Situation 41: Square Roots

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Prompt

A teacher asks her students to sketch the graph of $f(x) = \sqrt{-x}$. A student responds, “That’s impossible! You can’t take the square root of a negative number!”

Commentary

The student’s comment communicates a common misunderstanding that “-*x*” always represents a negative number, rather than signifying “opposite *x*.” Teachers must understand this notion of opposites, as well as having a deep understanding of functions: their graphs, reflections, and domains.

Focus 1 addresses the issue of domain, as well as the notion of “-*x*” being “opposite *x*” rather than always a negative number. In Focus 2, the graph of $f(x) = \sqrt{-x}$ is examined by considering it as a horizontal reflection of $g(x) = \sqrt{x}$. Focus 3 presents a numerical approach: create a table of values. Though not a proof, it provides evidence that $f(x) = \sqrt{-x}$ does, in fact, exist, and that its domain is $x \le 0$.

Mathematical Foci

Mathematical Focus 1

The domain of the square root function $f(x) = \sqrt{x}$ is all nonnegative real numbers (D: $x \geq 0$). To find the domain of any square root function, then, one must consider x-values for which the radicand is greater than or equal to zero. For example, if the function were $f(x) = \sqrt{x+2}$, the domain can be found algebraically:

$$x + 2 \geq 0$$
$$x \geq -2$$

In the case of $f(x) = \sqrt{-x}$, an algebraic approach for finding the domain is to set $-x \geq 0$:

$$-x \geq 0$$
$$x \leq 0$$

In other words, the function $f(x) = \sqrt{-x}$ does exist and its domain is $x \leq 0$ (see graph in Focus 2).

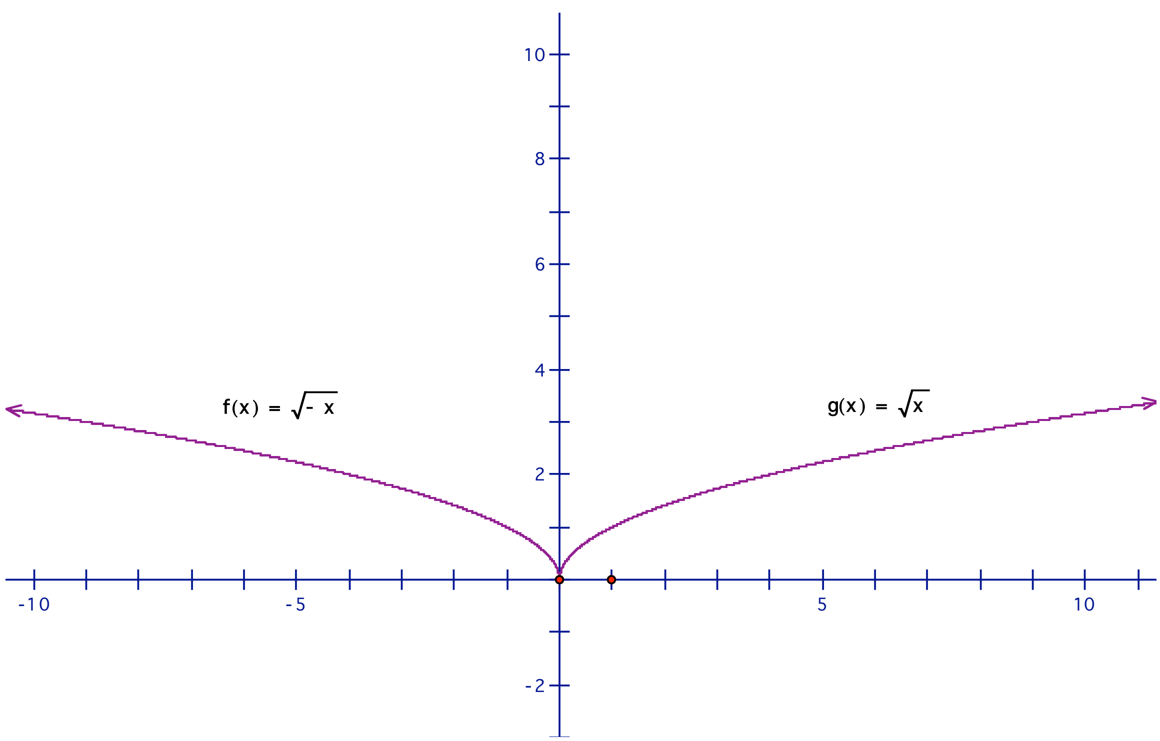
Mathematical Focus 2

Using a transformation of the graph of the known function $g(x) = \sqrt{x}$, the less familiar function, $f(x) = \sqrt{-x}$, can be generated. This requires a look into translations of graphs of functions:

When the graph of -*h*(*x*) is compared to the graph of *h*(*x*), it can be seen that the two graphs are reflections of each other about the horizontal axis. This is because by graphing the opposite of *h*(*x*), each point in the positive part of the range of *h*(*x*) becomes negative, and each point in the negative part of the range of *h*(*x*) becomes positive. Since zero has no opposite, *h*(*x*) = 0 and - *h*(*x*) =0 are the same on both graphs.

Comparing the graphs of *h*(*x*) and *h*(-*x*) also reveals a reflection: this time the graphs are reflections of each other about the vertical axis. Rather than the range values being negated (as in -*h*(*x*)), the domain values are negated, resulting in a reflection of *x*-values about the vertical axis. Again, since zero has no opposite, *h*(0) and *h*(-0) are the same point.

Specifically, the graph of the function $f(x) = \sqrt{-x}$ is a reflection of the graph of $g(x) = \sqrt{x}$ about the vertical axis, as is shown in the following figure.



The graph illustrates that $f(x) = \sqrt{-x}$ does exist, and its domain appears to be $x \leq 0$.

Mathematical Focus 3

The results from a numerical approach to $f(x)=\sqrt{-x}$ echo what has been examined algebraically and graphically. Below is a chart of values of $f(x)=\sqrt{-x}$ for various *x*-values:

x	$\sqrt{-x}$
-4	$\sqrt{-(-4)}=2$
-3	$\sqrt{-(-3)}=\sqrt{3}$
-2	$\sqrt{-(-2)}=\sqrt{2}$
-1	$\sqrt{-(-1)}=1$
0	$\sqrt{-0}=0$
1	$\sqrt{-1}=i$ (not a real number)
2	$\sqrt{-2}$ (not a real #)
3	$\sqrt{-3}$ (not a real #)
4	$\sqrt{-4}=2i$ (not a real #)

The results show that $f(x)=\sqrt{-x}$ exists, and suggest that its domain is $x \leq 0$.