

Situation 03: Inverse trig functions

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Prompt

Three prospective teachers have planned a unit of trigonometry as part of their work in a “methods” course on the teaching and learning of secondary mathematics. They developed a plan in which students first encounter what they call “the three basic functions,” sine, cosine, and tangent. They indicated in their plan that they would next have students work with “the inverse functions,” apparently meaning the secant, cosecant, and cotangent.

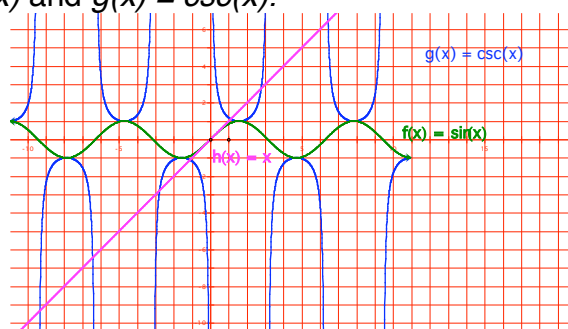
Commentary

Mathematical Paths

Mathematical Path 1

Mathematical path 1 is an example of a contradiction that might result from the manifestation in this scenario of the students’ concept of inverse function.

Suppose cosecant is the inverse function for the sine function. Then wouldn’t it be true that the reflection of $\sin(x)$ over the line “ $y=x$ ” be $\csc(x)$? Let’s look at the graphs of $f(x) = \sin(x)$ and $g(x) = \csc(x)$.



Notice that g is not the inverse of f since the graph of g is not the reflection of the graph of f over the line $y = x$. [Follow-up could include a discussion of why this procedure works.] Notice that $f(x) = \sin(x)$ won’t have an inverse that is a function since multiple input values have the same output value.

Mathematics used for Mathematical Path 1

- Understand the univalent property of functions.
- Understand the relationship between the inverse of a function and the reflection of that function over the line $y = x$.

Mathematical Path 2

The problem seems centered on knowing about the entity of inverse. Connections can be made to the notion of inverse from abstract algebra. When we think about inverses, we need to think about the operation and the elements on which the operation is defined. One might think of $csc(x)$ as an inverse of $sin(x)$ because $csc(x)$ is the reciprocal of $sin(x)$. For any value of x such that $csc(x) \neq 0$, the number $csc(x)$ is the **multiplicative** inverse for the number, $sin(x)$; multiplication is the operation in this case. But when we are looking for an inverse **function**, the elements are functions and the operation is composition. There are several ways that inverses are used in school mathematics, including reciprocals, opposites, and inverse functions.

Mathematics used for Mathematical Path 2

- Conceptual understanding of an inverse as related to the associated operation and set.

References

None