MAC-CPTM Situations Project

Situation 16: Area of Sectors of a Circle

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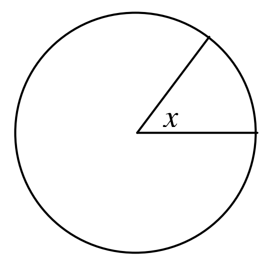
Prompt

An honors geometry class had the following problem for homework. A student asked simply, "How do you do this?"

Complete the following table for a circle with radius 3:

Central Angle x	30°	60°	90°	120°	150°
Area of sector					



Commentary

What could this problem afford beyond a series of simple calculations? If the question is passed off as simply a chance to practice computations, then perhaps something is missed. The problem could have been chosen to provide an opportunity to use a spreadsheet, write a program for a hand calculator, or to look for patterns in the completed table. The completed table could suggest exploring a graph of the five number pairs. This prompt allows for the introduction of radian measure as a more efficient representation of the sector's area formula. Students can also explore possible values for the area using both the table and programming features of a graphing calculator. In addition, software applications such as Excel are also useful in calculating the area. Such uses of technology allow the students to find any value within the domain and range of the continuous function represented by the formula. All of these are means to explore the underlying functional relationship of sector areas and their central angles.

Mathematical Foci

Mathematical Focus 1

The formula for the area of a sector can use either degree or radian measure.

The formula for the area of a sector, $A_x = \frac{x}{360}\left(\pi r^2\right)$, is straightforward, though cumbersome. Converting the central degree measure *x,* to radians θ, gives $A_\theta = \frac{\theta}{2} r^2$. This formula is less cumbersome and has the same form as the familiar area formula for a circle. The use of radian measure would require the students not only to use a modified version of the original formula but it would also require that they convert the central angles noted in the chart to radian measure as well. Using both degree and radian measure could also confirm that each approach leads to the same set of area measures.

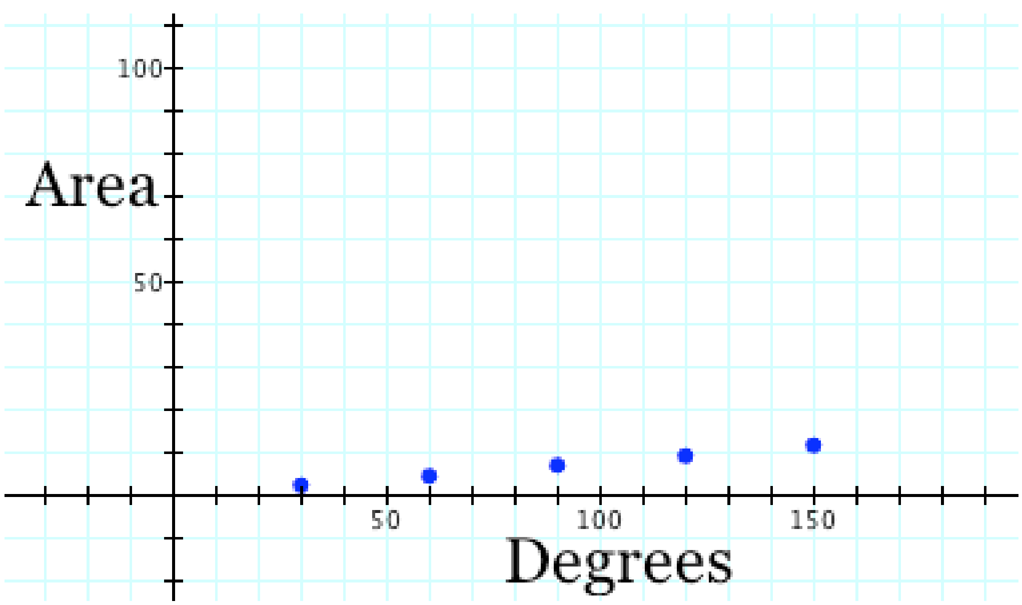
Mathematical Focus 2

The five number pairs (angle, area) identify five points in the Cartesian plane.

Using either formula, the areas of the five chosen sectors can be calculated readily with a scientific calculator.

Central Angle *x*	30°	60°	90°	120°	150°
Area of sector	2.356 sq. in.	4.712 sq. in.	7.069 sq. in.	9.42 sq. in	11.781 sq. in.

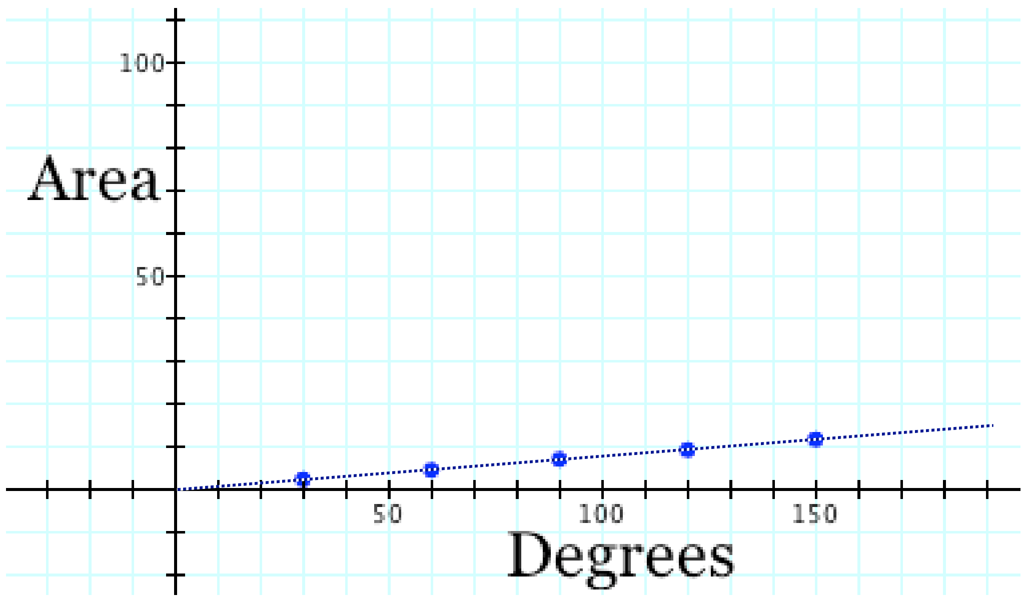
The plotting these points on a graph can be done either by hand or with technology.



__Mathematical Focus 3__

The areas of the sectors of a circle of radius r are a continuous linear function of the measures of the central angles.

Adding a line to the graph will suggest the need to confirm that the relationship may be linear. Considering the slope between pairs of points can do this or it can be



done by examining either of the formulas and writing them as either

$$y = \frac{x}{360}(\pi r^2)$$

where x is the degree measure of the angle, r is the radius, and y is the area; or

$$y = \frac{x}{2}(\pi r^2)$$

where x is in radian measure of the angle, r is the radius, and y is the area. Each of these is a linear function.

Mathematical Focus 4

Using a graphing calculator such as the TI-84, students can use the graphing and table features of the calculator to find the area of any sector and plot the table of values.

These features of a hand held graphing calculator are useful tools when rapid and accurate calculations are needed in order to explore mathematical relationships. One exploration, for example, might be to examine how the slope of the linear function changes as the radius of the circle is changed. The slope of the linear function will be different when the domain of the function is from degree measures than when the domain is from radian measures.

Mathematical Focus 5

This problem could be an opportunity to write a program for a hand-held calculator, to use an existing program, or to generate and explore the data in the table.

A simple TI-84, program, for example, can be generated to output the area of a sector given the central angle and the radius. The program can be structured to accept either degree measure or radian measure as input.

Mathematical Focus 4

Spreadsheet technology, such as Excel can be used to generate and explore the data in the table.

The functionality of spreadsheet operations to repeat operations with fill operations makes the spreadsheet particularly attractive for this problem. Further, once the data in the table is generated, the spreadsheet features for displaying graphs can be used to suggest a linear relationship.