

MAC-CPTM Situations Project

Situation 31: Expanding Binomials

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Prompt

In a high school Algebra I class, students were given the task of expanding $(x+5)^2$. A student responds, "That's easy! Doesn't $(x+5)^2 = x^2 + 25$?" What is the relationship between $(x+5)^2$ and $x^2 + 25$?

Commentary

Mathematical Foci

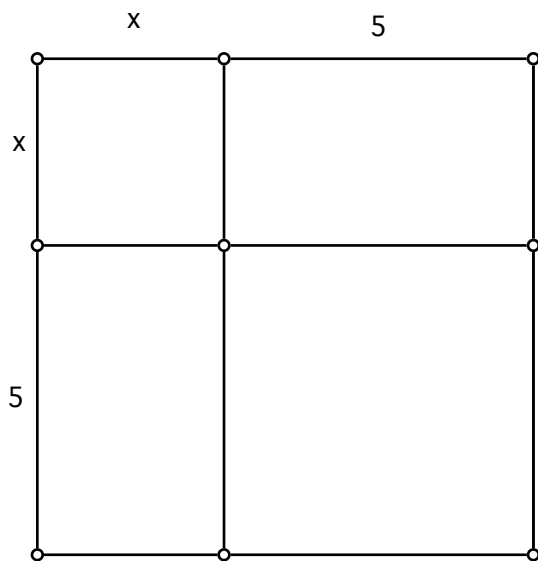
Mathematical Focus 1

Symbolic multiplication of binomials can be seen in terms of applications of the distributive property of multiplication over addition for real numbers. Once the students recognize that $(x+5)^2 = (x+5)(x+5)$ then discuss how to multiply the two binomials using the distributive property. Symbolic manipulation using field properties is at the basis of this path.

A distinction should be made concerning the differences between the $(x+5)^2 = x^2 + 25$ and $(x+5)^2 = (x+5)(x+5) = x^2 + 10x + 25$.

Mathematical Focus 2

The following diagram is connected to path 1 but is different because the treatment of symbols in this focus involves symbolic understanding and in the other path the symbol treatment is symbolic manipulation via field properties. The following diagram gives meaning to each of the three or four terms in the expanded form. Different pieces of the diagram represent different components of the symbolic expressions. The figure can be used to explore the original conjecture and find an appropriate answer for $(x+5)^2$ as well as to refute x^2+25 as an appropriate response.



Mathematical Focus 3

One possible path is to try to verify the equation by choosing specific values for x to substitute into the expression. It is necessary to understand what it means to justify a conjecture, and the fact that it only takes one counterexample to disprove a conjecture. In other words, two algebraic expressions are equivalent only if they produce the same values over the domains of interest. Assuming the convention that the set of all real numbers is the domain for a polynomial, substituting a real number into one polynomial expression and evaluating the result should produce the same value as substituting that number into the other expression and evaluating the result. It also helps to know some numbers are “special” – in the case of polynomials, a quick look at the constants involved gives a clues as to whether using 0 as a test value can be helpful. Of course, this use of 0 as a poor choice connects to the role of 0 as an additive inverse and as a non-member of the set of real numbers that constitutes the group under multiplication.

Mathematical Focus 4

One possible path is seeing the equation in terms of two functions, $f(x) = (x + 5)^2$ and $g(x) = x^2 + 25$. Generating the two graphs will show the differences between those graphs. These differences could be used as a way to identify that the expressions are not equivalent. This requires an understanding of functions and their graphs as well as equivalence, specifically how equivalence can be shown or refuted by graphical representation.