Situation: Functional Representation
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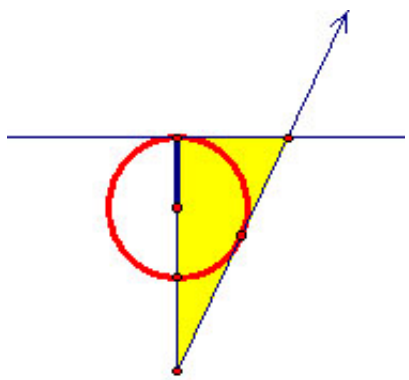
1 Prompt

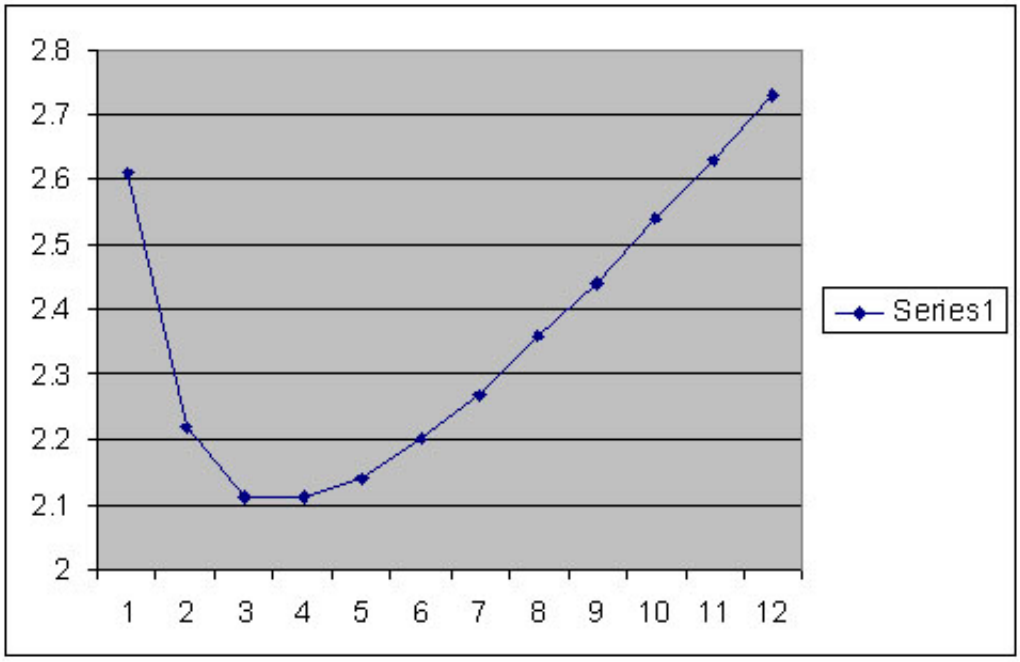
An Algebra II class which is working on the concept of function examines a geometric construction. A circle with a fixed radius of 0.90 cm is tangent to a horizontal line. Construct a ray that's perpendicular to the horizontal line going through the center of the circle. There is a point on that ray which is outside the circle. Draw a line tangent to the circle which forms a triangle.



When examining the area of the triangle as a function of the distance between the point on the ray and the circle, the data yields the following:

Distance From Circle	0.26	0.5	0.76	1.0	1.26	1.5
Area of Triangle	2.61	2.22	2.11	2.11	2.14	2.2

Distance From Circle	1.74	2.00	2.25	2.5	2.74	3.00
Area of Triangle	2.27	2.36	2.44	2.54	2.63	2.73



What function $f(x)$ can we use to fit our data?

2 Commentary

The function's most unusual trait is its asymmetry. Students can use GSP to find more points and make their line smoother.

Mathematical Focus 1: Asymptotes

There are three types of asymptotes which exist in rational functions:

Horizontal Asymptotes: For example, $y = \dfrac{3x+5}{x-2}$ has a horizontal asymptote at $y = 3$.

Vertical Asymptotes: Here, $y = \dfrac{1}{x-2}$ has a vertical asymptotes at $x = 2$.

Slant Asymptotes: The function $y = \dfrac{x^2 - x - 2}{x+2}$ has a slant asymptote at $y = x - 3$

After the dip at around what looks like 3.5 on the graph, the function seems to become linear. Thus, we have evidence of a possible slant asymptote. To figure out what it will be, we need to find the equation of the line. It also makes sense that the graph has a vertical asymptote at $x = 0$, since the area of the triangle approaches infinity the closer the point gets to the circle.

Mathematical Focus 2: Equation of a Line

Since the Slant Asymptote is essentially a line, we can find its equation by taking two of the points on the ray. For example, we can take (3, 2.73) and (2.74, 2.63) and find the slope:

$$\frac{2.73 - 2.63}{3 - 2.74} = \frac{0.1}{0.26} = 0.38$$

And then the equation of the line using the point (3, 2.73):

$$y = 0.38x + b$$

$$2.73 = 0.38 \cdot 3 + b$$

$$b = 1.59$$

So the equation of the Slant Asymptote seems to be $y = 0.38x + 1.59$. What does this mean? That means that the quotient of the numerator of $f(x)$ (our original function) with its denominator is $y = 0.38x + 1.59$f