Situation 40: Powers
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Prompt

During an Algebra I lesson on exponents, the teacher asked the students to calculate positive integer powers of 2. A student asks the teacher, "We've found 2^2 and 2^3. What about $2^{2.5}$?"

Commentary

Mathematical Foci

Mathematical Focus 1

The value for $2^{2.5}$ can be estimated based on the values for 2^2 and 2^3. It is necessary to understand that the value for $2^{2.5}$ will not be halfway between 2^2 and 2^3. Due to the nature of exponential growth, the value for $2^{2.5}$ will be closer to 2^2 than it will be to 2^3. A visual representation could be helpful with this approach, that is, utilizing blocks as units to estimate the relationship between the exponent and the height of the blocks.
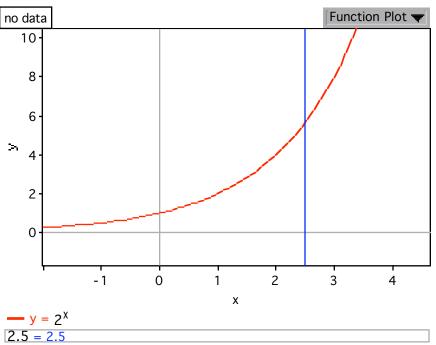
Mathematical Focus 2

The value for $2^{2.5}$ can be explored using properties of exponents. The expression $2^{2.5}$ can be rewritten as $2^2 2^{0.5} = 2^2 2^{\frac{1}{2}}$. Two raised to the exponent of one-half is equivalent to the square root of 2. $2^2 \cdot 2^{0.5} = 2^2 \sqrt{2} \approx 4(1.414) = 5.656$. So $2^{2.5} \approx 5.656$.

Mathematical Focus 3

The value for $2^{2.5}$ can be explored using properties of rational exponents. The expression $2^{2.5}$ can be rewritten as $2^{\frac{5}{2}}$. This quantity can be represented as

$\left(2^5\right)^{\frac{1}{2}} = \sqrt{2^5} = \sqrt{32} \approx 5.656$ or $(2^{\frac{1}{2}})^5 = (\sqrt{2})^5 \approx 1.414^5 \approx 5.656$.

Mathematical Focus 4

One possible approach to finding the value of $2^{2.5}$ is to examine the graph of the function $f(x) = 2^x$. One can estimate from the graph the value of the function at $x = 2.5$ in at least two different ways. First, one can look at the intersection of the function graph with the vertical line $x = 2.5$ in the following graph to see $f(x) \approx 5.5$.

Second, one can trace along the function graph to obtain $f(x) \approx 5.656$ when $x = 2.5$.

no data | Function Plot ▼

$y = 2^X$

2.5 = 2.5