

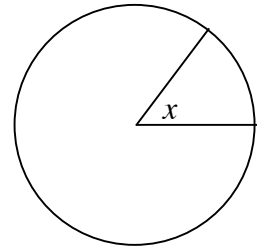
**Situation 16: Area of Sectors of a Circle**  
**Prepared at University of Georgia**  
**Center for Proficiency in Teaching Mathematics**  
**Revised 09/27/05 – Dennis Hembree**

## Prompt

An honors geometry class had the following problem for homework. A student asked simply, “How do you do this?”

Complete the following table for a circle with radius 3:

Central Angle $x$	$30^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$150^\circ$
Area of sector					



Question: Perhaps using a spreadsheet or writing a simple program for a hand-held computer such as the TI-83 best approaches this problem. What could this problem afford beyond a series of simple calculations?

## Commentary

### Mathematical Foci

#### **Mathematical Focus 1**

Measurement units.

The formula for the area of a sector,  $A_x = \frac{x}{360}(\pi r^2)$ , is straightforward, though cumbersome. Converting the central degree measure  $x$ , to radians  $\theta$ , gives  $A_\theta = \frac{\theta}{2}r^2$ . This formula is less cumbersome and has the same form as the familiar area formula for a circle. Of course, the issue of conversion to radians is now shifted from the formula to the central angle entries in the table. In most American geometry courses, students would not have seen radian measures and this is an opportunity for entry to the topic.

### Mathematical Focus 2

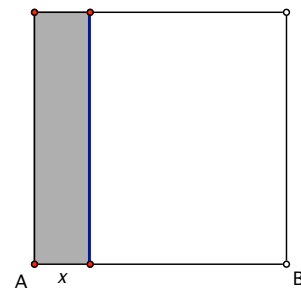
Function.

Plotting the points  $(x, A_x)$  on the Cartesian plane suggests questions about continuity, domain and range, and other analyses of the resulting graphical situation. For this particular problem with  $r = 3$ , we are plotting the continuous linear function  $A_x = \frac{\pi}{40}x$  with domain  $0 \leq x \leq 360$ . Plotting the calculated, rounded values is unlikely to produce a regression line with the exact slope and a y-intercept of zero.

### Mathematical Focus 3

Similarity.

It may be surprising that a linear change in one measurement of an object does not produce a quadratic change in the area of the object. Such a statement about quadratic change refers to comparing *similar* figures. This problem situation does not compare similar figures, but rather investigates fractional portions of a given, fixed figure, much like asking for the area of the shaded portion of this square as  $x$  increases along side AB.



### Mathematical Focus 4

Algebraic representation.

What are appropriate algebraic representations for the area function in this problem? The answer depends on the use of the representation. The simplified form  $A_x = \frac{\pi}{40}x$  is the *cleanest* of the representations in that it is in a familiar form for a linear equation. The more complex form  $A_x = \frac{x}{360}(\pi 3^2)$  retains the structure of the problem setting. A calculated, approximate equation such as  $A_x = .0785x$  is a user- and calculation-friendly representation. The *value for use* of each of these representations depends on the context of the problem and intent of the user/teacher.