranges over its domain, the length of the side of a square. We can then think of  $g(x) = 9$  as representing a constant function. When the constant function intersects the area function,  $f(x)$ , we know the value of the area function will be 9 and hence that these intersection points will represent when the area of our square is 9. I have included the graph of the constant function in the GSP representation.

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