Situation: Area of an Intersection of two Circles
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Center for Proficiency in Teaching Mathematics
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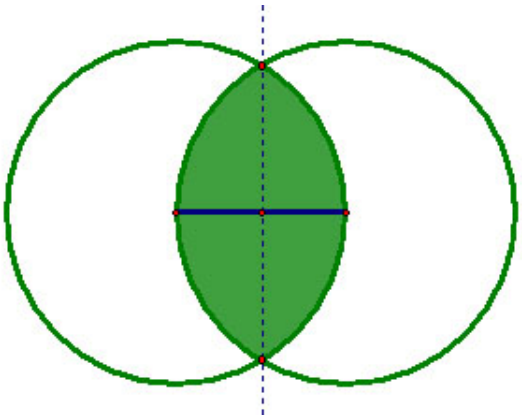
Prompt

The week before this happened, students in an Informal Geometry class worked on finding the area of compound figures - things such as a rectangle with a semi-circle at one end. The class then moved to some geometric constructions. While working on finding the midpoint of a segment, one of the students remarked:

> "This looks like a perfect Venn Diagram. What is the area of that in the middle?"

The student then shaded in the middle region as such:



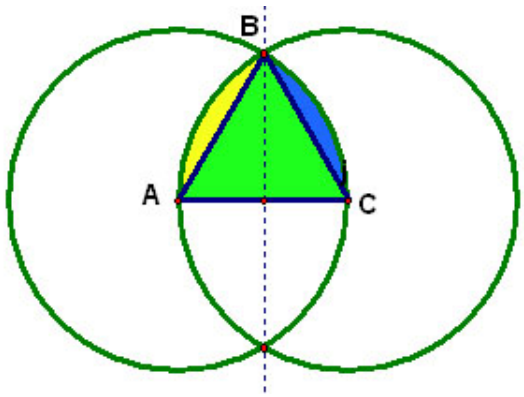
Commentary

To talk about area, we need for our original segment to have a measurable length. For the sake of the problem, let's say the original segment is 10 inches long.

Mathematical Focus 1: Additive Properties of Area

The area of the shaded region can be thought of as the sum of the blue, green, and yellow areas. This additive property of area means that if we

find the individual areas of those sections we can find the total area of the whole figure.



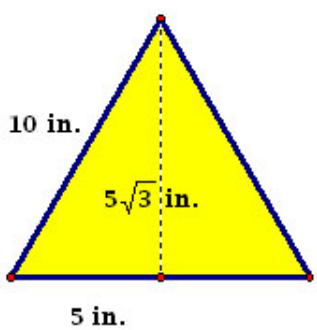
Area of a Sector of a Circle

By construction, the sector of the circle represented by the green and yellow sections seen above has a measure of 60°. Thus if the radius of the circle is 10 inches long, the area of the sector is:

$$\frac{100\pi}{6} = \frac{50\pi}{3}$$

Area of an Equilateral Triangle

The area of the equilateral triangle, represented by the yellow area above, can be found using the Pythagorean Theorem.



The area of it is $5 \cdot 5\sqrt{3} = 25\sqrt{3}$. Therefore, the area of just the green section is the difference between the sector and the equilateral triangle, or:

$$\frac{50\pi}{3} - 25\sqrt{3} = \frac{25(2\pi - 3\sqrt{3})}{3}$$

Symmetry

Due to symmetry across our original line, the area of the whole green area we originally needed is:

$$4 \cdot \frac{25(2\pi - 3\sqrt{3})}{3} + 2 \cdot 25\sqrt{3} =$$

$$\frac{200\pi - 150\sqrt{3}}{3} \text{ in}^2$$