Situation 30: Translation of Functions

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Prompt

During a unit on functions, the transformation of functions from their parent function is discussed in a class. For example, if the parent function is $y = x^2$, then the child function $y = x^2 + 4$ would have a vertical translation of 4 units. When the class encounters the function $y = (x - 2)^2 + 3$, one student notes that the vertical translation of +3 "makes sense," but the horizontal translation to the right of 2 does not "make sense" with a – 2 within the function. As a teacher, how would you explain this?

Commentary

Note: Foci 1 and 2 use a parabola as a parent function only for demonstration purposes. One may substitute any function for the parabola.

Mathematical Foci

Mathematical Focus 1*: Graphical Representation*

One way to investigate this problem is to examine graphs of parent functions and their respective children. Included are four QuickTime movies of $y = x^2$ undergoing different translations, both vertically and horizontally. Looking at the various movies, students can predict the general form of a parabola $y = (x - h)^2 + k$, where h and k predict the proper translation.

- Movie 1
- Movie 2
- Movie 3
- Movie 4

Mathematical Focus 2*: Numerical Representation*

Another way to explore horizontal translations is with a numerical representation. With the parent function as $y = x^2$ again, students can investigate what happens with the values. It's obvious to see that the values of the child function $y = (x-1)^2$ have shifted right when compared to the parent function.

x	Parent x^2	Child (x-1)^2	Child (x-2)^2
-5	25	36	49
-4	16	25	36
-3	9	16	25
-2	4	9	16
-1	1	4	9
0	0	1	4
1	1	0	1
2	4	1	0
3	9	4	1
4	16	9	4
5	25	16	9

x	x^2	(x+1)^2	(x+2)^2
-5	25	16	9
-4	16	9	4
-3	9	4	1
-2	4	1	0
-1	1	0	1
0	0	1	4
1	1	4	9
2	4	9	16
3	9	16	25
4	16	25	36
5	25	36	49

***Mathematical Focus 3**: Transformation of coordinate axes*

This is from Smail, L. L. (1953). *Analytic Geometry and Calculus*. Appleton-Century-Crofts, New York.

Let OX and OY be the original axes and let $O'X'$ and $O'Y'$ be a new set of axes, having a new origin O' and parallel to the old axes and having the same senses.

Let the coordinates of the new origin with respect to the old axes be (h, k). Let P be any point whose coordinates are (x, y) referred to the old axes and (x', y') when referred to the new axes. Then $x = \overline{OM} = \overline{OH} + \overline{HM} = \overline{OH} + \overline{O'M'} = h + x'$ $y = \overline{MP} = \overline{MM'} + \overline{M'P} = \overline{HO'} + \overline{M'P} = k + y'$
Hence, if a translation of axes is made to a new origin O' whose coordinates with respect to the old axes are (h, k), the relation between the old and new coordinates of any point is given by $x = x' + h$ and $y = y' + h$.

Another way to think about this is that the function is not the object that gets translated. If the axes are translated right and up, this means the function looks like it is translated to the left and down. If the parent function $y = f(x)$ goes through a translation of axes, then the new child function is $y - k = f(x - h)$ which leads to $y = f(x - h) + k$.