Situation 41: Square Roots

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Prompt

A teacher asks her students to sketch the graph of $f(x) = \sqrt{-x}$. A student responds, “That’s impossible! You can’t take the square root of a negative number!”

Commentary

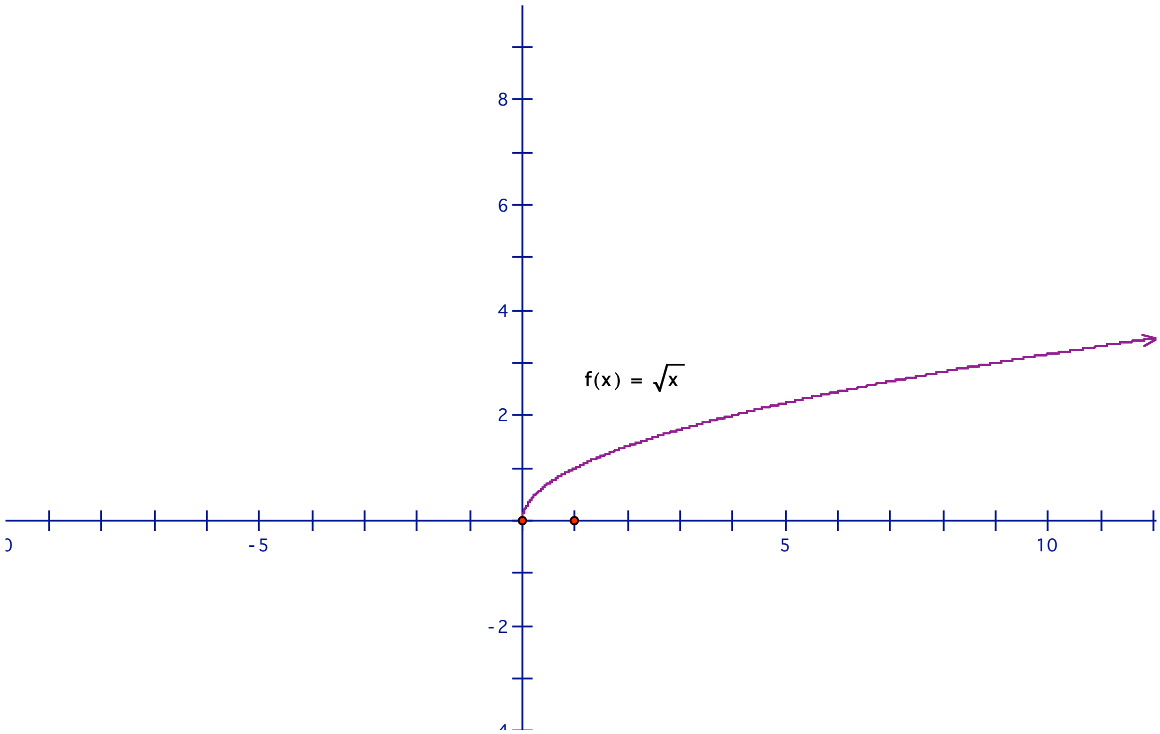
The student’s comment communicates a common misunderstanding that “-*x*” always represents a negative number, rather than signifying “opposite *x*.” Teachers must understand this notion of opposites, as well as having a deep understanding of functions: their graphs, reflections, and domains.

Focus 1 addresses the issue of domain, as well as the notion of “-*x*” being “opposite *x*” rather than always a negative number. In Focus 2, the graph of $f(x) = \sqrt{-x}$ is examined by considering it as a horizontal reflection of $g(x) = \sqrt{x}$. Focus 3 presents a numerical approach: create a table of values. Though not a proof, it provides evidence that $f(x) = \sqrt{-x}$ does, in fact, exist, and that its domain is $x \leq 0$.

Mathematical Foci

Mathematical Focus 1

The domain of the square root function $f(x) = \sqrt{x}$ is all nonnegative real numbers (*D*: $x \geq 0$).



To find the domain of any square root function, then, one must consider *x*-values for which the radicand is greater than or equal to zero. For example, if the function were $f(x) = \sqrt{x+2}$, the domain can be found algebraically:

$$x + 2 \geq 0$$
$$x \geq -2$$

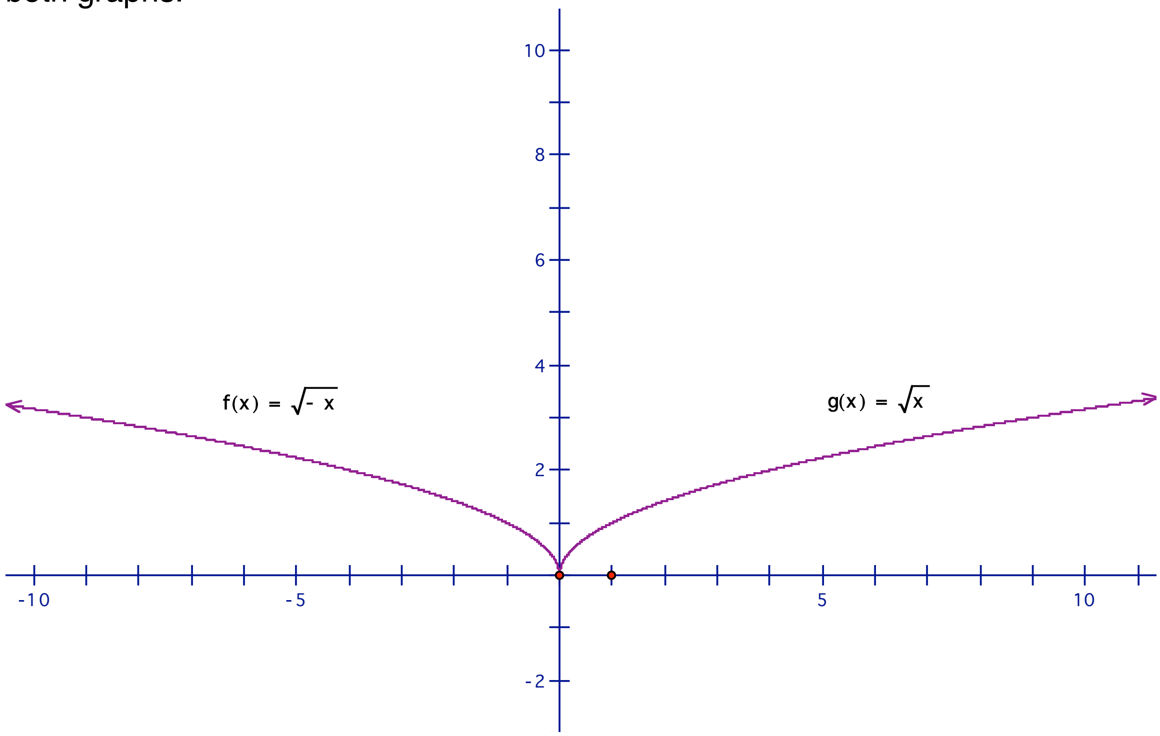
In the case of $f(x) = \sqrt{-x}$, an algebraic approach for finding the domain is to set $-x \geq 0$:

$$-x \geq 0$$
$$x \leq 0$$

In other words, the function $f(x) = \sqrt{-x}$ does exist and its domain is $x \leq 0$ (see graph in Focus 2).

Mathematical Focus 2

Use a transformation of the graph of the known function, $g(x)=\sqrt{x}$, in order to generate a graph of a less familiar function, $f(x)=\sqrt{-x}$. If the graph of $g(x)=\sqrt{x}$ is reflected about the vertical axis, the result is the graph of $f(x)=\sqrt{-x}$ as is shown in the following figure. It is important to recognize that the point (0, 0) is on both graphs.



Mathematical Focus 3

Verify that the function $f(x)=\sqrt{-x}$ makes sense by testing a few specific negative values and a few specific positive values for *x*. It might help to choose numbers whose absolute values are perfect squares, such as these shown on the following chart:

x	$\sqrt{-x}$
-4	$\sqrt{-(-4)}=2$
4	$\sqrt{-4}=2i$ (not a real number)
-1	$\sqrt{-(-1)}=1$
1	$\sqrt{-1}=i$ (not a real number)
0	$\sqrt{-0}=0$

The results for *x*-values –4, 4, -1, 1, and 0 suggest the function's domain contains all non-positive numbers.