MAC-CPTM Situations Project

Situation 41: Square Roots

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Prompt

A teacher asked her students to sketch the graph of $f(x) = \sqrt{-x}$. A student responded, "That's impossible! You can't take the square root of a negative number!"

Commentary

This situation addresses several key concepts that occur frequently in school mathematics: opposites, negative numbers, domains and ranges of functions. These concepts are represented symbolically, graphically and numerically. Mathematical focus 1 contrasts the terms "opposite" and "negative" and highlights multiple interpretations of a single symbolic representation. Mathematical focus 2 examines the implications of restricting the domain and range of a function to the set of real numbers. Mathematical focus 3 uses a transformation of the graph of $g(x) = \sqrt{x}$ to generate the graph of $f(x) = \sqrt{-x}$.

Mathematical Foci

Mathematical Focus 1

In Algebra, - x is a notation that represents the opposite of x

Mathematical terms have precise meanings. The symbol "-" is commonly read as both negative and opposite. However, a negative number is a *kind* of number, while the opposite of a number describes the *relationship* of one number to another. For example, negative 6 (-6) indicates a number < 0, and the number opposite of positive 6 (+6) indicates the additive inverse of +6, which is negative 6 (-6). Using the variable x to represent a number does not indicate the kind of number (e.g. positive, negative, zero). In this way, –x represents the opposite or additive inverse of x, which could be positive, negative, or zero.

Mathematical Focus 2

The domain of the function $f(x)=\sqrt{-x}$ is critical in determining where the function is defined and in determining a particular value of the function if it exists.

Often in school curricula, it is assumed that the domain and range of a function are restricted to real numbers. This implicit assumption could contribute to the statement "You can't take the square root of a negative number." If the range and domain of $f(x)=\sqrt{-x}$ are restricted such that $f(x),\ x\in R,$ the function is defined only for $x\leq 0$. If the range and domain of $f(x)=\sqrt{x+2}$ are restricted such that $f(x), x\in R,$ the function is defined only for $x\leq -2$. If the domain includes all real numbers and the range is not restricted, the function is defined within the set of complex numbers.

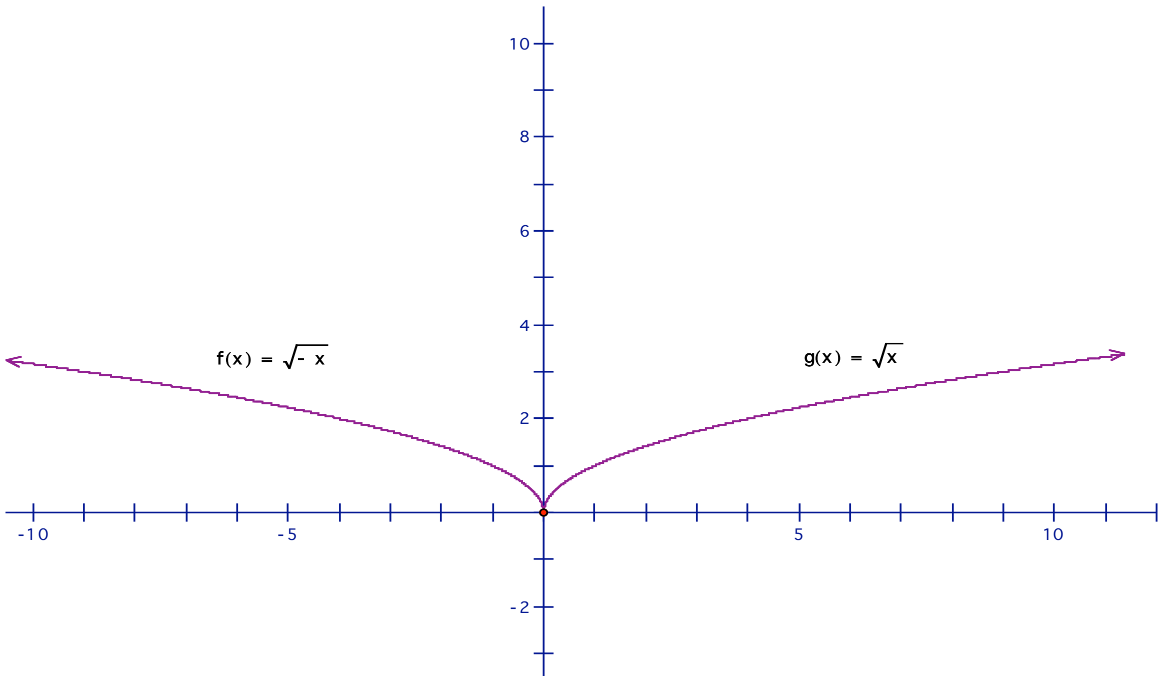
A numerical approach highlights the importance of the restrictions on *x* in determining where $f(x)=\sqrt{-x}$ is defined

x	$\sqrt{-x}$
-4	$\sqrt{-(-4)}=2$
-3	$\sqrt{-(-3)}=\sqrt{3}$
-2	$\sqrt{-(-2)}=\sqrt{2}$
-1	$\sqrt{-(-1)}=1$
0	$\sqrt{-0}=0$
1	$\sqrt{-1}=i$
2	$\sqrt{-2}=i\sqrt{2}$
3	$\sqrt{-3}=i\sqrt{3}$
4	$\sqrt{-4}=2i$

The results show that $f(x)=\sqrt{-x}$ exists. If $x\leq 0$, the radicand ≥ 0 and $f(x)$ is defined over the real numbers. If $x>0$, the radicand < 0 and $f(x)$ is an imaginary number.

Mathematical Focus 3

The graph of the real-valued function $f(x) = \sqrt{-x}$ *is a reflection of the graph of the real-valued function* $g(x) = \sqrt{x}$ *about the vertical axis.*



The graph illustrates that $f(x) = \sqrt{-x}$ does exist, and that its domain is $x \le 0$. The range of $f(x)$ is the same as that of $g(x)$. That is, $f(x) \ge 0$.