

MAC-CPTM Situations Project

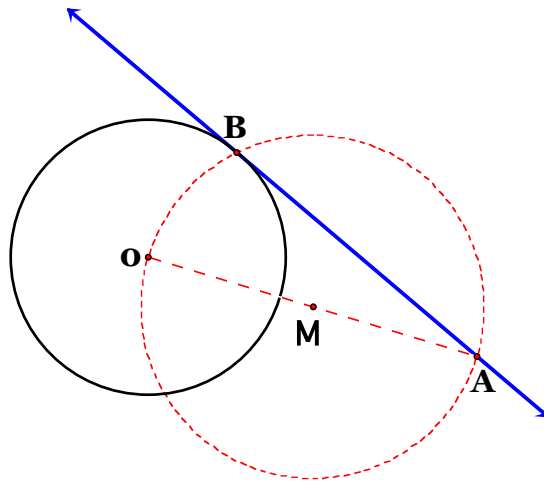
Situation 53 (PN9): Constructing a Tangent Line

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Prompt

A student in a Geometry class was shown how to construct a tangent to a circle from a Point A outside the circle.



1. First, construct the segment OA and find its midpoint M.
2. Next, using M as the center of a new circle, construct a circle of radius MA.
3. Label one point of intersection between the two circles as B. Construct the line AB that is also a tangent line to Circle O.

After seeing this, a student asked how we know that this is in fact the tangent line or how do we know that $\angle OBA$ is a right angle?

Commentary

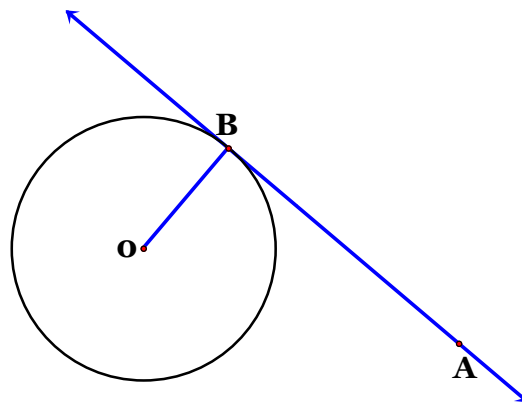
Essentially, this question is asking why the chosen method of construction works. In addressing this question, it may be useful to review the definition of tangency as well as why the radius of a circle is perpendicular to the line of tangency. To prove that the line of tangency and its corresponding radius form a right angle, the inscribed angle theorem can be used as well as the properties of isosceles triangles.

Mathematical Foci

Mathematical Focus 1

The radius of the circle is perpendicular to the line of tangency.

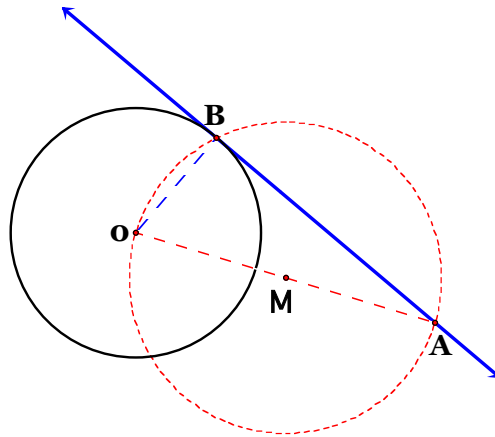
A line is **tangent** to a circle if it intersects the circle in exactly one point. Because the shortest distance between two points is a straight line, the radius of the circle marks the shortest distance between this point of intersection and the center of the circle. This implies that the radius is perpendicular to the line of tangency. Therefore, in the figure below, if line BA is tangent to Circle O, then the radius OB intersects the line BA at a 90 degree angle and angle OBA is a right angle.



Mathematical Focus 2

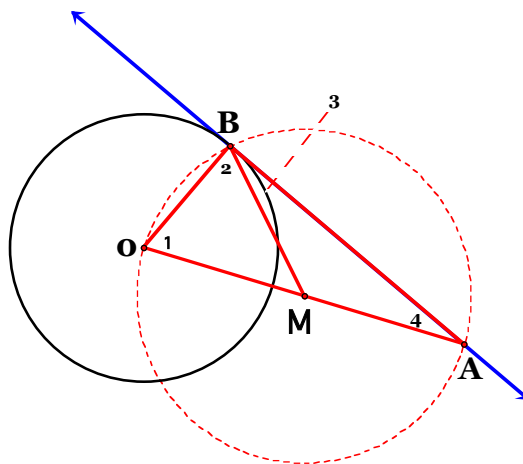
If an inscribed angle in a circle spans the diameter of that circle, that angle will be a right angle. Conversely, given a right angle, any two points O and A, each on one of the two different rays of the angle, are the endpoints of a diameter of a circle that passes through the vertex of the right angle.

The inscribed angle theorem states that the measure of an angle inscribed on a circle is one-half the measure of the arc it intercepts. As a result, if the intercepted arc is a semicircle, the measure of the inscribed angle would be 90 degrees. A line tangent to the circle passes through the vertex of such an angle. In the figure below, angle OBA is inscribed on Circle M and intercepts an arc marked off by the endpoints, O and A, of the diameter of the circle. Consequently, angle OBA measures 90 degrees and line BA is tangent to circle O.



Mathematical Focus 3

The radii of the constructed circle form two isosceles triangles.



Since we know that segments MB, MO and MA are radii of Circle M, we know that they are congruent. This indicates that triangles OBM and MBA are isosceles triangles. Because we know these triangles are isosceles, we can state that their base angles are congruent. Using the notation in the diagram, we can state that...

$$\angle 1 \cong \angle 2 \quad \text{and} \quad \angle 3 \cong \angle 4.$$

In addition, we know that triangles OBM and MBA comprise the larger triangle OBA. Since the sum of the interior angles of a triangle is 180 degrees, we can state that

$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 180^\circ.$$

Using substitution, we can conclude that

$$2m\angle 2 + 2m\angle 3 = 180^\circ$$

therefore,

$$m\angle 2 + m\angle 3 = 90^\circ.$$

Using the Angle Addition Postulate, we can state that angles OBM (angle 2) and MBA (angle 3) comprise angle OBA; hence, angle OBA is a right angle because it measures 90 degrees.