Situation 16: Area of the Sector of Circle

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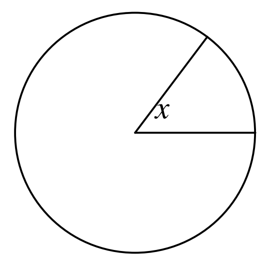
I will describe a portion of a lesson I observed in the classroom of Jeanette Phillips and Jay Jones on February 09, 2005 (note that Jay wrote about this same lesson as one of his “situations” for EMAT 4950). The class was Honors Geometry immediately after lunch. Several students asked about the following homework problem from the text (The assignment was to do every third problem. This problem was number 30, however, it was based on information in problem 29 and most students had decided to skip number 30 for that reason):

30) Complete the following table for a circle with radius 3:

Central Angle x	30°	60°	90°	120°	150°
Area of sector					



Jay was at the front of the room and Jeanette sat in a student desk near the middle of the room (among the students). Students’ initial questions seemed to be simply “how do I do this?” Jay simply repeated the question back to the students and someone responded with a formula. Jay asked students around the room to calculate various entries for the table and students agreed on the submitted answers.

I believe a demonstration of MKT occurred next, when Jay suggested the students plot the data points on a Cartesian plane. I am not sure if he knew the plot would be linear, or if he just thought the situation might be worth pursuing. Jay asked a few questions, such as, “What does the point (60, 4.712) represent?” and “Is this situation continuous or discrete?” When a student responded that the graph should be a line, Jay suggested the students calculate slopes between various pairs of points to see if the data seemed to really lie on a line. May of the students had calculated the areas in terms of π, which generated much discussion about how to mathematically represent the slope. Eventually, the class agreed on $y = \frac{2.3}{\pi}x + 0.1$. Again, I see this conversation as MKT on Jay’s part. I hoped Jay would continue to pursue the situation, since the y-intercept was not zero and the domain should be restricted to be $0 \le x \le 360$, but I do not know if he thought of this. Jeanette changed this line of questioning by what happened next.

Jeanette asked the class if they could have written this equation from the geometric situation rather than rely on the data. I see this as another instance of MKT. This line of questioning was not prepared in advance, but rather happened spontaneously as she observed the situation evolve. Jeanette led most of the following derivation rather than have students develop it. The students certainly participated in her discussion as she questioned them constantly throughout. Starting with $A = \frac{x}{360}\pi r^2$ for the area of a sector, one gets $y = \frac{x}{360}\pi \cdot 3^2 = \frac{x}{40}\pi$. At this point Jeanette really pushed the students on several issues. One was simply, "why is this a linear equation?" Students eventually agreed that the equation was of the form $y = mx + b$. Another challenge by Jeanette was to compare the empirical and experimental results for the equation of the line relating central angle and area. Quite a bit of time was spent discussing relative merits of the expression $\frac{\pi}{40}$ (or even $\frac{1}{360}\pi 3^2$) versus its approximation 0.0785. There was some degree of disbelief on the students' part that Jeanette thought $\frac{1}{360}\pi 3^2$ was a perfectly acceptable final form to represent the slope and would get full credit in an assessment situation. She made some pedagogical comments related to the purpose of assessment and what an item such as the one under discussion was intended to assess (not arithmetic or calculator skills, in her opinion). Her comments were in line with the College Board's AP Calculus scoring guidelines. The students seemed to be interested in this rather strange (to them) view of mathematics compared to an emphasis on "correct" answers, or rather, answers in some standardized form. The discussion never addressed the "problem" of the non-zero y-intercept or exactly what the slope means in this case. Here's my own extension question: What does the radius need to be in order for the slope of this line to be one? In other words, for what size circle is the following is true? : As the central angle changes from 0 to 360 degrees, the area of the resulting sector changes at exactly the same rate. Is there any other significance to this particular radius?